Université d'Orléans

Master International Economics

Competition policy and game theory

Exercise Set 2 : game theory 's basic elements

Fall 2019

You should know the definition of a mixed strategy and of Nash equilibrium.

A mixed strategy of a player is a distribution about its strategies. Instead of choosing a pure strategy, a player can commit to choose a mixed strategy. That extend the set of strategy, and in certain situations, allow to find an equilibrium when there were no equilibrium in pure strategy. Remark that a pure strategy is a (degenerated) mixed strategy	A set of strategies (that could be mixed strategies) is a Nash equilibrium whenever there is no unilateral deviation of the players.

1 Rock-Paper-Scissor

This is about showing that the famous Rock-Paper-Scissor game has only one equilibrium in mixed strategy, in which each player plays at random.

	r	p	s
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

Rock-Paper-Scissor

1) Prove that if one player chooses the mix strategy putting the same weight on R, P, S, then the payoff of each of the two player is zero.

2) Prove that we are at an equilibrium when both players chooses the mix strategy putting the same weight on the three actions.

3) Prove that there is only one equilibrium of the game.

2 Four finite Games

Compute Nash Equilibrium for each following game When the strategy space for the players are $S_1 = S_2 = [0,1]$ and with the pay-off functions :

a) ▷ $g_1(x,y) = 5xy - x^2 - y^2 + 2$
$g_2(x,y) = 5xy - 3x^2 - 3y^2 + 5$

b) ▷ $g_1(x,y) = -2x^2 + 7y^2 + 4xy$
$g_2(x,y) = (x+y-1)^2$

c) ▷ $g_1(x,y) = 5xy - x - y + 2$
$g_2(x,y) = 5xy - 3x - 3y + 5$

d) ▷ $g_1(x,y) = -2x^2 + 7y^2 + 4xy$
$g_2(x,y) = (x-y)^2$

3 Stop at the pedestrian crossing

Should a pedestrian cross at the zebra crossing in a country when motorists arrive on the zebra crossing. We show that there exists an economic answer to that question by modelizing a basic problem in game theory.

Consider the folllowing game with two players, a pedestrian and a car : The payoffs (pedestrian, car) are as follows : if the pedestrian crosses and the car passes (-1, *alpha*) ; if the pedestrian crosses and the car does not pass (1.0) ; if the pedestrian does not cross and the car passes (0,1) ; if the pedestrian does not cross and the car does not pass (0, 0).

1) Represent that game in normal form and find all the Nash equilibria, depending on the value of the parameter α. Interpret what you obtain.