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Journal of Economic Theory 169 (2017) 707–716

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Redistribution by means of lotteries ☆

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Received 12 May 2016; final version received 28 March 2017; accepted 3 April 2017
Available online 6 April 2017

Abstract

A government designs anonymous income transfers between a continuum of citizens whose income valuation is privately known. When transfers are deterministic, the incentive constraints imply equal treatment independently of the government's taste for redistribution. We study whether random transfers may locally improve upon the egalitarian outcome. A suitable Taylor expansion offers an approximation of the utility function by a quasilinear function. The methodology developed by Myerson to deal with incentive constraints then yields a necessary and sufficient condition for the existence of a socially useful randomization. When this condition is met a large set of lotteries are locally improving. A special menu made of two lotteries only is of interest: all the agents with low risk aversion receive the same random transfer, financed by a deterministic tax paid by the high risk aversion agents.

JEL classification: H21; H23; H26

Keywords: Lerner egalitarianism; Random redistribution; Incentives

☆ We benefited from the comments of Andrew Chesher, Philippe Jehiel, David Martimort, Jean-Marc Tallon and seminar participants in Cairo. Funding is acknowledged from the European Research Council (reference ERC-2010-AdG-269440-WSCWTBDS).

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http://dx.doi.org/10.1016/j.jet.2017.04.002

1. Introduction

It is known that in a second-best world a principal may find it valuable to propose random contracts to the agents. For instance, in the presence of asymmetric information, risk can be used to relax the incentive constraints (Laffont and Martimort, 2002). In Gauthier and Laroque (2014) we give a necessary and sufficient condition for useful/useless randomization near a deterministic optimum, but our previous analysis only applies to well-behaved problems where the constraints are qualified, i.e., the gradients of the binding constraints at the optimum are linearly independent. In this note we deal with a case where the constraints are not qualified. We consider a government that allocates a given sum of money deterministically between potential recipients with different income valuations. As observed by Lerner (1944) only equal sharing can be implemented if valuations are not observed by the government and recipients always prefer more income to less. The constraint set reduces to a single point and qualification is not met.

In this setup a random allocation may allow the government to screen individuals according to their attitudes toward risk. Pestieau et al. (2002) provide a necessary condition for local randomized redistribution to improve upon the equal sharing outcome. However their proof uses a Taylor expansion where the variance of the lotteries is negligible and so it does not give tools to design the optimal differential risk exposure. One contribution of our note is to provide a class of expansions where the noise component is first-order non-negligible. An appealing feature of this approach is that it enables us to apply the standard quasilinear toolkit of contract theory developed by Myerson (1982). This yields a necessary and sufficient condition for the existence of locally improving stochastic allocations. The social weights put on the agents with the smallest risk aversions must be large enough that a transfer in their favour more than compensates for the extra randomness required to meet the incentive constraints.

There are many ways to design the locally improving randomizations. Still it turns out that there is no loss in generality in limiting the attention to simple schemes that work as follows. The agents have to choose between two (small) deviations from the status quo. One is a certain tax, the other is a random transfer with positive expectation and variance. The deviations are built so that all the agents with a risk aversion larger than a threshold choose to pay the certain tax, while the agents with a smaller risk aversion take the other (risky) option.

The paper is organized as follows. Section 2 lays down the deterministic framework, which is extended to a random environment in Section 3. Section 4 presents an example. Finally Section 5 states and proves the main propositions.

2. Deterministic redistribution

A government allocates a total fixed income $\overline{y}$ to agents who differ in their utilities for income. There is a continuum of agents with total unit mass. The utility of a type θ agent is $u(y,\theta)$ when her income is y, y in $\mathbb{R}^+$. The parameter θ is distributed on a closed bounded interval Θ, with a positive continuous probability density function $f(\cdot)$ and cumulative distribution function $F(\cdot)$. Utility is twice continuously differentiable, increasing and concave in income: $u'_y(y,\theta) > 0$ and $u''_{yy}(y,\theta) < 0$ for all y and θ.

Each agent is supposed to know her own type. In the first-best the government also knows this type and allocates $y^*(\theta)$ to agent θ. Denoting $a(\theta)$ the social weight of type θ, the menu $(y^*(\theta))$ maximizes

$$\int_{\Theta} a(\theta)u(y(\theta),\theta)\,\mathrm{d}F(\theta) \tag{1}$$

subject to the feasibility constraint

$$\int_{\Theta} y(\theta)\,\mathrm{d}F(\theta) \le \overline{y}. \tag{2}$$

The first-best optimum $(y^*(\theta))$ is characterized by (2) at equality and the first-order conditions $a(\theta)u'_y(y^*(\theta),\theta) = a(\hat\theta)u'_y(y^*(\hat\theta),\hat\theta)$ for all θ and $\hat\theta$ in Θ. The tastes for redistribution reflected in $(a(\theta))$ and heterogeneity in utility typically lead to an unequal division of income at the first-best.

As observed by Lerner (1944) the implementation of the first-best solution crucially requires that the government observes agents' types. Otherwise the optimal income distribution maximizes (1) subject to (2) and the incentive constraints

$$u(y(\theta),\theta) \ge u(y(\hat\theta),\theta) \text{ for all } \theta \text{ and } \hat\theta.$$

Since utility is increasing in income, these constraints are satisfied if and only if $y(\theta) = y(\hat\theta)$ for all θ and $\hat\theta$. Feasibility then gives

$$y(\theta) = \overline{y} \text{ for all } \theta. \tag{3}$$

In this Lerner (deterministic second-best) optimum, income is equalized independently of the tastes for redistribution.

3. Random redistribution

We extend the power of the government and allow for randomized redistribution. The government can now design a menu of lotteries, such that every individual must choose some lottery in this menu. Given the random draw from the lottery, there is commitment from both the government and the players to conform to the outcome. Suppose also that a law of large number holds, so that with independent draws the cost of the lottery is equal to its mathematical expectation.

A menu of lotteries $(\tilde y(\theta))$ improves upon the reference Lerner optimum if

$$\int_{\Theta} a(\theta)\mathbb{E}\big[u(\overline{y}+\tilde y(\theta),\theta)\big]\,\mathrm{d}F(\theta) > \int_{\Theta} a(\theta)u(\overline{y},\theta)\,\mathrm{d}F(\theta).$$

In addition, the menu must satisfy the feasibility constraint

$$\int_{\Theta} \mathbb{E}\big[\tilde y(\theta)\big]\,\mathrm{d}F(\theta) = 0,$$

and the incentive constraints

$$\mathbb{E}\big[u(\overline{y}+\tilde y(\theta),\theta)\big] \ge \mathbb{E}\,[u(\overline{y}+\tilde y(\hat\theta),\theta)] \text{ for all } \theta \text{ and } \hat\theta.$$

We consider small randomizations around the deterministic Lerner optimum. In the previous literature, e.g., Gjesdal (1982), Pestieau et al. (2002) or Gauthier and Laroque (2014), small randomizations obtain whenever the support of the lotteries in the menu $(\tilde y(\theta))$ is close to 0. In a Lerner setup this characterization is not well suited because the noise component then appears as a second-order negligible term that cannot be explicitly exploited by the government. Indeed, with such a characterization of small lotteries, a Taylor expansion yields

$$\mathbb{E}\big[u(\overline{y}+\tilde y(\theta),\theta)\big] = u(\overline{y},\theta) + m(\theta)u'_y(\overline{y},\theta) + o\,(\tilde y(\theta)^2),$$

where $m \equiv \mathbb{E}\big[\tilde{y}\big]$ stands for the mean of a typical lottery. If one considers first-order terms and neglects other higher order terms, the incentive constraints for type θ reduce to

$$m(\theta) \geq m(\hat{\theta}) \text{ for all } \hat{\theta}. \tag{4}$$

Reproducing the same exercise for type $\hat{\theta}$ immediately implies $m(\theta) = m(\hat{\theta})$ for all θ and $\hat{\theta}$, and by the feasibility constraint $m(\theta) = 0$ for all θ. It is unclear how introducing a nonzero variance could then improve welfare.

One contribution of our note is to provide a class of small randomizations that make the variance of the transfer a first-order term. Let $v \equiv \operatorname{var}\big[\tilde{y}\big]$ be the variance of a typical lottery. At the Lerner optimum, $\tilde{y}(\theta) = m(\theta) = 0$ and $v(\theta) = 0$ for all θ. We consider a family of menus $(\tilde{y}_\lambda(\theta))$ indexed by a real parameter λ, $\lambda \geq 0$. The lottery designed for a type θ has mean and variance $(\lambda m(\theta), \lambda v(\theta))$. Small randomizations obtain when λ is small enough. For this class of small randomizations a second-order Taylor expansion yields

$$u(\overline{y} + y, \theta) = u(\overline{y}, \theta) + u'_y(\overline{y}, \theta) y + \frac{1}{2} u''_{yy}(\overline{y}, \theta) y^2 + o(y^2),$$

for all y close to 0, which implies

$$\begin{aligned}\mathbb{E}\big[u(\overline{y} + \tilde{y}_\lambda(\theta), \theta)\big] = u(\overline{y}, \theta) + u'_y(\overline{y}, \theta)\lambda \left[m(\theta) - \frac{r(\theta)}{2}\left(\lambda m(\theta)^2 + v(\theta)\right)\right] \\ + \mathbb{E}\left[\lambda^2 o(\tilde{y}^2)\right],\end{aligned}$$

where

$$r(\theta) = -\frac{u''_{yy}(\overline{y}, \theta)}{u'_y(\overline{y}, \theta)}$$

is the coefficient of (absolute) risk aversion of a type θ individual evaluated at the Lerner outcome. When λ is close enough to 0, we have

$$\mathbb{E}\big[u(\overline{y} + \tilde{y}_\lambda(\theta), \theta)\big] \simeq u(\overline{y}, \theta) + u'_y(\overline{y}, \theta)\lambda \left[m(\theta) - \frac{r(\theta)}{2} v(\theta)\right].$$

Therefore the menu of lotteries $(\tilde{y}_\lambda(\theta))$ improves upon the deterministic Lerner optimum if and only if the change in the social objective is positive,

$$\lambda \int_\Theta \alpha(\theta) \left[m(\theta) - \frac{r(\theta)}{2} v(\theta)\right] \mathrm{d}F(\theta) > 0, \tag{5}$$

where $\alpha(\theta) = a(\theta) u'_y(\overline{y}, \theta)$ is the marginal social weight of type θ. Such a menu must also satisfy the feasibility constraint

$$\int_\Theta m(\theta)\, \mathrm{d}F(\theta) = 0, \tag{6}$$

and the incentive constraints

$$m(\theta) - \frac{r(\theta)}{2} v(\theta) \geq m(\hat{\theta}) - \frac{r(\theta)}{2} v(\hat{\theta}) \text{ for all } \theta \text{ and } \hat{\theta}. \tag{7}$$

With this class of small randomizations the variances of income transfers explicitly enter the incentive constraints (7). In the absence of noise these constraints reduce to (4) and prevent any income redistribution. Randomized income transfers expand the set of possible redistribution schemes consistent with incentive compatibility.

For small randomizations the menu of lotteries $(\tilde{y}_\lambda(\theta))$ leads to incentive constraints with the quasilinear shape used in the toolkit of contract theory developed by Myerson (1981). Usual textbook arguments (see, e.g., chapter 2 in Salanié, 2005) lead to define

$$U(r) = \sup_{\theta \in \Theta} m(\theta) - \frac{r}{2} v(\theta).$$

The function U is convex nonincreasing in r, almost everywhere differentiable with $v(\theta(r)) = -2U'(r)$, where $\theta(r)$ is the best report of an agent whose risk aversion is r. Moreover $m(\theta(r))$ is equal to $U(r) + rv(\theta(r))/2$. This construction makes it clear that the government can only screen agents according to their risk aversion. Since there is no loss of generality in working with direct menus parametrized in r, r in $\mathcal{R} = \left[r^{\inf}, r^{\sup}\right]$, in the remainder of the paper we use $(m(r), v(r))$ to describe the menus of lotteries, with a slight abuse of notation. The problem in (5), (6) and (7) can be restated as looking for a profile $(m(r), v(r))$ such that

$$\int_{\mathcal{R}} \alpha(r) \left[m(r) - \frac{r}{2} v(r) \right] \mathrm{d}G(r) > 0, \tag{8}$$

$$\int_{\mathcal{R}} m(r) \mathrm{d}G(r) = 0, \tag{9}$$

and

$$m(r) - \frac{r}{2} v(r) \geq m(\hat{r}) - \frac{r}{2} v(\hat{r}) \text{ for all } r \text{ and } \hat{r}, \tag{10}$$

where $G(r)$ stands for the proportion of agents with risk aversion less than r at the Lerner optimum, and $\alpha(r)$ is the average social weight of the agents with risk aversion r. We denote $A(r)$ the aggregate social weight of the individuals of risk aversion smaller than r,

$$A(r) = \int_{r^{\inf}}^{r} \alpha(z) \mathrm{d}G(z).$$

The weights are normalized so that $A(r^{\sup}) = 1$.

Although the set of allocations consistent with incentive compatibility is larger than what is seen in the deterministic Lerner world, incentive constraints restrict possible redistribution. These restrictions, which follow from standard arguments, are given in Lemma 1.

Lemma 1. *Consider a family of menus $(\tilde{y}_\lambda(r))$ with mean variance $(\lambda m(r), \lambda v(r))$, for $\lambda \geq 0$. Suppose that they satisfy the incentive constraints (10). For two types r and $\hat{r}$ such that $r \leq \hat{r}$, we have $m(r) \geq m(\hat{r})$ and $v(r) \geq v(\hat{r})$. The equality $v(r) = v(\hat{r})$ implies $m(r) = m(\hat{r})$. The contribution of an agent with type θ to the change in social welfare from the Lerner optimum is $\lambda\alpha(r)U(r)$, where*

$$U(r) = m(r) - \frac{r}{2} v(r)$$

is nonincreasing in r.

The incentive constraints imply that randomized income transfers can only redistribute welfare toward least risk aversion types, independently of the social tastes ($\alpha(r)$) for redistribution. Low risk aversion types receive transfers with higher means, and these transfers must be random not to attract higher risk aversion types.

From Lemma 1 we also see that the incentive constraints imply that the variance $v(r)$ is nonincreasing in r. The minimal variance therefore is $v(r^{\sup})$. A direct examination of the system (8), (9) and (10) shows that any solution $(m(r), v(r))$ with $v(r^{\sup}) > 0$ can be improved upon by keeping the function m unchanged and translating the function v by $-v(r^{\sup})$. Extra randomness, above the requirements associated with incentives, is of no use, and in the remainder of the paper we shall work with

$$v(r^{\sup}) = 0.$$

4. A simple reform

This section considers a menu made of only two choices. The first one is a lottery $\tilde{y}$ that has mathematical expectation $\underline{m}$ and variance $\underline{v}$, a positive number. The second one is a sure transfer $\overline{m}$. For the small lotteries considered in Lemma 1 the agents who prefer the lottery $\tilde{y}$ have risk aversion $r \le r^*$ where

$$\overline{m} = \underline{m} - \frac{r^*}{2}\underline{v}. \tag{11}$$

We limit our attention to cases where $G(r^*)$ is positive. Otherwise all agents choose $(\overline{m}, 0)$, and feasibility implies that $\overline{m}$ is equal to zero, so that the outcome is the Lerner optimum. From (8) the menu improves upon Lerner if

$$\int_{r^{\inf}}^{r^*} \alpha(r)\left(\underline{m} - \frac{r}{2}\underline{v}\right) \mathrm{d}G(r) + \int_{r^*}^{r^{\sup}} \alpha(r)\overline{m}\,\mathrm{d}G(r) > 0. \tag{12}$$

Using the feasibility constraint,

$$G(r^*)\underline{m} + [1 - G(r^*)]\overline{m} = 0,$$

and the definition of the threshold r^* given in (11) we get

$$\overline{m} = -G(r^*)\frac{r^*}{2}\underline{v} \quad \text{and } \underline{m} = [1 - G(r^*)]\frac{r^*}{2}\underline{v}.$$

Reintroducing the expressions of $\overline{m}$ and $\underline{m}$ into (12), the change in social welfare brought by the menu of lotteries is positive if and only if

$$\frac{1}{2}K(r^*)\underline{v} > 0,$$

where

$$K(r) = r\,[A(r) - G(r)] - \int_{r^{\inf}}^{r} z\alpha(z)\,\mathrm{d}G(z). \tag{13}$$

Since $\underline{v}$ is positive, we have:

Lemma 2. *A sufficient condition for a random redistribution to improve upon the Lerner deterministic outcome is*

$$K(r^*) > 0. \tag{14}$$

The term $A(r^*)$ represents the aggregate social gain that results from giving one unit of money to each individual whose risk aversion is less than r^*, while $G(r^*)$ is the total cost of these transfers (the size of the population is normalized to 1). Hence the difference between $A(r^*)$ and $G(r^*)$ is positive when individuals with risk aversion smaller than r^* have higher social values than the others. It is clear from (13) and (14) that this pattern of social weight makes useful randomization more likely.

The last sum in $K(r^*)$ depends on the shape of the left tail of the risk aversion distribution. In the special case where all the individuals with a risk aversion less than r^* are risk neutral (there is a positive mass of agents at $r^{\inf} = 0$ and no agent has $0 < r < r^*$), this sum is 0, so that randomization is useful whenever the social weight of the risk neutral agents is larger than 1.

5. Improving randomizations

We consider now general menus $(m(r), v(r))$ and provide necessary and sufficient conditions for a local improvement.

Proposition 1. *Consider a profile of small lotteries associated with the mean-variance menu $(m(r), v(r))$, satisfying the feasibility and incentive compatibility requirements. If the function K is continuously differentiable, the profile induces a change in social welfare upon the Lerner optimum equal to*

$$\frac{1}{2}\int_{\mathcal{R}} K'(r)v(r)\,\mathrm{d}r, \tag{15}$$

where $K'(r) = A(r) - G(r) - rg(r)$ is the first derivative of $K(r)$.

Proof. Appealing to the powerful tool developed by Myerson (1981) one can check that the profiles $(U(r))$ satisfying incentive constraints are such that

$$U(r) = U(r^{\inf}) - \int_{r^{\inf}}^{r} \frac{v(z)}{2}\,\mathrm{d}z, \tag{16}$$

where $U(r^{\inf})$ is derived from feasibility requirements,

$$U(r^{\inf}) = \frac{1}{2}\int_{\mathcal{R}} \big[[1 - G(r)]\,v(r) - rv(r)g(r)\big]\,\mathrm{d}r. \tag{17}$$

Thus the change in social welfare is

$$\int_{\mathcal{R}} \alpha(r)\left[U(r^{\inf}) - \int_{r^{\inf}}^{r} \frac{v(z)}{2}\,\mathrm{d}z\right]\mathrm{d}G(r). \tag{18}$$

Integration by parts gives

$$\int_{\mathcal{R}} \alpha(r) \int_{r^{\inf}}^{r} v(z)\, \mathrm{d}z\, \mathrm{d}G(r) = \int_{\mathcal{R}} v(r) \int_{r}^{r^{\sup}} \alpha(z)\, \mathrm{d}G(z)\, \mathrm{d}r = \int_{\mathcal{R}} [1 - A(r)]\, v(r)\, \mathrm{d}r,$$

where the last step uses the normalization made on social weights (their sum over the whole population is 1). From this relation and (17), the inequality (18) is equivalent to (15). □

A pure utilitarian government, with $\alpha(r)$ constant equal to 1, has $A(r)$ equal to $G(r)$, so that $K(r)$ is nonpositive. It has no possibility to improve welfare with lotteries. More generally, a high social value put on those who have least risk aversion is needed to get a social improvement upon the deterministic Lerner outcome: there is no useful (local) randomization when $A(r) \leq G(r)$ for all r. This confirms the insights obtained from the sufficient condition given in Lemma 2.

Proposition 1 can be used to test whether a given menu satisfying (9) and (10) yields a welfare improvement. It is however difficult to exploit immediately (15) to discuss the shape of improving menus since incentive constraints put restrictions on the variance profile. By Lemma 1 the variance $v(r)$ must decrease with risk aversion r, and so it only has at most countably many discontinuities. From now on, following the literature since Guesnerie and Laffont (1984), we limit our attention to variance profiles which have at most a finite number of discontinuities.[1] To make these restrictions explicit in (15), we let $\mathcal{D}$ stand for the set of risk aversions where a discontinuity in the variance profile occurs. Accounting for the fact that the discontinuities have a zero contribution to the sum, (15) becomes

$$\frac{1}{2} \int_{\mathcal{R} \setminus \mathcal{D}} K'(r) v(r) \mathrm{d}r. \tag{19}$$

For $r_i \in \mathcal{D}$, let

$$v(r_i^-) = \lim_{\substack{r \to r_i \\ r < r_i}} v(r) \quad \text{and } v(r_i^+) = \lim_{\substack{r \to r_i \\ r > r_i}} v(r).$$

Integration by parts on the typical interval $\left[r_i^+, r_{i+1}^-\right]$ yields

$$\int_{r_i^+}^{r_{i+1}^-} K'(r) v(r) \mathrm{d}r = [K(r) v(r)]_{r_i^+}^{r_{i+1}^-} - \int_{r_i^+}^{r_{i+1}^-} K(r) v'(r) \mathrm{d}r.$$

Using $K(r^{\inf}) = 0$ and $v(r^{\sup}) = 0$ allows us to rewrite (19) as

$$\frac{1}{2} \left[\sum_{\mathcal{D}} K(r_i) \left[v(r_i^-) - v(r_i^+) \right] - \int_{\mathcal{R} \setminus \mathcal{D}} K(r) v'(r) \mathrm{d}r \right]. \tag{20}$$

This yields another formulation of (15).

[1] Guesnerie and Laffont (1984) seem to be the first to point out the need for such a condition, see their footnotes 3 and 6. This restriction seems to be mild, and to the best of our knowledge there is no example in the literature where it binds. It is widely used, see e.g. Laffont and Martimort (2002), page 135.

Proposition 2. *Consider a profile of small lotteries associated with the mean-variance menu $(m(r), v(r))$ satisfying the feasibility and incentive compatibility constraints where the variance $v(r)$ has a finite number of discontinuities. Let $\mathcal{D} = \{r_i \mid i \in I\}$ be the set of risk aversion r where a discontinuity in $v(r)$ occurs. A lottery profile $(m(r), v(r))$ induces a change in social welfare upon the Lerner optimum given by (20).*

The condition (20) can be easily interpreted using a marginal argument. Let r^* be a point where K is positive, if any. At this point we are free to change $-v'$ provided that it stays positive. This allows to introduce a discontinuity on v. Starting from $r^{\sup}$ we have v unchanged on $[r^*, r^{\sup}]$ and increased by Δ for r smaller than r^*. This transformation translates into a change of welfare of

$$-\frac{\Delta}{2}\int_{r^{\inf}}^{r} z\alpha(z)\,\mathrm{d}G(z).$$

This is the last term in the expression of K, scaled by the factor $\Delta/2$. This first step is incomplete, since the modified menu is not incentive compatible. In particular the utility of type r^* has decreased by $r^*\Delta/2$ while the utilities of all types greater than r^* are kept at their initial values. We have to make type r^* indifferent between what she gets and what her immediate neighbour $r^* + \varepsilon$ gets, which by construction is unchanged from the initial condition. This can only be done through a deterministic transfer of money which compensates her for the randomness: it should be equal to $r^*\Delta/2$. Now all the types smaller than r^* are exposed to the same change in randomness than r^*, and so each one must also be given $r^*\Delta/2$ to keep incentive compatibility with type r^*. Summing up these transfers, we get that they cost

$$G(r^*)r^*\frac{\Delta}{2}$$

from the budget constraint, while they increase social welfare by

$$A(r^*)r^*\frac{\Delta}{2}.$$

The normalization $A(r^{\sup}) = 1$ implies a marginal cost of public funds equal to 1. Thus, all things considered, the change in social welfare is equal to $K(r^*)\Delta/2$: the change is improving provided $K(r^*)$ is positive.

From Proposition 2, a natural candidate for an improving menu involves $v'(r) < 0$ on the set $K(r) > 0$ and $v'(r) = 0$ otherwise. But this is by no means the only possible choice: there is a large family of possibly complicated menus that locally improve upon the Lerner outcome when $K(r) > 0$ for some r. Indeed, by Lemma 1, any nonpositive v' would fit on the set $K(r) > 0$. In addition, the overall inequality (20) can be satisfied with v' negative small in absolute value on the set $K(r) \le 0$. However it is immediate from (20) that one can always consider reforms analyzed in Section 4, where v has a single point of discontinuity at a point r^* where $K(r^*) > 0$. For such reforms one can set $v'(r) = 0$ for every $r \neq r^*$, so that the change in social welfare (20) reduces to

$$\frac{1}{2}K(r^*)v(r^*).$$

The agents then separate into two subpopulations, with risk aversions below or above r. The former all are assigned a lottery, while the later pay a nonrandom tax.

Remark 1. The observation that there is no loss to consider the class of simple reforms analyzed in Section 4 can be used to assess the effectiveness of the randomization. The tax supported by the risk averse agents is $\mid \overline{m} \mid = G(r^*)r^*\underline{v}$. In the case where the government only cares about the individuals with least risk aversion, the gain in social welfare is $K(r^*)\,\underline{v}/2$. The effectiveness of the reform, measured as the ratio of the gain in welfare per unit of taxation is equal to $(1/G(r^*) - 1)$, which is infinite for $G(r^*)$ close enough to zero.[2] Such reforms thus yield a large welfare gain compared to the size of the perturbation.

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[2] We thank the Associate Editor for suggesting this computation.