Université d'Orléans

Master International Economics

Competition policy and game theory

Exercise Set 1 : game theory 's basic elements

► exercise set 1 a brief correction

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You should know some definitions about an objet we call a game : simultaneous and dynamic games, normal form and extensive form game, and the main solutions concepts.

In a game, a finite (sometimes infinite) number of rational agents have to take decisions, Which affects the welfare of every body. A game is described by the list of players, the rules of the games, i.e., the allowed actions of the players and the interactions the payoff resulting in any realizable history. The actions choices are either simultaneous or asynchronous.	A main feature of game theory is payoff's interdependency. That is for each player, her payoff could depend not only on the action she takes but also on the actions taken by the other players. Then, when choosing its action, the player should anticipate what the other players would choose. More precisely, we define for each player what we call her strategies, i.e., her unilateral decision of what she will do in any *node* she has to take an action. Analyzing a game is then to make the list of consistent a set of strategies.	The solution concept you should know is Nash Equilibrium : a set of strategies is a Nash Equilibrium is there is no unilateral deviation of any player. That is, when considering the equilibrium set of strategies, none of the player could increase her payoff by changing unilaterally her deviation.

1 Analyzing the equilibrium in a simultaneous game

This is about a simultaneous game in which two players A and B, called *firms* have to choose a price to sell at least one unit of good. For each firm the choice set is continuous, equal to $\mathbb{R}_+$

Let consider the following competition game between two firms, A and B. Both of them share a market in which there is a continuum of agents. Each buyer reservation price is equal to 1. Each firm 's marginal cost is equal to $c > 0$. The game is simultaneous : whenever $1 \geq p_A$ and $p_A < p_B$, firm A wins all the market, $q_A = 1$ whenever $1 \geq p_A = p_B$, there is a tie break rule : the market is divided among the competitors and $q_A = 1/2$. Firm i 's payoff is :

$$\pi_i = q_i(p_i - c)$$

1) Prove that (c, c) is one equilibrium of the game

When each firm chooses (c, c), the market is divided in two, each firm sell $1/2$, and get zero profit : $\pi_i = \frac{1}{2}(c - c)$.

To prove that (c, c) is an equilibrium, we have to prove that there is no profitable unilateral deviation. A necessary condition for a player A 's to be profitable is to set a price $p_A > c$. However, by such a strategy, player A would loose the whole market, ending up at zero profit : such a deviation is not profitable ; the argument is similar for player B. Then, there is no profitable deviation : (c, c) is an equilibrium of this game.

2) Prove that there is only one equilibrium of the game, that induces zero profit.

ROADMAP : We prove first that there is no asymetric equilibrium for instance with $p_A > p_B$ and then that (p, p) is not an equilibrium when $p > c$, which allow to conclude that there is only one equilibrium of the game (c, c), given the preceding question.

First, let consider a set of actions (p_A, p_B) with $p_A > p_B$. If both players conform their behavior to this action set, then, B wins the market and A's profit is null.

1. **If $p_B \geq c$, then, consider the following deviation for player B : instead of p_B, B proposes $\frac{p_A + p_B}{2} > p_B$: B increases her prices but not that much, and still wins the market. Her profit increases by**

$\Delta\pi_B = \left(\frac{p_A+p_B}{2} - p_B\right) * 1 = \frac{p_A-p_B}{2} > 0$. This deviation is profitable for B and then, the set of actions (p_A, p_B) with $p_A > p_B$ cannot be an equilibrium.

2. If $p_B < c$ B make losses, and setting its price to c will allow him a higher (null) profit. Any case, There is a deviation profitable for B .

Second, let consider a set of actions (p,p) with $p > c$. If both players conform their behavior to this action set, then, they share the market and their profit is $\pi_A = \pi_B = \frac{1}{2}(p-c)$. We prove that this cannot be an equilibrium. Indeed, let consider the following deviation by player A. Instead of proposing p, A proposes $\frac{1}{3}c + \frac{2}{3}p < p$: A wins the market and its profit is $\left((\frac{1}{3}c + \frac{2}{3}p) - c\right) * 1 = \frac{2}{3}(p-c)$, a profit which level is unambigously greater than $\frac{1}{2}(p-c)$, which proves that the considered unilateral deviation is profitable, and by extension that a set of actions (p,p) with $p > c$ cannot be an equilibrium of that game

In conclusion, the only equilibrium of that game is (c,c)

2 Three finite Games

In seaching for the Nash equilibria of a game, you have to analyze the rationality of each player by eliminating the strategy they would never choose, because they are dominated, contingent on the strategies of the other playersConsider the three following games (player A 's action $\in \{a_1, a_2, a_3, a_4\}$, player B 's action $\in \{b_1, b_2, b_3, b_4\}$) :

	b_1	b_2	b_3	b_4
a_1	1,2	3,4	5,6	7,8
a_2	9,10	11,12	13,14	15,16
a_3	17,18	19,20	21,22	23,24
a_4	25,26	27,28	29,30	31,32

Left

	b_1	b_2	b_3	b_4
a_1	19,2	15,10	13,16	1,20
a_2	17,28	11,4	3,12	29,18
a_3	9,24	5,30	31,6	27,14
a_4	7,22	33,26	23,32	21,8

Center

	b_1	b_2	b_3	b_4
a_1	1,32	2,31	3,30	4,29
a_2	5,28	6,27	7,26	8,25
a_3	9,24	10,23	11,22	12,21
a_4	13,20	14,19	15,18	16,17

Right

1) Compute the Nash equilibrium of the left game. Be very precise on the followed methodology.

Left Game If we look at the payoffs of player A that are odd numbers, starting from 1 to 31, we observe that when he plays strategy a_4 the payoffs are greater. More precisely, it is immediate to see that strategy a_4 is a dominant strategy

Moreover, something similar happen to the payoffs of player B : It happen that strategy b_4 is a dominant strategy

Then (a_4, b_4) is the unique Nash Equilibrium of this game. The resulting payoffs are 13 for player A and 20 for player B.

2) Compute the Nash equilibrium of the right game. Be very precise on the followed methodology.

Right Game ROADMAP Looking quietly to the right game, it appears that $a4$ is a dominant strategy for player A and that that $b1$ is a dominant strategy for player B. When those two assertions are proved, it follows that there is one equilibrium in dominant strategies $(a4, b1)$ inducing a payoff of 13 for player A and a payoff of 20 for player B.

$a4$ is a dominant strategy for player A, as,

1. $a4$ is the best choice of player A whenever A anticipates that player B plays $b1$: (13>9>5>1),
2. $a4$ is the best choice of player A whenever A anticipates that player B plays $b2$ (14>10>6>2),
3. $a4$ is the best choice of player A whenever A anticipates that player B plays $b3$ (15>11>7>3),
4. $a4$ is the best choice of player A whenever A anticipates that player B plays $b4$ (16>12>8>4),

Similarly, for player B, 32 is the highest payoff he can achieve when he anticipates that player 1 plays $a1$, with the choice of $b1$, 28 is the highest payoff he can achieve when he anticipates that player 1 plays $a2$, with the choice of $b1$, 24is the highest payoff he can achieve when he anticipates that player 1 plays $a3$, with the choice of $b1$ and 20 is the highest payoff he can achieve when he anticipates that player 1 plays $a4$, with the choice of $b1$.

3) Compute if there is some Nash equilibrium in the center game. Be very precise on the followed methodology.

Center Game A priori, there is no dominant strategy for agent A, neither for agent B. Then we inspect the rationality of each agent, contingent on the strategy of the other agent.

Agent A rationality : **considering step by step the different strategies of player B we cross the cells that would induce a deviation for player A, that never corresponds to an equilibrium choice for player A.**

Agent B rationality : **considering step by step the different strategies of player A we cross the cells that would induce a deviation for player B, that never corresponds to an equilibrium choice for player B.**

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