Multiple Lenders, Strategic Default and Covenants*

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Abstract

PROPOSITION 6 REVISITED.

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1 Introduction

2 The model

3 Market equilibria with covenants

4 Market design under the threat of default

We now take a normative perspective and address whether an institution can be designed to cope with the entrepreneur's threat to overborrow and improve equilibrium outcomes. An obvious way to restore efficiency would be to simply forbid covenants. This requires however that the regulator is able to observe all details of investors' offers, as well as all non traded contracts, to enforce an appropriate set of punishments. Also, were this extreme policy implemented, it could be detrimental to aggregate investment and to the entrepreneur's welfare. For instance, when moral hazard is strong, only the monopolistic allocation is sustained at equilibrium without covenants, while the competitive allocation can only be sustained with covenants.

The history of financial regulation offers many examples of subsidy mechanisms to mitigate counterparty default risk. In futures markets, central clearinghouses set up and manage guarantee funds to provide insurance against counterparty default (Kroszner (1999)).[1] Similar mechanisms have been at work in credit markets. Before the creation of the Federal Reserve System in 1913, the New York Clearing House Association (NYCHA) could allow its members to issue loan certificates, the payment of which was guaranteed by the clearinghouse itself, that is, by the whole banking industry (Gorton (1985)).[2]

Inspired by these examples, we study below whether an institution can be designed to mitigate the inefficiencies arising in our model. To do so, we focus on a situation in which the anti-competitive role of covenants is exacerbated, that is, we set $\pi_L = 0$. We construct a mechanism in which investors collectively provide transfers to a fund that guarantees a subsidy to the

[1] Decentralised markets also organise default arrangements. For instance, on the forward market at the Paris Bourse in the nineteenth century -an OTC market-, the stock brokers organised a Common Fund to rescue defaulting brokers (Riva and White (2011)).

[2] More anecdotally, during the Comptoir d'Escompte crisis in 1889 in Paris, the Banque de France granted a loan to the insolvent Comptoir d'Escompte, asking other banks to provide a guarantee to absorb the losses on this loan (Hautcoeur et al. (2014)).

entrepreneur. The transfers are contingent on the amount of funds raised from each of the investors $(I_1, I_2, ... I_N)$, and the subsidy is only provided if all investors accept to participate.

After the subsidy fund's rules are set, the following extensive form game takes place.

1. a) Each investor i offers a menu M_i of contracts that can include any class of financial covenants.[3]

   b) Investors observe offers and sequentially decide whether to pay their transfers or not.

2. The entrepreneur selects a contract in each menu and chooses effort.

3. Uncertainty over the investment project realizes, and payoffs are distributed according to financial contracts and to the subsidy scheme.

Thus, we extend our general game by adding one interim step at which investors decide whether to participate in the subsidy fund. The following proposition highlights the role of the subsidy fund in resolving indeterminacy and in disciplining investors' behaviors.

Proposition 6. *If $\pi_L = 0$, there is a subsidy mechanism such that (I^c, R^c) is the unique aggregate allocation supported in a symmetric equilibrium of the induced game. If covenants are precluded, and investors are therefore restricted to post menus of bilateral contracts, the same subsidy mechanism supports the monopolistic allocation (I^m, R^m) as the only equilibrium one.*

When investors use covenants, the fund provides an equilibrium selection device: only the competitive allocation survives. In contrast, the fund cannot reduce the monopolistic rents arising with bilateral contracts.

The intuition for Proposition 6 is the following. Consider first that covenants are allowed, and suppose that an allocation different from the competitive one is supported at equilibrium. In the absence of a subsidy fund, we know from Proposition 3 that the entrepreneur's threat to default hinders deviations by an investor who tries to exploit the pro-competitive effect of covenants and undercut competitors. The fund is designed to ensure the existence of such a profitable deviation. To achieve this result, the fund must exhibit the following properties. The final subsidy provided to the entrepreneur must be sufficiently large to alleviate the threat of default, and to induce $e = H$ at the deviation stage. In addition, each investor must be willing to

[3] That is, the analysis applies to both the informational structures of Subsections 4.1 and 4.2.

pay his individual transfer. To ensure participation, the mechanism exploits the gains from trade arising after contracts are posted, when the entrepreneur's behavior is fully anticipated. When investors foresee that, absent the subsidy, the entrepreneur will default, they are willing to pay their transfer as long as i) the required payment is smaller than their anticipated loss in case of default, and ii) the corresponding subsidy induces the borrower to choose $e = H$. We construct a subsidy fund that satisfies both requirements. As a consequence, no aggregate allocation different from (I^c, R^c) can be supported in a symmetric equilibrium.[4] The corresponding transfers are simple: investors' payments are linear and increasing in the aggregate investment unless $I = I^c$, in which case they are null. This leaves (I^c, R^c) as the unique equilibrium aggregate allocation. In the bilateral contracts case, only the monopolistic allocation is supported at equilibrium. The fund cannot generate additional equilibria though: an investor can always neutralize its effect by choosing not to participate.

Our subsidy fund shares many features with existing financial institutions. First, contributing to this fund is not mandatory: investors can refuse to pay transfers, and still offer financial contracts. Relatedly, the European banking union lets non-Euro-area banks free to participate to the Single Resolution Mechanism. Second, the financial contributions of investors increase in the volume of investment they propose. In practice, the guarantee funds of clearinghouses are financed by members' contributions that depend on the volume of trades they generate, and therefore on their risk exposure.[5] An important difference with actual mechanisms is that our subsidy scheme does not involve a direct transfer from some banks to others. It is therefore immune from the criticism that such bailouts create moral hazard and induce the insured agents (banks or traders) to take excessive risk. Indeed, subsidizing the entrepreneur when she invests too little does not create incentives for banks to lend too much.

Our analysis therefore offers a rationale for market-based regulatory schemes that make banks or financial intermediaries liable for the negative externalities stemming from their strategic

[4] The subsidy fund therefore allows to select equilibria by exploiting investors' sequential rationality: when deciding to participate to the transfer mechanism, investors are willing to pay to avoid the entrepreneur's subsequent default.

[5] These are recurrent features in the history of guarantee funds: at the Paris Bourse in the nineteenth century, most of the Common Fund's revenue came from a stamp tax on the paper used by brokers for their operations (White (2007)). In the US, the Board of Trade Clearing Corporation created in 1925 built its reserve fund from the clearing fees charged to its members (Kroszner (1999)).

interactions. Note that, at equilibrium, the entrepreneur receives no subsidy: it is the threat to provide a subsidy that disciplines credit markets, enhancing competition and decreasing rents. While it is hard to assess to what extent actual guarantee mechanisms cope with counterparty externalities, some authors argue that they do have a disciplinary role. In his analysis of the 1987 financial markets crash, Bernanke (1990) points at the effectiveness of the clearinghouse institution as an insurance company. Riva and White (2011) find evidence that during the nineteenth century, the number of brokers' defaults at the Paris Bourse decreases as the size of the guarantee fund relative to the volume of transactions increases. This suggests that guarantee funds may reduce counterparty risk. Whether these insurance systems can also affect the volume of trades and the size of the market remains an open question.

FOR ONLINE PUBLICATION

ADDITIONAL APPENDIX A

PROOF OF PROPOSITION 6:

The Mechanism. A mechanism consists in a system of transfers from the investors to the entrepreneur and a randomizing device. Both the transfers and the device are contingent on the observable investment levels $(I_1, I_2, ...I_N)$ chosen by the entrepreneur, and on the investors' decisions to participate. The device selects a "pivotal" investor who makes no payment to the entrepreneur after the offers, the decisions of the investors and the decision of the entrepreneur . Specifically, letting $K \leq N$ be the number of investors who provide a loan to the entrepreneur, any investor j such that $I_j > 0$ is pivotal with probability $1/K$. The pivotal investor will be denoted by κ. Clearly, if the entrepreneur raises funds from one investor only, the latter is pivotal with probability one. Investors learn who is pivotal after all relevant decisions are made, and before payments occur.

The schedule of transfers from investor i to the entrepreneur when the project succeeds is denoted $T_i(.)$. Transfers are equal to zero in case of failure. If the entrepreneur raises $I = \sum_{i=1}^{N} I_i$, then the contribution of each non pivotal investor i is

$$T_i(I) = \begin{cases} \frac{1}{\pi_H}\frac{1}{N}BI & \text{if } I \in [0, I^c), \\ 0 & \text{if } I \geq I^c, \end{cases} \tag{20}$$

whenever all investors agree to participate. In that case, if the project succeeds, the entrepreneur therefore receives the subsidy $T(I) = \frac{1}{\pi_H}\frac{N-1}{N}BI$. The schedule $T_i(.)$ is set equal to zero in all other cases.

No Transfer at Equilibrium. We first establish that the entrepreneur receives no transfer at equilibrium. To show this result, consider an equilibrium aggregate allocation (I^*, R^*) with $I^* > 0$, and the equilibrium surplus $(\pi_H G - 1)(I^* + A)$. The corresponding entrepreneur's payoff cannot fall below her reservation utility $(\pi_H G - 1)A$, from what it follows that the investors' aggregate profit cannot be greater than $(\pi_H G - 1)I^*$. Now, assume that investors decide to pay a transfer at equilibrium. Given that $B > \pi_H G - 1$, the aggregate profit earned by the non pivotal investors is bounded above by $\frac{N-1}{N}(\pi_H G - 1)I^* < \frac{N-1}{N}BI^* = \pi_H T(I^*)$, as the equilibrium is symmetric . That is, for the set of non pivotal investors, the total transfers exceed the maximal equilibrium aggregate profit of investors. Therefore, upon providing a transfer, at least one investor makes losses, and refuses to participate. This contradicts the assumption that investors pay a positive transfer at equilibrium.

Remark 1. *The above result obtains regardless of the investors' strategy space (bilateral contracts or covenants).*

We now consider the situation in which covenants are allowed and show that, given the subsidy mechanism, only the competitive allocation is supported at equilibrium. The proof is developed in three steps.

1. Constructing a profitable deviation. Given $(T_1(.), T_2(.), ..., T_N(.))$, we construct a deviation from each symmetric equilibrium supporting an aggregate allocation $(I^*, R^*) \in \mathcal{F} - \{I^c, R^c\}$, that is profitable for investor1 whenever $e = H$ is chosen.

We denote $\alpha = \min\left(G - \frac{1}{\pi_H}; \frac{N-1}{N}\frac{I^c - I^*}{B(I^c+A)}A; \dfrac{A}{\pi_H G - 1}; \dfrac{1-B}{B}I^*\right)$. When $(I^*, R^*) \in \mathcal{F} - \{I^c, R^c\} - \{0, 0\}$, $\alpha >$

Suppose that investor k deviates to the menu $M_k' = ((0,0), (I_k', R_k'(.)))$ with $I_k' = I^* + \varepsilon - \frac{N-1}{\pi_H G-1}\dfrac{\pi_H R^* - I^*}{N}$, and $\varepsilon \in (0, \alpha)$. The function $R_k'(.)$ is such that

$$R_k'(I) = \begin{cases} R^* + \dfrac{\varepsilon}{\pi_H} + \varepsilon^2 - G\frac{N-1}{\pi_H G-1}\dfrac{\pi_H R^* - I^*}{N} & \text{if } I = I_k', \\ G(I + A) & \text{if } I \neq I_k'. \end{cases}$$

The pair $(I_k', R_k'(I_k'))$ satisfies $U(I_k', R_k'(I_k'), H) = U(I^*, R^*, H) + (\pi_H G - 1)\varepsilon - \pi_H \epsilon^2 > U(I^*, R^*, H)$ and $\pi_H R_k'(I_k') - I_k' = \frac{\pi_H R^* - I^*}{N}\left[N - (\pi_H G - 1)\frac{N-1}{\pi_H G-1}\right] + \pi_H \epsilon^2 = \dfrac{\pi_H R^* - I^*}{N} + \pi_H \epsilon^2$, and $\Psi' = R_k'(I_k') - \left(G - \frac{B}{\pi_H}\right)(I_k' + A) = R^* - \left(G - \frac{B}{\pi_H}\right)(I^* + A) - \frac{B}{\pi_H}\frac{N-1}{\pi_H G-1}\dfrac{\pi_H R^* - I^*}{N} + \frac{\varepsilon}{\pi_H}\left(B - (\pi_H G - 1) + \pi_H \varepsilon\right)$. Then, the condition $\Psi' < 0$ [6] implies that $(I_k', R_k'(I_k')) \in int(\mathcal{F})$.

[6] Indeed, as it is shown below, $(I_k', R_k'(I_k')) \in int(\mathcal{F})$ is true when $(I^*, R^*) \in \Psi - \{I^c, R^c\}$, when $(I^*, R^*) \in \{\pi_H I^* - R^* = 0\} - \{I^c, R^c\}$, and by extension whenever $(I^*, R^*) \in \mathcal{F} - \{I^c, R^c\}$

- Notice first that $\frac{\varepsilon}{\pi_H}\left(B - (\pi_H G - 1) + \pi_H \varepsilon\right) \leq B\frac{\varepsilon}{\pi_H} \leq \frac{1}{\pi_H}\frac{N-1}{N}\frac{I^c - I^*}{(I^c+A)}A$.
- Notice that, whenever $(I^*, R^*) \in \Psi$, then $\pi_H R^* - I^* = \pi_H(G - \frac{B}{\pi_H})(I^* + A) - I^* = \big(B - (\pi_H G - 1)\big)(I^* + A) + A = A\frac{I^*+A}{I^c+A} - A = \frac{I^c - I^*}{I^c+A}A >$, implying $\Psi' \leq \frac{1}{\pi_H}\frac{N-1}{N}\frac{I^*+A}{I^c+A}A\big(-\frac{B}{\pi_H G-1} + 1\big) < 0$
- Notice also that at the limit case $\pi_H I^* - R^* = 0$, then, $R^* - \left(G - \frac{B}{\pi_H}\right)(I^* + A) = \frac{1}{\pi_H}I^* - \left(G - \frac{B}{\pi_H}\right)(I^* + A) = \frac{1}{\pi_H}\big(B - (\pi_H G - 1)\big)(I^* + A) - \frac{1}{\pi_H}A = -\frac{1}{\pi_H}\frac{I^c - I^*}{I^c+A}A$, implying $\Psi' \leq -\frac{1}{\pi_H}\frac{I^c - I^*}{I^c+A}A\left(1 - \frac{N-1}{N}\right) < 0$ when $I^* \neq I^c$

For $\varepsilon = 0$, $(I'_k, R'_k(I'_k))$ lies at the intersection of the entrepreneur's equilibrium indifference curve with the investor 1's equilibrium isoprofit line.

It follows that, since $(I^*, R^*) \neq (I^c, R^c)$, we get that $(I'_1, R'_1(I'_1)) \in int(\mathcal{F})$ for ε small enough.

It is immediate to see that, if $e = H$ is chosen, the entrepreneur can get a payoff above the equilibrium one by selecting the contract $(I'_k, R'_k(.))$ only, therefore trading with investor k alone. In addition, if $(I'_k, R'_k(.))$ is the only contract selected by the entrepreneur, then investor k ends up being the pivotal one $(\kappa = k)$, which ensures that he does not pay any transfer. One should then notice that by choosing this contract alone, and if the transfer is paid, the utility he would obtain (the left hand side in equation (*)) is greater than the utility that the entrepreneur would get with strategic default (the right hand side in equation (*)), which is without transfer, as $\pi_L = 0$):

$$U(I'_k, R'_k(I'_k), H) + \pi_H T(I'_k) > B(\hat{I} + A) - A + B(I'_k - \frac{1}{N}\hat{I}). \tag{*}$$

However, given the definition of $T(I'_k)$, equation (*)) is equivalent to:

$$U(I'_k, R'_k(I'_k), H) > \frac{1}{N}\left[B(I'_k + A) - A\right] + \frac{N-1}{N}\left[B(\hat{I} + A) - A\right]$$

a true condition that follows from the fact that $U(I'_k, R'_k(I'_k), H) > \max\big(B(I'_k + A) - A; B(\hat{I} + A) - A\big)$, the first inequality because $(I'_k, R'_k(I'_k)) \in int(\mathcal{F})$, the second one because $U(I'_k, R'_k(I'_k), H) > U^* \geq B(\hat{I} + A) - A$ when $\varepsilon \in (0, \alpha)$.

Equation * follows from the following more demanding condition [because of $U^* \geq B(\hat{I} + A) - A$]

$$(\pi_H G - 1 - \pi_H \varepsilon)\varepsilon + \frac{N-1}{N} B I'_k > B(I'_k - \frac{1}{N}\hat{I}) \Longleftrightarrow (\pi_H G - 1 - \pi_H \varepsilon)\varepsilon > \frac{1}{N} B\left(I'_k - \hat{I}\right),$$

which is true, as the left hand side is greater than 0 while the right hand side, smaller than 0. Indeed, $\left(\pi_H G - 1 - \pi_H (G - \frac{1}{\pi_H})\right)\varepsilon = 0$, when $\varepsilon < \alpha$; and $(\pi_H G - 1)(I'_k - \hat{I}) < (\pi_H G - 1)(I'_k + A) - B(\hat{I} + A) = U^* + (\pi_H G - 1 - \pi_H \varepsilon)\varepsilon - (U^* + A) \leq (\pi_H G - 1)\varepsilon - A < 0$, the first inequality because $\pi_H G - 1 < B$, the second inequality being trivial, the last one because $\varepsilon < \alpha$.

~~It follows that, as long as $\varepsilon \in (0, \alpha)0$, M'_k is a profitable deviation for investor k when $e = H$ is chosen. Overall, his profit increases by $\pi_H \varepsilon^2$.~~ Je le mettrai plus tard cette remarque sur le profit, qui a ultime ment sa place dans le paragraphe 3, et on a déjà mentionné la valeur $\pi_H \varepsilon^2$, on le verra tranquillement après le paragraphe 2 qui a considérablement diminué

2. The entrepreneur's choices. We now analyze the continuation game induced by the deviation to M'_k. Proceeding backward, we first consider the entrepreneur's choices when all investors accept to finance the subsidy. We already know, as it has been argued with equation (*) that the entrepreneur can get at the deviation stage more than at the equilibrium, by choosing $e = H$ and $(I'_k, R'_k(.))$ than the utility she would get bay choosing $e = L$. It is then immediate, that at the deviation, if the transfer have been accepted by the board of investors, the entrepreneur will choose $e = H$, and given the covenants associated to the contract of the deviator, she will only select $(I'_k, R'_k(.))$.

We analyze the behavior of the entrepreneur when the investors do not accept to finance the subsidy. Following the preceding analysis, and particularly because of the covenants associated to $(I'_k, R'_k(.))$, he will either only select $(I'_k, R'_k(.))$ and $e = H$ (when condition (21a) is satisfied) or strategically default (when condition (21b) is not satisfied)

$$U(I'_k, R'_k(I'_k), H) > B(\hat{I} + A) - A + B(I'_k - \frac{1}{N}\hat{I}) \tag{21a}$$

$$U(I'_k, R'_k(I'_k), H) < B(\hat{I} + A) - A + B(I'_k - \frac{1}{N}\hat{I}) \tag{21b}$$

$$U(I'_k, R'_k(I'_k), H) = B(\hat{I} + A) - A + B(I'_k - \frac{1}{N}\hat{I}) \tag{21c}$$

In summary, the entrepreneur will always choose $e = H$ and only select $(I'_k, R'_k(.))$, except in the case where both condition (21b) and to decision not to pay the transfer by the entrepreneur are satisfied.

Notice that we implicitly addeed the extra condition on ε that condition (21c) is never satisfied, making then impossible the entrepreneur to be indifferent between choosing $e = H$ or $e = L$.

3. The investors' participation subgame. We now consider the financing subgame, following the deviation. We split the analysis by considering differently the case in which either (21a) or (21b) is satisfied. We show in the first case that it admits a unique equilibrium in which no investor pay and in the second case that it admits a unique equilibrium in which every investor accepts to pay the transfer.

In case condition (21a) is satisfied, the entrepreneur will always choose $e = H$ and only select $(I'_k, R'_k(.))$, independently of the decision of the board of investors of financing the transfer or not. In that case, for each entrepreneur, the unique optimal decision at any node is to decide not to finance the transfer, getting 0 at the deviation stage, instead of the possible payment of the transfer. *It follows that, as long as $\varepsilon \in (0, \alpha)$, M'_k is a profitable deviation for investor k when $e = H$ is chosen. Overall, his profit increases by $\pi_H \varepsilon^2$.*

In case condition (21b) is satisfied, we first analyze the optimal behavior of a player at a node, given some assumption of the behavior of the other players at the other nodes, then, we solve the game backward.

Let first consider the decision at any node of a non-deviating player j, under the assumption that he had observed all the players $j' < j$ have accepted to pay the transfer, and that he anticipates that in case he accepts, the unique optimal decision of her follower is to accept to play. Observe first that if he accepts to pay, then, the contract of the deviator will be uniquely traded at the deviation stage, and the player j anticipates that in that case he will not be pivotal. Paying is therefore the unique optimal choice if $-\pi_H T_N(I'_k) > -\hat{I}_j$. Using (20), together with the fact that $\hat{I}_j = \frac{\hat{I}}{N}$ in a symmetric equilibrium, the inequality can be rewritten as

$$\frac{B}{N} I'_k < \frac{\hat{I}}{N} \Longleftrightarrow I_{k'} < \frac{1}{B} I^*, \tag{22}$$

that is satisfied as,

$$I_{k'} - \frac{1}{B} I^* = \varepsilon + I^*(1 - \frac{1}{B}) - \frac{N-1}{\pi_H G - 1} \frac{\pi_H R^* - I^*}{N} \leq \varepsilon - I^*(\frac{1-B}{B}) < 0$$

The last inequality being satisfied, as $\varepsilon < \alpha$ ~~$BI'_k < B\hat{I} \leq \hat{I}$, the first inequality coming from (A.44), the second from $B \leq 1$.~~

Let now consider the decision at any node of the deviating k, under the assumption that he had observed all the players $j' < j$ have accepted to pay the transfer, and that he anticipates that in case he accepts, the unique optimal decision of her follower is to accept to play. Observe that if he accepts to pay, then he will anticipate that her contract will be uniquely traded and she will be pivotal, making positive profits, while if he refuses, then, $e = L$ will be chosen, implying losses.

The backward analysis is then trivial: all players choose to accept the transfer. It follows that M'_k induces the entrepreneur to choose $e = H$, and constitutes a profitable deviation for investor k.

To show that the competitive allocation (I^c, R^c) can be supported at equilibrium, it is enough to consider the strategies exhibited in the proof of Proposition 3. It is then straightforward to show that no investor can exploit the subsidy mechanism to construct a profitable deviation.

The proof of steps 1-3 extends to the case in which investors can write covenants contingent on the initial debt I_0 (see the additional Appendix A). ■

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PROOF OF PROPOSITION 6 (continued): The proof extends to the case in which investors can write covenants contingent on the initial debt I. The system of transfers to the entrepreneur and the randomizing device are unchanged, and, again, there is no transfer on the equilibrium path. Then, the proof follows the same logic as in the proof of Proposition 6. Only the construction of a profitable deviation needs minor adaptations that we present below.

Take any aggregate allocation $(I^*, R^*) \neq (I^c, R^c)$ supported in a symmetric equilibrium. Let $\left(\frac{I}{N}, \frac{R(I,I^*)}{N}, \frac{I^+(I)}{N}, \frac{R^+(I,I^*)}{N}\right)$ be the equilibrium trades of each lender with, $I + I^+(I) = I^*$ and $R(I, I^*) + R^+(I, I^*) = R^*$.

Let investor k deviate to the menu $M'_k = ((0,0,0,0), (I'_k, R'_k(.), 0, 0))$ where I'_k and $R'_k(.)$ are defined as in the proof of Proposition 6, that is, $I'_k = I^* + \varepsilon - \frac{N-1}{\pi_H G-1} \frac{\pi_H R^* - I^*}{N}$ with $\varepsilon \in (0, \alpha)$, and

$$R'_k(I) = \begin{cases} R^* + \dfrac{\varepsilon}{\pi_H} + \varepsilon^2 - G\frac{N-1}{\pi_H G-1}\dfrac{\pi_H R^* - I^*}{N} & \text{if } I = I'_k, \\ G(I + A) & \text{if } I \neq I'_k. \end{cases}$$

As in the first part of the proof, we get that M'_k is a profitable deviation for investor k whenever $e = H$ is chosen: if $(I'_k, R'_k(.), 0, 0)$ is the only contract selected by the entrepreneur, then investor k ends up being the pivotal one, which guarantees him the opportunity not to pay any transfer. Overall, his profit increases by $\pi_H \varepsilon^2$.

It remains to check that, following the deviation to M'_k, the entrepreneur chooses $e = H$ provided that all investors agree to pay their transfers. As in the proof of Proposition 6, this leads to consider the inequality (analogous to $(*)$) :

$$U(I^*, R^*, H) + (\pi_H G - 1 - \pi_H \varepsilon)\varepsilon + \pi_H T(I'_k) > B((\hat{I} + \hat{I}^+(\frac{I}{N}) + A) - A + B\big(I'_k - \frac{1}{N}(\hat{I} + \hat{I}^+(\frac{I}{N}))\big)$$

which is equivalent to

$$U(I'_k, R'_k, H) > \frac{1}{N}\big[B(I'_k + A) - A\big] + \frac{N-1}{N}\big[B(\hat{I} + \hat{I}^+(\frac{I}{N}) + A) - A\big]$$

where $\frac{\hat{I}}{N}$ and $\frac{1}{N}\hat{I}^{+}(\frac{I}{N})$ denote the largest initial investment and the largest additional investment offered in any equilibrium menu for an initial debt I, respectively. The rest of the proof ~~is unchanged~~ follows the same lines with respect to the case in which the entrepreneur's debt is only observed ex post. In particular, following the deviation the entrepreneur will always choose $e = H$, except when both the decision not to pay the transfer by the entrepreneur and the following condition $U(I'_k, R'_k(I'_k), H) < B(\hat{I} + \hat{I}^{+}(\dfrac{I}{N}) + A) - A + B(I'_k - \frac{1}{N}(\hat{I} + \hat{I}^{+}(\dfrac{I}{N})))$, analogous to (21b), are satisfied. Also, the condition $I'_k > \dfrac{1}{B}\big(\hat{I} + \hat{I}^{+}(\dfrac{I}{N})\big)$, analogous to (22), is satisfied for the same reason, as $BI'_k < I^*$.

■