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Testing unilateral and bilateral link formation

Margherita Comola

Marcel Fafchamps

Testing Unilateral and Bilateral Link Formation[*]

Margherita Comola [†] Marcel Fafchamps[‡]

Paris School of Economics Oxford University

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Abstract

We propose a test of whether self-reported network data is best seen as an actual link or willingness to link and, in the latter case, whether this link is generated by an unilateral or bilateral link formation process. We illustrate this test using survey answers to a risk-sharing question in an African village. We find that bilateral link formation fits the data better than unilateral link formation, but the data are best interpreted as willingness to link rather than an actual link. We then expand the model to include self-censoring and find it to fit the data significantly better than willingness to link. This suggests that, in our data, the data generating process behind self-reported links is a hybrid between an actual link and willingness to link.

JEL codes: C12; C52; D85

Keywords: pairwise stability; self-reported link; self-censoring; risk sharing

1. Introduction

It is increasingly recognized that many important economic phenomena, such as trade, information diffusion, and learning, take place within social networks (e.g. Granovetter 1985, Jackson 2008) and that the architecture of these networks can affect the efficiency and equity of the resulting allocation (Vega-Redondo 2006). We also now know that the mechanism through which links are created has a profound influence on the equilibrium architecture of purposely formed networks. In particular, Bala and Goyal (2000) have shown that unilateral and bilateral link formation result in fundamentally different network structures – see also Goyal (2007). The consent rule (unilateral versus bilateral) within a group may also shape the aggregate outcome, as Charness and Jackson (2007) have shown in an experimental set-up.

Bilateral link formation refers to situations in which the consent of both nodes is needed for a link to be formed; it is a natural assumption for voluntary exchange. Unilateral link formation arises whenever one node can form a link without the express consent of the other; it is a natural assumption for information access networks, e.g., the Internet. It may also arise in market exchange when legal or social norms make it unlawful for one party to refuse to trade.[1]

The contribution of this paper is primarily methodological. The econometric analysis of social networks is still novel, and there often is a lack of clarity on the implicit assumptions necessary to estimate network models. The ultimate aim of this paper is to shed some light on the way self-reported network data should be interpreted, and how discordant responses should be treated. We find that in our case some models fit the data better than others. Other data may yield different conclusions.

We propose a simple methodology for testing whether self-reported network data reflect a

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[1] In many developed countries anti-discrimination laws typically make it unlawful for a retailer to refuse to sell to a specific customer.

simple willingness to link or an existing link and, in the latter case, whether this link is generated by an unilateral or bilateral link formation process. Building on the work of Comola (2007), we take pairwise stability as starting point for the estimation process. First introduced by Jackson and Wolinsky (1996), pairwise stability has established itself as a cornerstone equilibrium condition in the study of bilateral link formation processes (Goyal 2007). Using pairwise stability as starting point, Comola (2007) uses a bivariate probit model to estimate a bilateral link formation model. We extend this approach by noting that, under unilateral link formation, the absence of a link is formally equivalent to a pairwise stable decision by *both* nodes *not* to form a link.

We illustrate our methodology using data on self-reported mutual insurance links. This is a natural choice given that many empirical studies of social networks have relied on self-reported data and that mutual insurance networks have received much attention in the literature (e.g. Scott 1976, Altonji, Hayashi and Kotlikoff 1992, Coate and Ravallion 1993, Townsend 1994, Fafchamps and Lund 2003). Every household in an African village was asked to give a list of households on whom they rely – or who rely on them – for help in cash, kind or labor. The question is intended to capture a link between two households i and j and should in principle be answered in the same way by both, irrespective of whether the assistance is one-sided or reciprocated. In practice, it is frequent that household i mentions j but j does not mention i.

This is open to multiple interpretations. One possibility is that respondents gave the names of households from whom they *wish* to seek assistance, not necessarily those who would provide it, should the need arise. In this case, answers are best understood as representing willingness to link, not an actual link. Another interpretation is that respondents provided information on actual links but their answers differ because of misreporting. It is unclear a priori which of these two alternatives is a better representation of the data.

To test between the two, we use the fact that actual links should satisfy equilibrium conditions; willingness to link need not. Two types of equilibrium conditions are considered, depending on whether link formation is bilateral or unilateral. It seem natural to expect mutual insurance links to require the agreement of both parties – and this is indeed how the economic literature has modeled informal risk sharing (e.g. Coate and Ravallion 1993, Kocherlakota 1996). In this case, link formation is bilateral and pairwise stability is a necessary condition for the network to be in equilibrium.[2]

It is also conceivable that social norms make it impossible for villagers to refuse assisting others. For instance, a son may not be able to refuse helping his father. Platteau (1996) argues that many agrarian societies, especially in sub-Saharan Africa, cultivate egalitarian norms, a point that has repeatedly been made by anthropologists and by casual observers alike.[3] In the presence of sharing norms, the link formation process is basically unilateral. In this case, a transformed version of the pairwise stability condition must hold in the sense that *both* nodes must agree *not* to form a link.

Comola (2007) has shown that, under the normality assumption, the restrictions imposed by pairwise stability take the form of a bilateral probit model with partial observability (Poirier 1980). In contrast, if answers only represent willingness to link, the relevant regression model is a simple probit. Building on this insight, we test whether willingness to link, bilateral, or unilateral link formation is most consistent with the responses given by surveyed households. This is achieved using the non-nested likelihood ratio test first proposed by Vuong (1989), that we adapt to correct for network dependence across residuals. When we peg the bilateral and unilateral models against each other, we find that the bilateral link formation model wins.

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[2]Pairwise stability does not fully characterise equilibrium in the village network; it is a local condition. One could therefore argue that our test is not efficient since it fails to impose all the structure of a village network equilibrium. The difficulty is that deriving global equilibria would require additional assumptions about the information each household has about all payoffs, etc, something we are reluctant to do.

[3]Barr and Stein (2008) provide some recent evidence to this effect.

However, both are outperformed by a simple willingness-to-link model.

We then relax the willingness-to-link model to allow for self-censoring. For instance, a respondent i may refrain from reporting his wish to seek help from j if he anticipates rejection. Alternatively, i may report links with individuals who he cannot refuse to help even though he prefers not to. Both cases involve self-censoring – of willingness to link in the first case, and of unwillingness to link in the second. We show that self-censoring can be represented as a bilateral probit model without cross-equation coefficient restrictions.

We find evidence of self-censoring in our data. This is important because self-censoring of reported willingness to link has long plagued the study of link formation (e.g. Hitsch, Hortacsu and Ariely 2005, Belot and Francesconi n.d., Fisman, Iyengar, Kamenica and Simonson 2008). In their study of internet dating, Hitsch et al. (2005) for instance note that the emails participants send to each other to initiate interaction may not reflect their true willingness to link if they refrain from making openings they know will be rejected. Belot and Francesconi (n.d.) make similar observations in their study of internet dating. Self-censoring is also present in matching processes in which participants can only list a limited number of preferred links – e.g., the University Centralised Application System (UCAS) in the UK: students can only list 5 universities of their choice, and hence do not list universities most likely to reject them. The methodology proposed here offers a way of testing the presence of self-censoring.

The paper is organized as follows. In Section 2 we provide a conceptual framework and describe our estimating and testing strategy. The data are described in Section 3. Estimation results are discussed in Section 4. Section 5 concludes.

2. Conceptual framework and testing strategy

In this section we begin by presenting the different estimation strategies used in the paper. As in Comola (2007) the starting point of our estimation strategy is pairwise stability as defined by Jackson and Wolinsky (1996). We then discuss the important issue of how to draw consistent inference by correcting standard errors for non-independent data. We conclude the section with a discussion of non-nested hypothesis testing with non-independent data.

Formally, for each pair of nodes ("dyad") ij, define $g_{ij}^i = 1$ if i reported a link with j, and 0 otherwise. Similarly define $g_{ji}^j = 1$ if j reported a link with i. Variables g_{ij}^i and g_{ji}^j provide a representation of the data. Their interpretation varies depending on what the data generation process is assumed to be. In subsection (2.1) we consider these data as an indication of willingness to link and we specify the corresponding data generation process. In subsections (2.2) and (2.3) we regard g_{ij}^i and g_{ji}^j as two different measurements of the same actual link g_{ij}. Subsection (2.2) specifies the data generation process if the link formation process is bilateral while subsection (2.3) focuses on the unilateral case.

2.1. Willingness to link

Here the response variables g_{ij}^i and g_{ji}^j are interpreted as the willingness of nodes i and j respectively to form the link g_{ij}. Formally, let the network be denoted by the symmetric adjacency matrix $g = [\, g_{ij}]$ with $g_{ij} = 1$ if the link ij exists and $g_{ij} = 0$ otherwise. Given the wording of the survey question, we cannot distinguish between help that is mutual and one-sided assistance that is given or received by the respondent. We therefore define a link g_{ij} to exist whenever i and j help each other, whether help is one-sided or mutual. It follows that $g_{ij} = g_{ji}$ by construction.

The utility that node i derives from network g is written $U_i(g)$. By a standard abuse of notation, let g_{-ij} denote the network g without the link g_{ij}, that is, with $g_{ij} = 0$. Similarly,

let g_{+ij} denote the network with the link g_{ij}, that is, with $g_{ij} = 1$. The gain to household i of forming the link g_{ij} is $U_i(g_{+ij}) - U_i(g_{-ij})$. We assume that this gain is a linear function of observables X_{ij} and a zero-mean residual ε_{ij}:

$$U_i(g_{+ij}) - U_i(g_{-ij}) = X'_{ij}\beta - \varepsilon_{ij} \tag{2.1}$$

$$U_j(g_{+ji}) - U_j(g_{-ji}) = X'_{ji}\beta - \varepsilon_{ji} \tag{2.2}$$

This the key maintained assumption on which the testing strategy rests.

Since the order in which i and j appear in the data is arbitrary, they must be interchangeable. This implies that the coefficient vector β must be the same in equations (2.1) and (2.2). Assuming that $(\varepsilon_{ij}, \varepsilon_{ji})$ are jointly normal, it follows that equations (2.1) and (2.2) can be estimated as a standard probit by stacking observations g^i_{ij} and g^j_{ji}:

$$\Pr(g^i_{ij} = 1) = \Pr\left(U_i(g_{+ij}) \geq U_i(g_{-ij})\right) = \Pr\left(\varepsilon_{ij} \leq X'_{ij}\beta\right)$$

$$\Pr(g^j_{ji} = 1) = \Pr\left(U_j(g_{+ji}) \geq U_j(g_{-ji})\right) = \Pr\left(\varepsilon_{ji} \leq X'_{ji}\beta\right) \tag{2.3}$$

2.2. Bilateral link formation

Let us now interpret g^i_{ij} and g^j_{ji} as two separate measurements of the same actual link g_{ij}. This implies that discrepancies in survey answers g^i_{ij} and g^j_{ji} must be due to misreporting. Since we have no reason to believe one answer more than the other, we give each measurements equal weight.

In order to specify the data generation process, we impose the partial equilibrium structure implied by pairwise stability. We first consider the bilateral link formation case. As in Comola

(2007) the starting point of our estimation strategy is pairwise stability as defined by Jackson and Wolinsky (1996). Pairwise stability imposes that the agreement of both nodes is needed for a link to be formed, and all profitable nodes are formed in equilibrium. This occurs if and only if:

$$\forall g_{ij} \;=\; 1, \; U_i(g_{+ij}) \geq U_i(g_{-ij}) \text{ and } U_j(g_{+ij}) \geq U_j(g_{-ij})$$

$$\forall g_{ij} \;=\; 0, \text{ if } U_i(g_{-ij}) < U_i(g_{+ij}) \text{ then } U_j(g_{-ij}) > U_j(g_{+ij})$$

This set of conditions implies that:

$$\Pr(g_{ij} = 1) = \Pr\left(U_i(g_{+ij}) \geq U_i(g_{-ij}) \text{ and } U_j(g_{+ij}) \geq U_j(g_{-ij})\right) \tag{2.4}$$

Using (2.1) and (2.2) equation (2.4) is equivalent to:

$$\Pr(g_{ij} = 1) = \Pr\left(\varepsilon_{ij} \leq X'_{ij}\beta \text{ and } \varepsilon_{ji} \leq X'_{ji}\beta\right) \tag{2.5}$$

where $(\varepsilon_{ij}, \varepsilon_{ji})$ are jointly normal.

Model (2.5) has a single dependent variable but two regressing equations. Such model, first proposed by Poirier (1980) and later on used by Comola (2007) to model network formation, is known as a partial observability bivariate probit. This is because the link g_{ij} can be understood as the product of two distinct and unobservable events, i's willingness to form the link ij and j's willingness to form the same link. Let us define these unobservable variables w^i_{ij} and w^j_{ji} such that $w^i_{ij} = 1$ if $\varepsilon_{ij} \leq X'_{ij}\beta$ and similarly for w^j_{ji}. Under pairwise stability, a link is formed only if both i and j are willing to form it, i.e., $g_{ij} = 1$ iff $w^i_{ij} = 1$ and $w^j_{ji} = 1$ or, more succinctly, iff $w^i_{ij}w^j_{ji} = 1$. The term 'partial observability' comes from the fact that we only observe the

product $w_{ij}^i w_{ji}^j$, not each of them separately. That is, whenever a link $g_{ij} = 0$ we can not observe whether one or both nodes are not willing to form it.

In practice, we have two measurements g_{ij}^i and g_{ji}^j of g_{ij}. The estimated model is thus:

$$
\begin{aligned}
\Pr(g_{ij}^i \;\; &= \;\; 1) = \Pr\left(\varepsilon_{ij} \leq X_{ij}'\beta \text{ and } \varepsilon_{ji} \leq X_{ji}'\beta\right) \\
\Pr(g_{ji}^j \;\; &= \;\; 1) = \Pr\left(\varepsilon_{ji} \leq X_{ji}'\beta \text{ and } \varepsilon_{ij} \leq X_{ij}'\beta\right)
\end{aligned}
\tag{2.6}
$$

Estimating β under the assumption of bilateral link formation thus boils down to maximizing the likelihood function implicitly defined by (2.6).

2.3. Unilateral link formation

An undirected network may also result from a process of unilateral link formation. This corresponds to the situation in which only one side's consent is sufficient for a link to be formed. Put differently, a link does not exist only if both nodes refuse to create it (Goyal 2007). As in the bilateral case, we let w_{ij}^i and w_{ji}^j represent the nodes' unobserved willingness to form link g_{ij}. Under unilateral link formation, $g_{ij} = 1$ whenever either of the two nodes wishes to form a link. It follows that $g_{ij} = 0$ only when both links do not wish to form the link. This simple observation forms the basis of our estimation strategy because it implies that, using a change of variable, the unilateral link formation model can also be estimated as a partial observability model.

To see how this is possible, we begin by noting that:

$$
\begin{aligned}
\Pr(g_{ij} = 0) \;\; &= \;\; \Pr\left(U_i(g_{+ij}) < U_i(g_{-ij}) \text{ and } U_j(g_{+ij}) < U_j(g_{-ij})\right) \\
&= \;\; \Pr\left(\varepsilon_{ij} > X_{ij}'\beta \text{ and } \varepsilon_{ji} > X_{ji}'\beta\right)
\end{aligned}
\tag{2.7}
$$

Let $h_{ij} \equiv 1 - g_{ij}$. We have $h_{ij} = 1$ iff $w_{ij}^i = 0$ and $w_{ji}^j = 0$ or, more succinctly, iff $(1 - w_{ij}^i)(1 - w_{ji}^j) = 1$. Estimation can proceed by applying a partial observability bivariate probit to the transformed system:

$$\Pr(h_{ij} = 1) = \Pr\left(-\varepsilon_{ij} \leq -X'_{ij}\beta \text{ and } -\varepsilon_{ji} \leq -X'_{ji}\beta\right) \qquad (2.8)$$

The dependent variable is still binary, and the partial observability feature ensures that the absence of a link $(h_{ij} = 1)$ is interpreted as implying that both nodes do not wish to form that link. As is clear from (2.8), estimated coefficients have the reverse sign compared to (2.5). This is because we are estimating individuals' willingness *not* to form a link.

Once again, we have two measurements h_{ij}^i and h_{ji}^j of h_{ij}. The estimated model is thus:

$$
\begin{aligned}
\Pr(h_{ij}^i &= 1) = \Pr\left(-\varepsilon_{ij} \leq -X'_{ij}\beta \text{ and } -\varepsilon_{ji} \leq -X'_{ji}\beta\right) \\
\Pr(h_{ji}^j &= 1) = \Pr\left(-\varepsilon_{ji} \leq -X'_{ji}\beta \text{ and } -\varepsilon_{ij} \leq -X'_{ij}\beta\right)
\end{aligned}
\qquad (2.9)
$$

Estimating β under the assumption of unilateral link formation thus boils down to maximizing the likelihood function implicitly defined by (2.9).

2.4. Standard errors

Dyadic data can seldom if ever be regarded as made of independent observations; residuals are typically correlated across some observations. This does not invalidate estimation itself: as long as regressors remain uncorrelated with residuals, coefficients can be estimated consistently. But uncorrected standard errors are inconsistent, invalidating inference.

Methods have been proposed to correct standard errors in non-independent data. These methods extend White's formula for robust standard errors to correlation across observations

(Conley 1999). For dyadic data, the most pressing concern is the correlation in the residual for observation g_{ij} with those pertaining to all observations involving nodes i and j. This is because i's decision to form a link with j potentially affects his or her decision to form a link with any other node. Fafchamps and Gubert (2007) propose a correction of standard errors that takes care of this form of cross-observation dependence of the form:

$$AVar(\widehat{\beta}) = \frac{1}{N-K}(X'X)^{-1}\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N}\frac{m_{ijkl}}{2N}X_{ij}u_{ij}u'_{kl}X_{kl}\right)(X'X)^{-1} \qquad (2.10)$$

where β denotes the vector of coefficients, N is the number of dyadic observations, K is the number of regressors, X is the matrix of all regressors, X_{ij} is the vector of regressors for dyadic observation ij, and $m_{ijkl} = 1$ if $i = k, j = l, i = l$ or $j = k$, and 0 otherwise. Formula (2.10) was developed for linear regressions where u_{ij} denotes the residual from observation ij. To apply it to maximum likelihood estimation, simply replace u_{ij} by the corresponding score l_{ij}. The only structure imposed on the covariance structure is that $E[u_{ij}, u_{ik}] \neq 0$, $E[u_{ij}, u_{kj}] \neq 0$, $E[u_{ij}, u_{jk}] \neq 0$ and $E[u_{ij}, u_{ki}] \neq 0$ for all k but that $E[u_{ij}, u_{km}] = 0$ otherwise. The standard errors reported in this paper are all based on formula (2.10).

It is conceivable that $E[u_{ij}, u_{km}] \neq 0$ for $i \neq k, m$ and $j \neq k, m$. This would arise, for instance, if i's willingness to form a link with j depends on whether k has a link with m. In this case, formula (2.10) is no longer sufficient to correct standard errors and more cross-terms should be added. Whether this is feasible depends on the data. If the researcher has observations from unlinked sub-populations (e.g., multiple villages), it is possible to allow for arbitrary cross-observation dependence by clustering standard errors at the level of each sub-population (e.g. Arcand and Fafchamps 2008, Barr, Dekker and Fafchamps 2008). In our data, we only have a single village so this option is not available. Bester, Conley and Hansen (2008) has suggested an approach to approximately eliminate bias in standard errors by dividing the data into large

blocks and clustering within blocks. Unfortunately this approach requires a large sample, which again is not our case.

2.5. Non-nested tests

Our aim is to test which one of models (2.3), (2.6) or (2.9) best accounts for the data. To this effect we proceed by pairwise comparisons. Vuong (1989) has proposed a framework for hypothesis testing in non-nested models. Say we want to test which of two alternative, non-nested models k and m fit the data best. Let $M = N(N-1)$ be the total number of dyadic observations. The original form of the Vuong test statistic is

$$V = \frac{M^{-1/2} LR(k,m)}{\hat{\omega}} \xrightarrow{d} N(0,1)$$

where $LR\,(k,m) \equiv L^k - L^m$ is the log of the likelihood ratio statistic and:

$$\hat{\omega}^2 = \frac{1}{M} \sum_{ij=1}^{M} \left[\log \frac{l_{ij}^k}{l_{ij}^m} \right]^2 - \left[\frac{1}{M} \sum_{ij=1}^{M} \log \frac{l_{ij}^k}{l_{ij}^m} \right]^2$$

where l_{ij}^k and l_{ij}^m are the observation-specific scores for each model k and m. This test can be implemented more simply by regressing the difference between scores on a constant:[4]

$$l_{ij}^k - l_{ij}^m = \alpha_{km} + v_{ij}^{km}$$

The t-value on the constant α_{km} is the Vuong statistic that tests whether model k outperforms model m. For inference to be valid, we correct the standard error of the constant $\hat{\alpha}_{km}$ for cross-dependence across observations using formula (2.10).

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[4]The Vuong test requires that the models have the same dependent variable. This condition is satisfied by construction for models (2.3) and (2.6). In spite of the change of variable from g_{ij}^i to $h_{ij}^i = 1 - g_{ij}^i$, it is also satisfied for model (2.9) because the scores are the same except for a sign change, which we correct for.

Essential for our identification strategy is that both nodes have provided a separate statement g_{ij}^i and g_{ji}^j regarding their mutual link g_{ij}. When testing unilateral versus bilateral link formation, identification is achieved from the symmetry between g_{ij}^i and g_{ji}^j. In the bilateral case, it is unlikely to observe a link when one of the nodes strongly wishes not to link. In the unilateral case, it is unlikely not to observe a link when one of the nodes strongly wishes to link. It is this difference between the two models that makes identification possible in test of bilateral versus unilateral link formation.

When we test either of the link formation models (2.6) or (2.9) against the willingness to link model (2.3), identification is achieved from the implicit symmetry assumption that, if responses g_{ij}^i and g_{ji}^j are two measurements of the same actual link g_{ij}, then both nodes i and j should be equally likely to report it. In contrast, if responses correspond to willingness to link, there may be systematic differences between the responses made by i and j. Systematic difference in responses would arise, for instance, if some features (e.g., popularity) make some nodes more attractive to others. If, for instance, j is more popular than i, then i may be willing to link with j while the reverse is not true. In this case, i would be reporting a willingness to link with j while j does not report a willingness to link with i. But if g_{ij}^i and g_{ji}^j correspond to statements about an actual link, then j would report a link with i even if j is not keen on the link – but cannot refuse it.

In the last section of the paper, we abandon this symmetry assumption and introduce a hybrid model that shares features from both (2.3) and (2.6) – or (2.9). The same methodology is then used to test this hybrid model against (2.3), (2.6), or (2.9). In this case, identification is achieved from the fact that (2.3), (2.6), and (2.9) are more restricted than the hybrid model. If the restriction is binding, then the hybrid model should dominate; if the restriction is not binding, then the hybrid model should not be able to fit the data better than the more restricted model.

3. The data

To illustrate our estimation and testing strategy we use survey data from a village community named Nyakatoke in the Buboka Rural District of Tanzania, at the west of Lake Victoria. The village is mainly dependent on farming of bananas, sweet potatoes and cassava for food, while coffee is the main cash crop. The community is composed by 600 inhabitants, 307 of which are adults, for a total of 119 households interviewed in five regular intervals during 2000. This dataset is ideal for our purpose because it is a census covering all 119 households in the village.[5] The data include information on households' demographics (composition, age, religion, education), wealth and assets (land and livestock ownership, quality of housing and durable goods), income sources and income shocks, transfers and network relations.

Each adult respondent was asked: *"Can you give a list of people from inside or outside of Nyakatoke, who you can personally rely on for help and/or that can rely on you for help in cash, kind or labor?"*. Aggregated at the level of each household, the responses to this question constitute variables g_{ij}^i and g_{ji}^j. In other words, $g_{ij}^i = 1$ if an adult member of household i mentions an adult member of household j in their response to the above question. Nyakatoke data have been analyzed by De Weerdt and Dercon (2006) and De Weerdt and Fafchamps (2007). These authors have shown that reported mutual insurance links g_{ij}^i and g_{ji}^j are strong predictors of subsequent loans and gifts, and that linked households give and receive much more from each other in times of illness.

Given the cultural context, it is not obvious how to interpret Nyakatoke villagers' responses to the risk sharing link question. One possible interpretation is that responses represent the respondent's desire to establish a link. This interpretation is particularly appealing when the responses are discordant, that is, when $g_{ij}^i \neq g_{ji}^j$. It is nevertheless possible that discordant

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[5] Everyone in the village agreed to participate in the survey, but there are some missing data for 4 households.

responses as due to measurement error and that the data describe, albeit with some error, actual links between villagers.[6]

The process by which links between villagers are formed can be bilateral or unilateral. Much of the economic literature on informal risk sharing in developing countries has assumed that households willingly enter in such arrangements (e.g. Kimball 1988, Coate and Ravallion 1993). Applied to social networks, this approach implicitly assumes that mutual insurance links follow a bilateral process. In contrast, much of the anthropological literature has emphasized the difficulty for individuals to abstract themselves from the moral and social obligation to assist others in need (e.g. Scott 1976, Platteau 1996). This point has been made by a number of economists as well, notably those studying remittance flows (e.g. Lucas and Stark 1985, Azam and Gubert 2006). Anderson and Baland (2002) provide evidence that individuals living in Kenyan slums put money in rotating savings and credit associations (ROSCAs) to avoid claims on their resources by spouse and relatives. Ambec (1998) and Banerjee and Mullainathan (2007) take these observations as starting point to model the saving behavior of poor households. This line of reasoning implies an unilateral mechanism of link formation. Testing these alternative data generation processes is the objective of this paper.

Because our dataset is small, we are limited in the number of regressors we can credibly include in the analysis. The covariates that appear in the regressions should be seen as illustrative of the type of variables one may want to include in an analysis of this kind. What matters most for our purpose is whether conclusions regarding bilateral or unilateral link formation are robust

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[6]Independently of whether the underlying network follows a bilateral or unilateral link formation process, it is necessary to decide how to treat discordant responses in the estimation itself. If respondents forget to mention some of their risk-sharing partners because they are involved in too many links to recall them all, we should treat any discordant pair as an existing link, i.e, as $g_{ij} = 1$. Doing so implicitly assumes that the main form of measurement error is omission, i.e., that respondents do not mention someone as a risk sharing partner unless the expectation of reciprocity is strong. Alternatively, discordant responses may arise because one of the two respondents mistakenly reported a link where none exists, i.e., discordant cases correspond to $g_{ij} = 0$. Without information on individual intent, we cannot disentangle the two.

to alternative choices of regressors. If we include too few regressors, the alternative models we wish to test will not account for much of the variation in the data, and we will not be able to tell them apart. Ultimately, all we want is a list of regressors that enables us to robustly test the models against each other. Since we are not interested in ascribing a causal interpretation to any of the regressors, all regressors should be viewed as controls proxying for their own effect plus any other correlated effect.

In this section we present our preferred list of regressors. At the end of the paper we discuss whether our results vary with alternative regressors. The covariates X_{ij} used in the regression analysis fall into three categories: variables that reflect the attractiveness of the potential partner j; variables proxying for homophyly, that is, the desire to link with similar households; and variables controlling for i's need to link.

Two regressors capture attractiveness. The first one, O_{ij}, is the overlap in productive activities between i and j. It is calculated as:

$$O_{ij} = \sum_{a=1}^{7} L_{ai} L_{aj}$$

where L_{ai} is the share of total time spent by adult members of household i in activity a.[7] Each L_{ai} is constructed using information collected on time use in seven broad income generating categories. Households whose productive activities overlap are expected to have more correlated incomes. Since less correlated incomes generate more opportunities for risk pooling, households with less overlap in activities with household i are in principle more attractive risk sharing

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[7] In the survey each adult individual mentions the productive activities he or she is involved into. These activities are divided in seven categories: casual labor, trade, crops, livestock rearing, assets, processing of agricultural products, and other off-farm work. Individuals can report multiple activities but are not asked about the relative importance of each activity. We have therefore no alternative but to assign equal weight to all listed activities. L_{ai} is calculated as follows. Say household i has n members, m of which report working full time in a and k report a and one other activity. Then $L_{ai} = \frac{1}{n}(m + \frac{k}{2})$. Individuals who do not report any involvement in an income generating activity are omitted from the calculation. Five households in the sample report no active member.

partners (. Fafchamps and Gubert 2007, De Weerdt and Fafchamps 2007) We therefore expect O_{ij} to have a negative sign.

We also control for the in-degree P_j^i of j, omitting any link between i and j to avoid spurious correlation. We think of P_j^i as a proxy for various unobservable characteristics – e.g., sociability, generosity, moral sense – that make j an attractive partner for many villagers. It is reasonable to assume that, other things being equal, all households in our sample would prefer to be linked to popular households. Of course, popular households may not wish to link to everyone, since this would mean assisting the entire village.[8] They may therefore be unwilling to link with unpopular households, a feature that is captured by pairwise stability and the bilateral link formation model.

A second set of regressors seeks to control for homophyly, that is, the desire to link with similar or proximate households. The literature has shown that social ties depend to a large extent on social and geographical proximity (e.g. Fafchamps and Gubert 2007, De Weerdt and Fafchamps 2007). To control for geographical proximity, we introduce a dummy that takes value one if i and j are neighbors, that is, live less than 100 meters apart.[9] Blood ties are controlled for using a kinship dummy that takes value one if i and j – or members of their household – are related.[10] Constructing this variable is particularly demanding in terms of data collection, a strong point of the Nyakatoke dataset. We also include a religion dummy taking the value of one if i and j have the same religion.[11]

To capture similarity in social status, we include as regressor the absolute difference in total

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[8] For a formalization of this idea, see for instance Vandenbossche and Demuynck (2009) 's model of risk sharing network formation. See Ellsworth (1989) for a detailed description of mutual assistance flows in a Burkinabe village, and of the role played by one 'holy man' as center of a village-wide redistribution network. There is no such central person in our village, however.

[9] Slight variation in the cutoff distance does not affect our main results.

[10] This includes parents/children, siblings, cousins, uncle/aunt/niece/nephew, grand-parents/grand-children, and other blood ties.

[11] Catholic, or Protestant, or Muslim – 41%, 39% and 20% of the village population respectively.

wealth (computed as the sum of land and livestock) $|w_j - w_i|$ between i and j.[12] If i prefers to link with someone of similar wealth, the coefficient of $|w_j - w_i|$ should be negative. To avoid spurious results, we borrow from Fafchamps and Gubert (2007) and include the sum of wealth $(w_i + w_j)$ to control for the possibility that wealthier individuals have, on average, more links.

The third set of regressors includes factors likely to make household i more interested in forming links. Some respondents report more links than others. This may be because they are pro-social or anti-social. To control for i's proclivity for forming – or reporting – mutual insurance links with others, we include i's out-degree as regressor, omitting any link with j. Wealthy households are less in need of mutual insurance. To capture this possibility, we include a dummy which is equal to one if household i in top 25% wealth percentile in the village. For similar reasons, we also include the number of adult members of household i. As De Weerdt and Fafchamps (2007) show, informal transfers in Nyakatoke respond to health shocks. Since they pool labor resources, larger households should find it easier to deal with health shocks than smaller ones – and hence are less in need of forming mutual insurance links with other villagers (Binswanger and McIntire 1987).

Descriptive statistics are reported Table 1. The first and second panels of the table present dichotomous and continuous variables, respectively. In the dataset there are 119 households, which make 119*118=14042 dyads in total. We see from the table that the proportion of pairs for which g_{ij}^i or $g_{ji}^j = 1$ is 7%. The proportion of discordant responses is large. Around one third of household pairs share the same religion. Wealth and the other continuous regressors display a healthy amount of variation in the data. Some regressors were rescaled to facilitate estimation.[13]

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[12]Data on land was collected in acres, but transformed in monetary equivalent using a conversion rate of 300000 tzs for 1 acre. This reflects the average local price in 2000, the time at which the data were collected.

[13]To minimize convergence problems that arise when using bivariate probit with partial observability.

Table 1: Descriptive statistics (n=14042)

g_{ij}^i, g_{ij}^j	$g_{ij}^i = g_{ij}^j = 1$			280
	$g_{ij}^i \neq g_{ij}^j$			700
	$g_{ij}^i = g_{ij}^j = 0$			13062
Neighbors	distance ij <100 m			4%
Same family	ij have blood ties			6%
Same religion	ij same religion			35%
Rich respondent	i in top 25th %			25%

continuous variables	mean	min	max	sd
Overlap in productive activities O_i	0.22	0	1	0.162
In-degree of household j (/10) (**)	0.52	0	2.30	0.445
$\|w_j - w_i\|^{(*)}$	0.44	0	2.80	0.524
$w_i + w_j^{(*)}$	0.91	0	5.56	0.678
Out-degree of household i (/10) (**)	0.52	0	1.90	0.304
No. adult members in household i (/10)	0.26	0.1	0.90	0.131

(*) 1 unit corresponds to 1 million Tanzanian Shillings.

(**) excluding the ij link

4. Empirical results

4.1. Model estimation

We now estimate models (2.3), (2.6) and (2.9). Each model includes the list of X_{ij} regressors presented in Table 1. For each set of results the z-values reported in the last column are based on dyadic standard errors corrected using formula (2.10).

We begin by reporting the estimation results obtained when we assume that responses to the

risk sharing question capture willingness to link, as explained in subsection (2.1). Coefficients estimates are reported in Table 2 using probit. They suggest that respondents prefer to link with popular households who live nearby, are related, and share a similar level of wealth. The coefficient of $w_i + w_j$ is positive and marginally significant, suggesting that willingness to link is higher among wealthy households. Other regressors are not significant.

Table 2: Willingness to link

Regressor	coefficient	dyadic z
Overlap in activities O_{ij}	-0.194	-0.85
Popularity P_j^i	0.508	7.71***
Neighbor dummy	0.760	5.17***
Blood ties dummy	0.987	5.86***
Same religion dummy	0.169	1.31
$|w_j - w_i|$	-0.250	-2.35**
$w_i + w_j$	0.249	1.74*
Out-degree of i	0.287	1.65*
Rich dummy of i	-0.004	-0.04
Nber adult members of i	0.105	0.26
Intercept	-2.659	-15.99***

We then turn to the bilateral link formation model (2.6).[14] Results are presented in Table 3. Several coefficient estimates are similar to those reported in Table 2. Popularity P_j^i remains strongly significant. Coefficient estimates are again suggestive of homophyly. Overlap in activi-

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[14]We also estimated the model with alternative assumptions about misreporting, e.g., assuming that discordant pairs are due to over-reporting only (i.e., $g_{ij} = g_{ij}^i g_{ji}^j$) or under-reporting only (i.e., $g_{ij} = 1$ whenever g_{ij}^i or $g_{ji}^j = 1$). These versions yield parameter estimates that are by and large comparable to those reported in Table 3. But the Vuong test cannot be used to compare these alternative versions to the willingness to link model (2.3) because they use different dependent variables.

ties O_{ij} is now marginally significant with the anticipated sign, suggesting a desire to link with individuals who have a different income profile.

Table 3: Bilateral link formation

Regressor	coefficient	dyadic z
Overlap in activities O_{ij}	-0.065	-1.92*
Popularity P_j^i	0.136	2.52**
Neighbor dummy	0.213	3.20***
Blood ties dummy	0.316	3.95***
Same religion dummy	0.042	1.95*
$\lvert w_j - w_i \rvert$	-0.057	-1.81*
$w_i + w_j$	0.051	1.33
Out-degree of i	0.037	1.05
Rich dummy of i	0.021	0.71
Nber adult members of i	0.213	2.30**
Intercept	-0.271	-1.74*
arc $\tan(\rho)$	-1.894	-3.59***

Next we present the results assuming that the data were generated by the unilateral link formation model (2.9). As explained in subsection (2.3), we transform household responses g_{ij}^i and g_{ij}^j into the equation-level dependent variables $h_{ij}^i \equiv 1 - g_{ij}^i$ and $h_{ij}^j \equiv 1 - g_{ij}^j$. Results are reported in Table 4. To facilitate comparison with Table 3, we report estimated coefficients $\widehat{\beta}$ directly, which means inverting the sign of the coefficient estimates obtained from estimating (2.9) with partial observability bivariate probit. In terms of coefficient estimates, results are similar to those reported in Table 3. Popularity P_j^i and activity overlap O_{ij} are both significant with the anticipated sign. Homophyly variables are all strongly significant while i's characteristics

are not.

Table 4: Unilateral link formation

Regressor	coefficient	dyadic z
Overlap in activities O_{ij}	-0.213	-2.04**
Popularity P_j^i	0.412	7.83***
Neighbor dummy	0.706	9.88***
Blood ties dummy	0.928	9.54***
Same religion dummy	0.155	2.83***
$|w_j - w_i|$	-0.198	-3.69***
$w_i + w_j$	0.171	1.80*
Out-degree of i	0.161	0.97
Rich dummy of i	0.107	1.17
Nber adult members of i	0.564	1.50
Intercept	-2.862	24.64***
arc $\tan(\rho)$	0.628	3.47***

4.2. Specification tests

We now turn to the main object of the paper, which is to compare the performance of the different models in accounting for the data. As explained in Section 2, we proceed by pairwise comparisons, adapting the non-nested Vuong test to the dyadic structure of the data. To compare two models k and m we calculate, for each observation ij, the log-likelihood contributions (or score) under the two models and we regress the difference $l_{ij}^k - l_{ij}^m$ on a constant, correcting the standard errors using formula (2.10). The t-value of the constant is the Vuong test corrected for dyadic non-independence. Since the distribution of the Vuong test is asymptotically normal, the relevant critical value for a 5% level of significance is 1.96. Note that the test works in two

directions: if $t > 1.96$ model k is to be preferred to model m; if $t < -1.96$ model m is to be preferred to model k. For values of t between -1.96 and 1.96 the test is inconclusive – both models fit the data equally.

Table 5 reports the result of the pairwise comparisons between the willingness-to-link model and the other two. When the bilateral and unilateral models are compared to each other, the bilateral model is found superior. But the Table unambiguously shows that the willingness-to-link model fits the data best.

Table 5: Vuong tests

Model k	Model m	Vuong test	Best fit
bilateral	unilateral	2.28**	bilateral
willingness to link	bilateral	2.34**	willingness to link
willingness to link	unilateral	3.34***	willingness to link

4.3. Self-censoring

Our results imply that responses given to the mutual insurance question are best understood as indicative of willingness to link than evidence of an actual link. Yet De Weerdt and Fafchamps (2007) have shown that g_{ij}^i is a strong predictor of gifts and transfers reported in subsequent survey rounds. Fafchamps and Gubert (2007) report similar findings with data collected in the Philippines using a similarly worded question.[15] Responses to the mutual insurance question may be more than just willingness to link.

In particular, we suspect that respondents did not report households with whom they would like to share risk but who are likely to turn them down. Self-censoring has been discussed in the economic literature on dating. In that literature, the researcher typically has access to information on willingness to date – e.g., answers to a direct question following speed dating

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[15]In fact the Philippines question was used as template for the Tanzania survey.

interviews (e.g. Belot and Francesconi n.d., Fisman et al. 2008), or emails sent to prospective partners on an internet dating site (Hitsch et al. 2005). In both cases, the authors worry that respondents may fail to list or contact desirable partners who are unlikely to accept them.[16]

A similar kind of self-censoring may also be at work in our data. In particular, household i may have liked to share risk with household j but expected j to refuse, and so failed to mention j as possible mutual insurance link. This corresponds to an alternative data generating process in which j can veto a link that i wants.

Such data generating process can be represented as follows. As before, g_{ij}^i is i's report of whether a link to j exists. This report is now thought of as made of two parts: (1) i's willingness to link with j, which we denote w_{ij}; and (2) i's expectation of whether the link would be accepted by j, which we denote e_{ij}. Expectation e_{ij} is thought of as made of two intermingled parts: j's willingness to link with i and j's inability to refuse a link with i even though j does not want to link with i. We observe $g_{ij}^i = 1$ if both $w_{ij} = 1$ and $e_{ij} = 1$. We observe $g_{ij}^i = 0$ if either $w_{ij} = 0$ or $e_{ij} = 0$ or both.

To illustrate what we have in mind, imagine that unpopular households wish to link to popular households ($w_{ij} = 1$) but popular households never wish to link with unpopular households ($w_{ji} = 0$). Yet popular households cannot refuse to help some of the unpopular ones, e.g., members of their church. In that case, unpopular household i will report $g_{ij}^i = 1$ with popular household j whenever i expects that j will not refuse to help ($e_{ij} = 1$) because of social norms or altruism. Formally we have:

$$\Pr(g_{ij}^i = 1) = \Pr(w_{ij} = 1 \text{ and } e_{ij} = 1) \tag{4.1}$$

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[16]Self-censoring has also been discussed in the context of matching models in which individuals can only rank a subset of their possible choices (e.g., schools or jobs). In such models, it is optimal for low ranked individuals not to 'waste' limited slots on options they are unlikely to get.

with

$$\Pr(w_{ij} \ = \ 1) = \beta x_{ij}$$

$$\Pr(e_{ij} \ = \ 1) = \gamma x_{ji}$$

Model (4.1) can be estimated using bivariate probit with partial observability. The only difference with model (2.5) is that we no longer impose that coefficients be the same in the two equations. Instead, we now estimate different coefficients β and γ for the two equations. As before, the estimator allows for non-independence between $\Pr(w_{ij} = 1)$ and $\Pr(e_{ij} = 1)$ (for instance because of unobserved individual effects common to both). Model (4.1), which we call the 'vetoed link' model, can be seen as a refined version of willingness to link which incorporates expectations about the potential partner's likely behavior.

Estimation results for the vetoed link model are presented in Table 6. Coefficient estimates for the w_{ij} equation have the same interpretation as before. Coefficient estimates for the e_{ij} equation capture two kinds of effects: j's willingness to link with i, and j's capacity to veto a link with i. Bilateral link formation model (2.6) is a restricted form of (4.1) with $\beta = \gamma$, which is equivalent to assuming that i's statement about the existence of a link with j only depends on j's willingness to link. Put differently, in model (2.6), i's statement regarding a link with j internalizes both i's and j's willingness to link. The symmetry of (2.6) is equivalent to setting $\beta = \gamma$ and implies that g_{ij}^i and g_{ji}^j have the same probability, i.e., both i and j are equally likely to report link g_{ij}. Model (4.1) allows g_{ij}^i and g_{ji}^j to have differing probabilities depending on the characteristics of i and j. The willingness to link model (2.3) corresponds to the case where $\gamma = 0$: i's answer only depends on i's desire to link with j. Model (4.1) sits between both extremes in that it allows g_{ij}^i and g_{ji}^j to have differing probabilities in a way that nevertheless

takes into account more than just own willingness to link. In model (4.1), if $\gamma < 0$ for a given regressor x_{ji}, this implies that x_{ji} is associated with a lower e_{ij} and thus a higher likelihood of 'veto' by j. A $\gamma > 0$ in contrast implies that the corresponding x_{ji} makes it harder for j to refuse to assist i.

We see that estimated coefficients in the w_{ij} equation are somewhat similar in terms of magnitude and statistical significance to those reported in earlier regressions: popularity P_{ij}^i is again strongly significant, and so are geographical proximity and a shared religion. The out-degree of i (omitting the ij link) is also statistically significant. In contrast, coefficients in the e_{ij} regression are quite different from those reported for the w_{ij} equation. Only three coefficients are statistically significant: the kinship dummy, j's out-degree, and the size of j's household. This means that kin are less likely to veto a link but the smaller j's household is and the larger j's out-degree, the more likely j will veto a link with i. This suggests that larger households have a duty to care for others, possibly because their size makes them better able to self-insure – and thus to assist others.

Table 6. Vetoed links model

	w_{ij} equation			e_{ji} equation	
Regressor	coefficient	dyadic z	Regressor	coefficient	dyadic z
Overlap in activities O_{ij}	0.281	1.50	Overlap in activities O_{ij}	-2.389	-1.23
Popularity P_j^i	0.462	7.32***	Popularity P_i^j	-0.034	-0.16
Neighbor dummy$_{ij}$	0.643	7.34***	Neighbor dummy$_{ij}$	0.454	0.29
Blood ties dummy$_{ij}$	0.963	5.73***	Blood ties dummy$_{ij}$	-0.306	-0.42
Same religion dummy$_{ij}$	0.205	2.97***	Same religion dummy$_{ij}$	-0.308	-1.26
$|w_j - w_i|$	-3.233	-1.06	$|w_j - w_i|$	7.914	0.36
$w_i + w_j$	0.271	1.09	$w_i + w_j$	-0.513	-0.60
Out-degree of i	0.259	3.59***	Out-degree of j	-0.845	-1.99**
Rich dummy of i	0.003	0.03	Rich dummy of j	0.103	0.52
Nber adult members of i	0.148	0.88	Nber adult members of j	4.672	2.25**
Intercept	-2.597	-12.85***	Intercept	2.700	2.54**
arc $\tan(\rho)$	-1.999	-4.00***			

By analogy with Section 2, it is also possible to define the 'dual' analogue of the vetoed link model. In this model, i reports his *un*willingness to link with j, except in cases when j can impose a link with i. This implies that i reports $g_{ij}^i = 1$ whenever i expects j to impose a link on i, even if i is not keen to link with j. In this model, we have:

$$\Pr(g_{ij}^i = 0) = \Pr(w_{ij} = 0 \text{ and } e_{ij} = 0)$$

with

$$\Pr(w_{ij} = 0) = -\beta x_{ij}$$

$$\Pr(e_{ij} = 0) = -\gamma x_{ji}$$

This model is the generalized equivalent of the unilateral link formation with $h_{ij} = 1 - g_{ij}$. It can be estimated in a fashion similar to (4.1), but the interpretation is slightly different. Here i reports a missing link ($g_{ij}^i = 0$) if i does not want to link *and* i expects that j cannot impose a link on i. But i reports a link whenever either i wishes to link with j *or* i expects that j can impose a link. We call this model the 'forced link' model since j can force a link that i does not want.

Regression estimates are shown in Table 7. As we did for Table 4, we report estimated coefficients $\widehat{\beta}$ and $\widehat{\gamma}$ directly, i.e., we invert their sign to facilitate comparison with Table 6. Interpretation of the coefficients of the w_{ij} equation is as before. In the case of the e_{ij} equation, $\gamma = \beta$ means that i expect j to force a link with i based purely on his/her willingness to link with i. This would arise for instance if i fully internalizes the unilateral link formation equilibrium. In contrast, if all $\gamma = 0$, g_{ij}^i is consistent with pure willingness to link. A $\gamma > 0$ means that the x_{ji} variable raises the likelihood that, in i's opinion, j's can force a link on i.

Coefficient estimates for the w_{ij} equation are fairly similar to those reported earlier in Table 2, except that $w_i + w_j$ is not marginally significant anymore and i's out-degree and i's rich dummy are now statistically significant. Coefficient estimates for the e_{ij} equation are different in sign and magnitude from those of the w_{ij} equation, a result that is consistent with our earlier finding that g_{ij}^i is not consistent with a unilateral link formation process. Several regressors have a significant coefficients in the e_{ij} equation, indicating factors that make it more (or less) likely

that j be willing and able to force a link onto i. Geographical proximity and blood ties appear with a strongly significant positive coefficient, indicating that it is difficult to deny assistance to kin and neighbors. The negative coefficient for i's popularity P_i^j indicates that the more popular i is, the less likely it is that j can impose a link onto i.

Table 7. Forced links model

	w_{ij} equation			e_{ji} equation	
Regressor	coefficient	dyadic z	Regressor	coefficient	dyadic z
Overlap in activities O_{ij}	-0.162	-0.62	Overlap in activities O_{ij}	-0.326	-0.98
Popularity P_j^i	0.707	7.31***	Popularity P_i^j	-0.496	-2.13**
Neighbor dummy$_{ij}$	0.436	2.07**	Neighbor dummy$_{ij}$	1.023	8.11***
Blood ties dummy$_{ij}$	0.788	4.86***	Blood ties dummy$_{ij}$	1.218	7.82***
Same religion dummy$_{ij}$	0.131	1.20	Same religion dummy$_{ij}$	0.219	1.47
$|w_j - w_i|$	-1.776	-1.85*	$|w_j - w_i|$	-5.052	-3.25***
$w_i + w_j$	0.172	1.48	$w_i + w_j$	0.130	0.72
Out-degree of i	0.534	4.15***	Out-degree of j	-0.509	-1.81*
Rich dummy of i	0.168	2.05**	Rich dummy of j	0.277	1.91*
Nber adult members of i	0.270	0.97	Nber adult members of j	0.925	2.75***
Intercept	-3.112	-10.82***	Intercept	-2.247	-10.12***
arc $\tan(\rho)$	1.081	1.29			

While these results are interesting in their own right, our primary interest is to test whether either of these models fits the g_{ij}^i data better than the pure willingness to link model. The Vuong test for the vetoed link and forced link models are presented in Table 8. Results show

that both significantly dominate the willingness to link model.[17] This is consistent with the idea that reported links g_{ij}^i are best interpreted as self-censored willingness to link. The last row of the Table also shows that we cannot distinguish between the vetoed link and forced link model: although the vetoed link provides a slightly better fit, the difference is not statistically significant. This is not entirely surprising given that the two models are fairly similar in terms of the underlying data generation process.

Table 8: Vuong test – vetoed links and forced links

model k	model m	Vuong test	best fit
vetoed links	willingness to link	3.47***	vetoed links
vetoed links	bilateral	3.58***	vetoed links
vetoed links	unilateral	4.05***	vetoed links
forced links	willingness to link	2.65**	forced links
forced links	bilateral	3.27***	forced links
forced links	unilateral	3.94***	forced links
vetoed links	forced links	0.70	both

5. Robustness analysis

To ascertain whether our findings are sensitive to the choice of regressors, we reestimate all models using different sets of explanatory variables. Results, not shown here to save space, indicate that when the included regressors have little predictive power – e.g., when the number of regressors is small – the comparison between models tends to be less conclusive. This is hardly

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[17]For comparison purposes, we also computed a standard likelihood ratio test to compare the vetoed link and bilateral link formation models since the latter is nested in/is a restricted form of the former. The value of the test is 87, which is well above the 1% critical value of 20.1 for a χ^2 distribution with 8 degrees of freedom. This confirms that the vetoed link regression dominates the bilateral link formation model. A similar comparison between the forced link and the unilateral link formation model yields a test statistic of 124, which clearly shows that the forced link model dominates. Neither of these test statistics corrects for dyadic correlation across observations, however.

surprising as the problem is common to all non-nested tests. The models are compared in terms of their ability to account for the data. When regressors have little predictive power, all models do rather poorly in predicting observed g_{ij}^i and hence cannot be distinguished.

In most cases, eliminating one or more regressors leaves the models' ranking unchanged but turns some pairwise comparison inconclusive. Dropping some regressors can nevertheless change the models' ranking. In particular, if we drop the in-degree P_j^i of j and/or the out-degree of i, non-nested comparisons indicate that willingness to link ranks lower than bilateral or unilateral link formation. Both self-censored models continue to dominate, however.

Finally, it worth mentioning that we have encountered the convergence difficulties that partial observability models are known for. Using a stepping algorithms for non-concave regions of the likelihood function alleviates part of the problem, but occasionally convergence may not be achieved. Also, in our experience the partial observability bivariate probit model is particularly sensitive to the choice of ad-hoc initial values and to multicollinearity, which in some extreme cases may result in the impossibility of computing standard errors.

6. Conclusion

The theoretical literature on networks has shown that the nature of the link formation process – e.g., whether unilateral or bilateral – has a strong effect on the resulting network architecture. In this paper we develop a methodology to test whether network data reflect a simple willingness to link or an existing link and, in the latter case, whether this link is generated by an unilateral or bilateral link formation process. Taking the equilibrium concept of pairwise stability as starting point, we propose a methodology to compare bilateral and unilateral processes. Central to this methodology is the observation that unilateral link formation requires that both nodes wish *not* to form a link for the link not to exist. This formal similarity between the bilateral and

unilateral link formation processes allows us to model them both as partial observability models and to compare them with the appropriate non-nested likelihood test.

We illustrate this methodology with data on informal risk-sharing networks in a Tanzanian village. The data is particularly well suited for our purpose because it covers all households in the community, and because the respondents are asked to enumerate all their network partners. The information provided by respondents is nevertheless open to several interpretations.

One possible interpretation is that responses capture an actual link. This interpretation is consistent with the observation made by De Weerdt and Fafchamps (2007) and Fafchamps and Lund (2003) who have shown that risk sharing links reported by survey respondents strongly predict subsequent inter-household transfers. It however remains unclear what process generates these links. The development literature is uncertain as to whether risk sharing networks should be seen as entirely voluntary, or whether social norms impose an element of moral or social pressure making it difficult for households to refuse helping others. If risk sharing is voluntary, link formation can be modelled as bilateral; if risk sharing is imposed by social norms, unilateral link formation is a more appropriate representation of the data generating process. Using a Vuong non-nested test, we find that the bilateral link formation model fits the data better than a unilateral one.

Another possible interpretation is that responses to a question about mutual insurance links capture the respondent's willingness to link, not an actual link. This may explain the large proportion of discordant answers whereby i reports a link with j although j does not report a link with i. We test a willingness-to-link model against the bilateral and unilateral link formation models and find that willingness to link fits the data best. This finding, however, is reversed if we drop the in-degree of j or the out-degree of i as regressors.

We then expand the data generating process to allow for self-censoring by respondents. We

investigate two forms of self-censoring. In the first one, which we call the vetoed link model, we allow respondents to form expectations about the other party's ability to refuse a link. In the second, which we call the forced link model, respondents anticipate that they may be unable to refuse certain links. We find that both models dominate the other three models, suggesting that self-censoring is present. But we are unable to distinguish between the vetoed and forced link models – both fit the data equally well.

While promising, the approach presented here suffers from a number of shortcomings. Test results are ultimately predicated on the assumption that the regressors used in the estimation are reasonable predictors of willingness to link. In the case of the self-censoring models, identification rests on exclusion restrictions that cannot be tested without additional data. The contribution of this paper should therefore be seen as primarily methodological. Stronger inference could be achieved if, in addition to information about links, the survey contained more direct evidence on respondents' willingness to link (or de-link) with other households. Should such data become available together with objective information on social links, the methodology presented here can yield a stronger test of bilateral versus unilateral link formation.

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