DINKs, DEWKs & Co.
Marriage, Fertility and Childlessness in the United States

Thomas Baudin* David de la Croix† Paula Gobbi‡

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Abstract

We develop a theory of marriage and fertility, distinguishing the choice to have children from the choice of the number of children. The deep parameters of the model are identified from the 1990 US Census. We measure voluntary and involuntary childlessness, and explain why (1) fertility of mothers decreases with education, for both married and single women; (2) childlessness exhibits a U-shaped relationship with education for both single and married; (3) the relationship between marriage rates and education is hump-shaped for women and increasing for men. We show how family patterns have been shaped by the rise in education and wage inequality, and by the shrinking gender wage gap.

Keywords: Fertility, Childlessness, Marriage, Education, Structural Estimation.
JEL Classification Numbers: J11; O11; O40.

*Centre de Recherche en Démographie, Université catholique de Louvain and EQUIPPE, Université Lille 3. E-mail: thomas.baudin@uclouvain.be

†IRES and CORE, Université catholique de Louvain. E-mail: david.delacroix@uclouvain.be

‡Paris School of Economics & IRES, Université catholique de Louvain. E-mail: paula.gobbi@uclouvain.be

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1 Introduction

American family patterns have changed over the course of the twentieth century. In this paper, we focus on one crucial transformation of family in the United States: the increasing discrepancy between marriage and motherhood. Nowadays, marriage no longer systematically implies a desire for parenthood, just as singleness no longer means childlessness. New types of families, such as DEWKs (Dually Employed With Kids), KOOPFs (Kids of One-Parent Families) or DINKs (Double Income No Kids) have become more common.[1] Childlessness is no longer necessarily a fate, it can also be a choice.

In this paper, we answer two questions. First, what are the incentives and constraints leading individuals to one type of family rather than another in terms of fertility outcomes? More specifically, when do married couples remain childless, and when do single women become mothers? Second, how do economic changes affect the proportion of these different families?

We propose to answer these questions using a theoretical framework that was built so as to reproduce three facts drawn from U.S. Census Bureau data for the year 1990: (*i*) the fertility of mothers decreases with education, for both married and single women, (*ii*) there is a U-shaped relationship between childlessness and education both for single and married women, and (*iii*) the relationship between marriage rates and education is hump-shaped for women and increasing for men. To the best of our knowledge, the first two facts have not been documented for both single and married women before, and there is no theory which accounts simultaneously for these three facts. These facts are discussed in more detail in Section 2.

Childlessness is an overlooked reality that cannot be explained by the current economic studies on fertility. At least three arguments show how important it is to distinguish between the extensive margin ($1-$ childlessness rate) and the intensive margin (fertility of mothers) of fertility. The first follows from a historical analysis of the evolution of both margins. The left panel of Figure 1 shows that from one generation to another, childlessness and the completed fertility of mothers can either be negatively or positively correlated, and that in the long run the relationship is surprisingly positive.[2] Second, the black line on the right panel of Figure 1 shows the relationship between childlessness and the completed fertility of mothers across education levels for a given cohort of American women (the sample is detailed in Section 2). We see that there exists an education threshold $\underline{e}$ for women below

[1]Each of these groups represents a different target for the marketing literature.
[2]A linear regression should lead to a positive relationship between childlessness and the completed fertility of mothers over time.

(resp. above) which childlessness and fertility are positively (resp. negatively) correlated as education changes. This means that the correlation between the two margins changes sign when education varies across individuals from a given country. To make this argument even stronger, the right panel of Figure 1 also shows the relationship for Brazil and Mexico.[3] For low education levels (or high fertility levels), the correlation between the fertility of mothers and childlessness is not negative in any of these countries contrary to what was expected. It is important to be aware of this because if the negative effect of education on childlessness is large enough, a policy aiming to reduce total fertility rates through the promotion of education might, in the short run, have the opposite effect through its positive effect on the extensive margin.

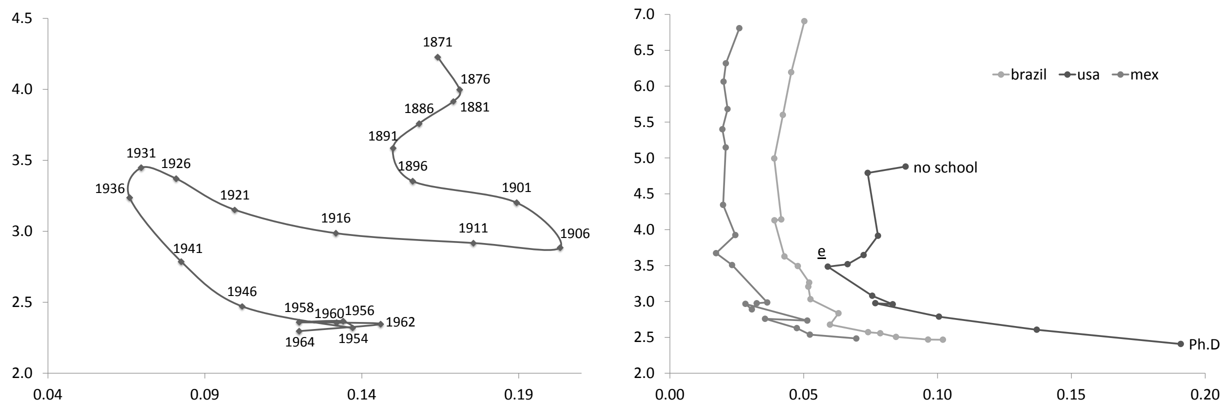

Figure 1: Completed Fertility of Mothers vs. Childlessness (married women). By Cohort in the USA (left panel), By Education levels in Brazil, Mexico and the USA (right panel)

Third, childlessness is key to understanding differences in average fertility across countries. Taking the international data of Sardon (2006) for the cohort born in 1955, the difference in completed fertility between each country and the U.S. can be decomposed into the extensive and intensive margins. This is shown in Table 1. In some countries, the total fertility is higher (lower) than it is in the U.S. because both margins are higher (lower). Thus, Ireland, Norway, the U.K. and Romania have a higher total fertility than the U.S., while Germany and the Netherlands have a lower fertility. For other countries, only one margin explains cross-country differences in total fertility. In Austria, Belgium and Denmark, motherhood rates are higher than in the U.S. but total fertility is lower because mothers have fewer children. In France and Sweden, on the contrary, the fertility of mothers is similar to U.S. levels, but motherhood rates are much higher. Hence, the driving force behind fertility differentials between France or Sweden and the U.S. for the 1955 cohort is childlessness. This

[3]We take Census data from IPUMS-International for Brazil 2000 and Mexico 2010. The samples consist of women aged 45-70 who are "married/in union".

	Fertility	Decomposition			Fertility	Decomposition	
	Gap with US	Intensive	Extensive		Gap with US	Intensive	Extensive
Austria	-10.2%	-11.5%	1.4%	Netherlands	-5.1%	-4.4%	-0.7%
Belgium	-7.1%	-8.3%	1.2%	Norway	4.1%	0.7%	3.4%
Denmark	-6.6%	-10.7%	4.1%	Sweden	3.0%	-1.1%	4.1%
France	8.1%	-1.3%	9.4%	UK	2.0%	1.4%	0.6%
Germany (W.)	-17.8%	-13.6%	-4.1%	Romania	15.7%	6.5%	9.3%
Ireland	35.5%	30.7%	4.8%				

Note: We use the following decomposition: if $z = ab$, then $\frac{z-z'}{z} = \frac{a(b-b')}{z} + \frac{(a-a')b'}{z}$ where a and b respectively denote motherhood rates and the fertility of mothers.

Table 1: Decomposition of Total Fertility Gaps into Intensive (Fertility of Mothers) and Extensive Margins (Motherhood)

confirms that both margins often play in opposite directions and that studying childlessness is necessary to understand international differences in fertility.

To summarize, the relationship between childlessness and fertility is not uniformly negative over time, across education groups, or across countries (which would be the case if childlessness was nothing more than low fertility). This reveals that economic incentives do not always lead the two margins to respond in the same direction, as expected based on the previous literature on fertility decisions. As average fertility is composed of both margins, ignoring which determinants affect childlessness can lead to unexpected consequences from a policy perspective. Hence, studying childlessness *per se* is of great interest.

From a social perspective, childlessness can either be a life choice for the "child-free", or a heavy burden for those who cannot experience parenthood. We claim that the U-shaped relationship between childlessness and the mother's education is driven by the coexistence of involuntary and voluntary causes. A woman will be involuntarily childless if she cannot procreate because of biological constraints leading to sterility or subfecundity; these constraints can either be innate, or acquired. We will call the first case "natural sterility" and the second "social sterility".[4] The definition of voluntary childlessness is more problematic; a restrictive position defines women who have never wanted to become mothers as voluntarily childless, while a broader way to define it includes those who have never tried to become mothers, given their particular living arrangement. We follow the broad definition.[5]

[4] "Acquired sterility" refers to the failure to conceive after bearing a first child.

[5] However, distinguishing between voluntary and involuntary childlessness is not as clear. Arguing about this would become more a matter of semantics and take us in a similar direction as the distinction between voluntary and involuntary unemployment. An educated woman who does not find a husband and is hence childless is not totally voluntarily childless because she might have liked to be married and be a mother. A

In this paper, women remain involuntarily childless when they are either naturally sterile, or, in the case of social sterility, when they do not have the minimum amount of commodities needed to be able to procreate. The existence of involuntary childlessness among disadvantaged groups in the United States is described in detail by McFalls (1979).[6] He argues that lower-income groups are more exposed to subfecundity causes than the rest of the population. Subfecundity factors that might affect the poor in developed countries are venereal diseases, malnutrition, psychopathological problems (drug abuse, stress, or psychoses) and some environmental factors (pollution). The poor also have less access to quality medical services, so that they are more subject to medical mistakes in abortions and cannot afford to buy fertilization services. Consequently, poor individuals are more affected by subfecundity factors because they do not have access to the same technologies as wealthier individuals. Educated women who are not naturally sterile remain childless because bearing and rearing a child takes time, and this opportunity cost is high for them in terms of foregone labor income.

Data from the National Survey of Family Growth for the years 1973 and 1976 which asked women questions about procreation (see Appendix A.3 for details), corroborate that, unlike voluntary childlessness, involuntary childlessness decreases with education. Among childless women, the proportion of voluntary childlessness increases with education, from 10% for women with no schooling to 70% for those with a Master's or Ph.D. degree. The proportion of involuntary childlessness decreases from 70% for the lowest education group to 20% for the highest one.

The model economy of this paper is composed of men and women who play a two-stage game. During the first stage, each individual is randomly matched with a partner of the opposite sex and decides whether to marry or not.[7] In the second stage, couples and singles make decisions about consumption and, if they can, fertility. Following a large literature initiated in the 1990s, we assume a collective negotiation process (see Chiappori and Donni (2009) for a literature review) to model the behavior of households. As shown by Chiappori (1988), this framework has considerable empirical support.

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large proportion of the childlessness cases among the poor are due to venereal diseases, so we could also argue that these women are childless because of risky sexual behavior and are partly voluntarily childless. So in either case, there is always a part of voluntary in involuntariness and of involuntary in voluntariness, except for "natural sterility" cases. Setting aside the discussion about what is voluntary, we keep this categorization only because it is the one used in other social sciences (see Morgan (1991) or Toulemon (1996)).

[6]The negative relationship between involuntary childlessness and income has already been documented in Wolowyna (1977), for 1971 Canadian Census data. Romaniuk (1980) provides a good discussion of the existence of high levels of involuntary childlessness in very poor societies.

[7]To simplify, we assume the match is done randomly and that there is no second round. Results with a positive degree of assortative matching are given in Appendix C.7. We do not consider same-sex souples or adoption, again for simplification purposes.

Marriage entails costs and benefits. For men, it opens the possibility of having children. As a counterpart, some of their time will be allocated to child-rearing. For women, a husband alleviates the time cost of raising children. Marriage also generates economies of scale since spouses share the expenses of household public goods. Being in a relationship also allows saving time, which is particularly precious for highly educated people. We accordingly assume that an individual has a lower time endowment when single than when married. Time endowment among singles may differ accross genders. This reflecs the fact that women (or men) may be more productive in domestic tasks, but also that there might be somebody else to help, in particular in the case of a single mother (a mother, a sister, a father, or a lover).

With this framework, we suggest that the hump-shaped relationship between marriage and education is related to the high childlessness rates for extreme education levels; mainly because marrying a woman who cannot or does not want to have children is less attractive for men. The relationship between childlessness and education is closer to a J-shape than a U-shape for married women because marriage works as an instrument against extreme poverty.

We abstract from unexpected births and cohabitation. Although unwanted births are important from a social and individual point of view (having an unwanted child can sharply reduce individual well-being), the percentage of women concerned is low. If the existence of unwanted births largely affected childlessness rates, our model should overestimate childlessness rates, in particular for low education levels.[8] Cohabitation covers two aspects: making decisions together, and/or receiving help from another person. Only the second aspect is taken into account in the model. For the period we consider, if cohabitation lasts, marriage is very likely to follow (Bumpass and Westoff (1970) estimate that between 1960-64 and 1985-89, around 80% of white women who got a first unmarried birth married within 10 years). It is therefore unlikely that a significant proportion of women whom we model as singles was involved in a long-term cohabitation relationship which lasted long enough to be

[8]When dealing with unintended births, one has to distinguish unwanted births (the mother did not want to have children at all or did not want a birth of that order) from mistimed births (the birth occurred too soon). Unwanted first births can reduce childlessness rates as some women would not have children at all if they were able to perfectly control their fertility. Using data from the National Fertility Study (NFS), Bumpass and Westoff (1970) estimate that only 5% of first births were reported as unwanted between 1960 and 1965. Using data from the National Survey of Family Growth (NSFG) in 2002, Mosher et al (2012) estimate that this percentage is 8,5%. Such an increase is consistent with the theory developed by Kennes and Knowles (2013) who investigate this issue from a theoretical and quantitative point of view. They show that the legalization and generalization of fertility control technologies have reduced the cost of sex outside of marriage and partly explain the sexual revolution (rise in the birth rate outside marriage and decrease in marriage rates). Both Bumpass and Westoff (1970) and Mosher et al (2012) find that income and education reduce the percentage of unintended births.

modeled by a cooperative decision process (see Appendix A.1 for more details).

To go beyond qualitative claims, the structural parameters of the model are identified using the three facts enumerated at the beginning. Average fertility decreases with mother's education, due to the higher opportunity cost of rearing children for more educated women. However, we show that Malthusian mechanisms, where education positively affects fertility, can appear for part of the population. As this positive effect concerns few individuals in the U.S., we do not observe it on an aggregate level. Concerning the U-shaped relationship between childlessness and education, we estimate that 2.5% of American women are socially sterile and 8.1% are voluntarily childless.

To further stress the interest of our methodology which allows to breakdown childlessness into three components, we run a historical experiment. This exercise shows that changes in education and in total factor productivity allow replicating the overall decreasing trend in fertility for the generations born between 1871 and 1964, while childlessness first decreased and then increased. We interpret the changes in childlessness first by a decline in childlessness created by poverty (involuntary) and, after the middle of the twentieth century, by an increase in childlessness created by a rise in the opportunity cost of having children (voluntary).[9]

According to the notion of "capabilities" (Sen (1993)), even though poor and rich childless individuals have the same "functioning" (being childless), they do not have the same "capability set". Indeed, whenever an individual does not have the opportunity to achieve a "valuable functioning" (in this case parenthood), this individual suffers from "capability deprivation". Fighting the causes of social sterility could then allow the set of capabilities of the poor to increase. We show that reducing wage inequalities and promoting gender parity on the labor market are powerful tools to limit the proportion of involuntary childlessness generated by poverty. A drop of 7% in the Gini coefficient allows reducing the percentage of socially sterile women by 25%.

To the best of our knowledge, this paper constitutes the first study of the determinants of marriage and childlessness in a unified framework. Gobbi (2011) studies the determinants and the evolution of marital voluntary childlessness during the twentieth century. Aaronson et al. (2011) focus on a quantity-quality approach and look at how the Rosenwald Rural Schools Initiative in the early twentieth century affected fertility along both the extensive and intensive margins. They show, in particular, that the expansion of schooling opportunities decreased the price of child quality, which decreased the proportion of women with the

[9] With close arguments and through a cross-country analysis, Poston and Trent (1982) show the existence of a U-shaped relationship between the development level of a country and its childlessness rate.

highest fertility rates as well as childlessness rates.We differ from these papers by looking at the role played by involuntary childlessness and marriage opportunities.

A growing literature is concerned both with family composition and fertility choices. However, it does not allow the three facts to all be explained together. Greenwood et al. (2003) and Regalia et al. (2011) analyze both marriage and fertility decisions in a dynamic programming framework where individuals can divorce. Instead of increasing the complexity of their set-up further to allow for different motives for childlessness, we develop a model abstracting from divorce and concentrating on the mechanisms behind fertility decisions.[10] Consequently, our work complements previous studies, while still replicating marriage rates in the U.S. for women who had completed their fertility life cycle in 1990. Our way of modeling is also different from theirs, due to our choice not to include divorce: we have a cooperative decision process inside the household, while Greenwood et al. (2003) use a Nash bargaining framework and Regalia et al. (2011) use a unitary decision model in which the woman chooses the number of children that the couple has.

The rest of the paper is organized as follows. Section 2 describes our stylized facts in detail. The theoretical model is described in Section 3, while Section 4 displays the identification strategy for the parameters of the model and provides simulation results. Section 5 runs counterfactual experiments to understand the changes that have occurred over the last 140 years. Our conclusions are presented in Section 6.

2 Three Facts from 1990 US Census

We use the 5% sample of the U.S. Census 1990, taken from the Integrated Public Use Microdata Series (IPUMS), and restrict our attention to the "ever-married, spouse present" and "never-married" women having completed their life-cycle fertility.[11] We look at women aged between 45 and 70 years old.[12] We divide the female population into 12 categories

[10] The models of Greenwood et al. (2003) and Regalia et al. (2011) allow for voluntary childlessness, but do not look at involuntary childlessness. However Regalia et al. (2011) study how changes in relative earnings affected the increase in the proportion of single mothers.

[11] The 1990 census is the last one for the U.S. to report completed fertility for women that are older than 44 years old. We drop from our sample women who are separated, divorced, widowed and married when their spouse is absent. The downwards relationship between fertility of mothers and education and the U-shaped relationship between childlessness and education hold for these women as well. These categories account for 30.5% of women. We exclude them from the sample because we do not know since when they are no longer with their partner.

[12] The oldest and the youngest women of the sample have decided to marry and to become mothers in somewhat different social and economic conditions. As shown in Appendix A.4, the facts presented in this section hold for each five-year cohort.

				Childlessness Rates		Completed Fertility of Mothers		Marriage Rates	
Nb	Education Level	e	N. obs.	Married	Single	Married	Single	Women	Men
1	No school	0	12122	0.088	0.755	4.880	3.897	0.699	0.716
2	Grade 1-4	3	14050	0.075	0.590	4.791	3.810	0.836	0.864
3	Grade 5-8	7	84243	0.078	0.631	3.916	3.480	0.909	0.909
4	Grade 9	9	38121	0.072	0.560	3.647	3.419	0.933	0.924
5	Grade 10	10	57213	0.066	0.588	3.519	3.324	0.945	0.928
6	Grade 11	11	49413	0.059	0.524	3.485	3.430	0.948	0.934
7	Grade 12	12	479703	0.076	0.781	3.079	2.549	0.948	0.937
8	1 year of college	13	178274	0.083	0.839	2.961	2.125	0.942	0.946
9	2 years of college	14	53428	0.077	0.825	2.976	2.257	0.945	0.946
10	Bachelor degree	16	99046	0.101	0.934	2.788	1.944	0.916	0.934
11	Master degree	17	56855	0.137	0.959	2.606	1.911	0.840	0.931
12	Doctoral degree	20	4612	0.191	0.957	2.408	1.743	0.755	0.926
all			1127080	0.081	0.787	3.160	2.935	0.930	0.930

Note: Standard errors are given in Appendix A.5. Marriage rates of men are rescaled in order to have an equal number of men and women. Each observation has a weight given by the Census and represents between 2 and 186 individuals. These weights are used to compute the facts.

Table 2: Facts from U.S. Census 1990

of education and report the average number of years of education, e, for each category as well as the number of unweighted observations (sum of singles and ever-married) per category. Table 2 summarizes the three stylized facts we focus on, computed using the weighted observations: completed fertility of mothers, childlessness rates of women and marriage rates of men and women for twelve education levels.

Fact 1: *Fertility of mothers decreases with education, for both married and single women.*

This fact is well known for married women. It is less known that, fertility of single women (conditionally on being mothers) also decreases with education. The negative relation between fertility and education has already been stressed in many papers without conditioning on both marital and motherhood status (see Becker (1981) pages 150-151, de la Croix and Doepke (2003) and Jones and Tertilt (2008)).[13]

Fact 2: *Childlessness exhibits a U-shaped relationship with education for both single and married women.*

The relationship between childlessness and education is not monotonic unlike the relationship

[13] See Appendix A.2 for more details on the single mothers with little education.

between fertility and education. We explain this U-shaped relationship by the existence of both involuntary and voluntary factors leading a woman to remain childless. For married women, the U-shaped relationship looks more J-shaped because marriage is used as a way of insuring against social sterility for women with the lowest levels of education. We retrieve here a feature already stressed in the introduction: there exists an education threshold for which childlessness is minimal (Grade 11 for both singles and married). The increasing side of the U-shaped relationship is easy to understand: highly educated women are more likely to be childless because their opportunity cost of raising children is high.[14]

Fact 3: *There is a hump-shaped relationship between marriage rates and education levels for women. Marriage rates (weakly) increase with education for men.*

Marriage rates are very high for intermediate levels of education: from Grade 5 to Bachelor degree, marriage rates are above 90%. For women, these rates are lower for extreme levels of education: less than 70% for women with no education and 75% for women having a PhD. Marriage rates are low for lowly educated men (72% for those without any education) but remain high for the most educated.

In Appendix A.6.1 we show that the three facts are true for Whites, Blacks, Natives, Asians and Hispanics separately. Appendix A.6.2 shows that the facts are also robust if we remove foreign born individuals from the sample. This means that they do not rely on the behavior of foreigners who have made their marriage and fertility decisions in another country and could be over-represented in some education categories.

3 Theory

3.1 The Model

We consider an economy populated by heterogeneous adults, each being characterized by a triplet: sex $i = \{m, f\}$, wage w^i, and non-labor income a^i. Marriage is a two stage game. During the first stage, agents are matched randomly with an agent of the opposite sex. They decide to marry or to remain single. A match will end up in a marriage only if the two agents

[14]For single women, the U-shaped pattern of childlessness with respect to education is robust if we merge the first three groups ("no school", "grade 1-4" and "grade 5-8"). Single women who are in the "no school" or "grade 5-8" group have an average childlessness rate that is significantly higher from the average rate of those in "grade 10". Childlessness rates of single women in all education groups are significantly different from that in "grade 11", except for those in "grade 9".

choose to marry. During the second stage of the game, agents decide how much to consume and how many children to have, if any. The utility of an individual of sex i is

$$u\left(c^i, n\right) = \ln\left(c^i\right) + \ln\left(n+\nu\right), \tag{1}$$

where c^i is the individual's consumption and n the number of children that he or she has. $\nu > 0$ is a preference parameter that allows for the existence of voluntary childlessness, as the individual utility remains defined when $n = 0$. In line with Iyigun and Walsh (2007), children are considered as a public good for the couple and there is no gender differences in preferences. Assuming homogeneity in preferences, both across and within genders, is a way to measure by how much economic incentives account for our three stylized facts without relying on differences in preferences. Differences in fertility, childlessness and marriage rates result from the structure of the marriage market and the heterogeneity on labor and non-labor incomes.

Each individual has a time endowment to be shared between working and child rearing. This time equals 1 for a married person, and $1-\delta^i$ for a single of sex i. This implies that marriage entails economies of scale in time (for instance, sharing domestic activities). Allowing for gender differences in the time endowment of singles does not only mean that women (or men) might be more productive in doing domestic tasks but also that there might be somebody else to help, in particular in the case of a single mother (a mother, a sister, a father, or a lover).

We assume that single women can have children whereas single men cannot. Having children entails time costs. First, there is a fixed cost, $\eta \in (0, 1)$ to becoming a parent. This is justified by the fact that the first child costs more in terms of time than the following children.[15] In addition to the fixed cost, there is a variable cost: each child needs $\phi \in]0, 1[$ units of time to be raised. If single, the mother has to bear the full time-cost alone. When married, the husband bears an exogenous part $(1-\alpha) < 1/2$ of the childrearing time, as in Echevarria and Merlo (1999). This makes single women more likely to be childless than married women, but among those who choose to become mothers, they reduce the average time cost per child by having more children.

We abstract from natural sterility until Section 3.5. But it is essential to model social sterility here, as it affects individual choices. We assume that in order to be able to give birth, a

[15] Turchi (1975) estimates that the mean number of hours spent per day on childrearing for a one-child family is 1.4, for the first 18 years. For a two-child family it is 0.99 per child. And for a three-child family, it is 0.93. See Table 6.4 in Folbre (2008) for similar results.

woman has to consume at least $\hat{c}$:

$$c^f < \hat{c} \Rightarrow n = 0. \tag{2}$$

This assumption has been amply justified in the Introduction by the fact that lower-income groups are more exposed to causes of subfecundity than the rest of the population. Notice that, unlike a goods cost of children, $\hat{c}$ does not depend on the number of children that the mother will bear. A proportional goods cost does not imply social sterility, as the mother could choose a low enough number of children that is compatible with her budget constraint.

Each adult draws a non-labor income $a^i > 0$ from a distribution $\mathcal{F}^i\left(\bar{m}_a^i, \sigma_a^i\right)$, independent of his/her education.[16] Non-labor income corresponds to the part of income that is uncorrelated with education.[17] The total non-labor income for a couple equals $a^f + a^m$. On the labor market, the wages per unit of time of men and women are respectively denoted w^f and w^m. Wages depend on education and gender only.

Each household has to pay a goods cost, μ, which is a public good within the household. This type of cost is commonly assumed in the literature and gives some incentive to form couples (see Greenwood et al. (2012)).

Given these assumptions, the budget constraints are as follows. For single men, consumption c^m equals income minus the household good cost:

$$c^m = (1 - \delta^m)w^m + a^m - \mu.$$

Single women can have children, their budget constraint is

$$c^f + \phi\left(1 + \eta(n)\right) w^f n = (1 - \delta^f)w^f + a^f - \mu, \tag{3}$$

with the fixed time cost being given by

$$\eta(n) = \begin{cases} \dfrac{\eta}{n} & \text{if } n > 0 \\ 0 & \text{if } n = 0. \end{cases} \tag{4}$$

Given the time constraint $\phi(1 + \eta/n)n \leq 1 - \delta^f$ the maximum number of children a single

[16] We relax the independence assumption in Appendix C.2. Allowing for a non zero correlation between education and non-labor income is not essential for the results.

[17] As shown in Jones et al. (2010), because of our log specification of the utility function, we need the non-labor income to be positive to generate the negative fertility-income relationship at the aggregate level. As they mention, this could be gifts, lottery income or bequests.

woman can have is

$$\underline{n}_{\mathrm{M}} = \frac{1-\delta^f-\phi\eta}{\phi}.$$

For couples, total non-labor income of the household net of cost is $a = a^m + a^f - \mu$. Their budget constraint is

$$c^f + c^m + \phi\,(1+\eta(n))\left(\alpha w^f + (1-\alpha) w^m\right) n = w^m + w^f + a. \tag{5}$$

The maximum fertility rate of a married woman equals

$$\bar{n}_{\mathrm{M}} = \frac{1-\alpha\phi\eta}{\alpha\phi}.$$

This is greater than the maximum fertility of a single woman ($\bar{n}_{\mathrm{M}} > \underline{n}_{\mathrm{M}}$) because husbands help in the raising of children. If spouses share the childrearing cost equally, the maximum number of children a woman can have equals $(2-\phi\eta)/\phi$.

To model couples' decision making, we assume a cooperative collective decision model following Chiappori (1988).[18] Spouses negotiate on c^m, c^f and n. Their objective function is

$$U(c^f, c^m, n) = \theta(w^f, w^m)\; u(c^f, n) + (1-\theta(w^f, w^m))\; u(c^m, n),$$

where $\theta(w^f, w^m)$ is the wife's bargaining power that depends on education and is given by

$$\theta(w^f, w^m) \equiv \frac{1}{2}\,\underline{\theta} + (1-\underline{\theta})\frac{w^f}{w^f+w^m}. \tag{6}$$

We specifically assume that the negotiation power of spouses is bounded from below, with a lower bound equal to $\underline{\theta}/2$, and positively related to their relative wage. As education will be used as a proxy for potential wages, negotiation power is in fact positively related to the relative education of the spouse.[19] The boundedness of the bargaining power function comes from the legal aspect of marriage: spouses have to respect a minimal level of solidarity inside marriage.

[18] Due to the presence of the fixed cost μ, there is always a surplus coming from marriage. By adopting a collective cooperative decision model, rather than a Nash bargaining process where potential spouses share the marriage surplus, we avoid marriage rates being equal to one. With a Nash bargaining process, and no frictions in the marriage market, everybody would get married in order to share the surplus. In this type of framework, the only way to allow for some proportion of singles would be to assume some negative shocks on the quality of the matching.

[19] See de la Croix and Vander Donckt (2010) for a discussion. An alternative consists in assuming a negotiation power that depends on the spouses' relative labor income rather than their relative education as in Iyigun and Walsh (2007). As suggested by Pollak (2005), we have also tried a specification that included non labor incomes in the bargaining power, results are provided in Appendix C.5

The different assumptions we have introduced imply some advantages to being married. It allows the cost of the household μ to be shared. It increases the time endowment of the participants by allowing to share domestic activities. Being married is the only way for men to have children. It allows women to reduce the opportunity cost of children (as long as $\alpha < 1$). Marriage gives at least one of the partners the opportunity to increase his/her consumption compared to the situation where he or she remains single.

3.2 Possible "Regimes"

We solve the problem backwards, first considering the consumption and fertility choice conditional on being married or not. Before studying (in Subsection 3.3) the precise conditions on wages and non-labor income under which the various constraints bind, we list the different possible "regimes", describind different living arrangements. Table 3 summarizes consumption and fertility choices in each regime.

3.2.1 Single Women

A single woman maximizes (1) subject to (2), (3), (4) and $0 \leq n \leq \underline{n}_{\mathrm{M}}$. She can be in different regimes, depending on which constraint is binding.

Regime I. (Interior solution) If no other constraint than the budget constraint is binding and the corner solution with no children does not dominate, the interior regime prevails. The equation of n_{I} shows that the effect of the woman's education on fertility is ambiguous. A higher w^f implies the usual income effect which raises fertility. But the increase in w^f also raises the time cost to rearing children, which reduces fertility through a substitution effect. If $a^f > \mu$, the substitution effect dominates and n_{I} decreases with w^f, while the reverse is true if $a^f < \mu$.

Regime II. (Social sterility) If her income does not allow the consumption level required to have children to be reached, the single woman lives in poverty and is prevented from choosing to have children.

Regime III. (Get fit to procreate) If her income is sufficiently high to escape Regime II but not high enough to have as many children as in Regime I, it may be optimal for the single woman to work and consume more in order to be able to procreate. A necessary

<u>Single Men</u>			
		$c^m = (1-\delta^m)w^m + a^m - \mu$	
<u>Single Women</u>			
Interior solution	$c^f_{\mathrm{I}} = \frac{w^f(1-\delta^f+\phi(\nu-\eta))+a^f-\mu}{2}$		$n_{\mathrm{I}} = \frac{1}{2}\left[\frac{w^f(1-\delta^f+\phi(\nu-\eta))+a^f-\mu}{\phi w^f}\right] - \nu.$
Social sterility	$c^f_{\mathrm{II}} = (1-\delta^f)w^f + a^f - \mu < \hat{c}$		$n_{\mathrm{II}} = 0$
Get fit to procreate	$c^f_{\mathrm{III}} = \hat{c}$		$n_{\mathrm{III}} = \frac{w^f(1-\delta^f-\phi\eta)+a^f-\hat{c}-\mu}{\phi w^f}$
Voluntary childlessness	$c^f_{\mathrm{IV}} = (1-\delta^f)w^f + a^f - \mu$		$n_{\mathrm{IV}} = 0$
Maximum fertility	$c^f_{\mathrm{V}} = a^f - \mu$		$n_{\mathrm{V}} = \underline{n}_{\mathrm{M}}$
<u>Couples</u>			
Interior regime	$c^f_{\mathrm{VI}} = \frac{\hat{\theta}y}{2} > \hat{c}$	$c^m_{\mathrm{VI}} = \frac{\left(1-\hat{\theta}\right)y}{2}$	$n_{\mathrm{VI}} = \frac{y}{2\phi\left(\alpha w^f + (1-\alpha)w^m\right)} - \nu > 0$
Social sterility	$c^f_{\mathrm{VII}} = \hat{\theta}\left(w^m + w^f + a\right) < \hat{c}$	$c^m_{\mathrm{VII}} = \left(1-\hat{\theta}\right)\left(w^m + w^f + a\right)$	$n_{\mathrm{VII}} = 0$
Eat and procreate	$c^f_{\mathrm{VIII}} = \hat{c}$	$c^m_{\mathrm{VIII}} = \left(1-\hat{\theta}\right)[y-\hat{c}]/(2-\hat{\theta})$	$n_{\mathrm{VIII}} = \frac{1}{2-\hat{\theta}}\left[\frac{y-\hat{c}}{\phi\left(\alpha w^f + (1-\alpha)w^m\right)}\right] - \nu$
Voluntary childlessness	$c^f_{\mathrm{IX}} = \hat{\theta}\left(w^m + w^f + a\right)$	$c^m_{\mathrm{IX}} = \left(1-\hat{\theta}\right)\left(w^m + w^f + a\right)$	$n_{\mathrm{IX}} = 0$
Eat & procreate a max	$c^f_{\mathrm{X}} = \hat{c}$	$c^m_{\mathrm{X}} = \left(\frac{2\alpha-1}{\alpha}\right)w^m + a - \hat{c}$	$n_{\mathrm{X}} = \bar{n}_{\mathrm{M}}$
Maximum fertility	$c^f_{\mathrm{XI}} = \hat{\theta}\left[\left(\frac{2\alpha-1}{\alpha}\right)w^m + a\right]$	$c^m_{\mathrm{XI}} = \left(1-\hat{\theta}\right)\left[\left(\frac{2\alpha-1}{\alpha}\right)w^m + a\right]$	$n_{\mathrm{XI}} = \bar{n}_{\mathrm{M}}$

Note: $y = w^f\left(1+\phi\alpha(\nu-\eta)\right) + w^m\left(1+\phi(1-\alpha)(\nu-\eta)\right) + a$, $\hat{\theta} = \theta(w^f, w^m)$

Table 3: Possible Regimes

condition for this regime to prevail is $a^f < \hat{c} + \mu$. Otherwise the woman would be able to consume more than $\hat{c}$ without working. $a^f < \hat{c} + \mu$ implies that n_{III} increases with the wage w^f. Regime III echoes Malthus's fertility theory (see e.g. Galor (2005)).

Regime IV. (Voluntary childlessness) When being childless yields the highest utility, the voluntary childlessness regime prevails. This regime arises when the opportunity cost of having children is high and the single woman fully specializes in labor market activities.

Regime V. (Maximum fertility) When the opportunity cost of having children is low and the non-labor income is high, it may be optimal for the woman to fully specialize in the production of children.

3.2.2 Couples

The problem for a couple is

$$\max_{c^f, c^m, n} \theta(w^f, w^m) \ln c^f + (1 - \theta(w^f, w^m)) \ln c^m + \ln(\nu + n)$$

subject to (2), (4), (5) and $0 \leq n \leq \bar{n}_{\text{M}}$.[20] Six living arrangements (regimes) are possible.

Regime VI. (Interior solution) If no constraint is binding and the corner solution with no children does not dominate we have the interior regime. An increase in the wage of the husband has an ambiguous effect on fertility:

$$\frac{\partial n_{\text{VI}}}{\partial w^m} > 0 \Leftrightarrow w^f > \frac{1 - \alpha}{2\alpha - 1} a. \tag{7}$$

In most of the literature (Galor and Weil (1996), etc.) $\alpha = 1$, then an increase in the wage of the husband has a pure income effect and $\partial n_{\text{VI}} / \partial w^m > 0$. When $\alpha < 1$, the income effect always dominates the substitution effect in families with very low non-labor incomes ($a < 0$). In families with higher non-labor incomes ($a > 0$), the substitution effect dominates only when the wage of the woman is low relative to the non-labor income of the couple.

[20] An alternative could be to add two participation constraints: $u^f(\textit{married}) \geq u^f(\textit{single})$ and $u^m(\textit{married}) \geq u^m(\textit{single})$. This implies that the partner with the highest negotiation power can have an interest in reducing his/her welfare in order to incite his/her match to accept marriage. However, because rationality is common knowledge, such a marriage contract would not be credible: as couples cannot divorce, the partner with the highest negotiation power has an incentive to deviate from the agreement.

In the case of an increase in the wife's wage, we find:

$$\frac{\partial n_{\mathrm{VI}}}{\partial w^f} < 0 \Leftrightarrow w^m > \frac{\alpha}{1-2\alpha} a.$$

As long as $\alpha > 1/2$, an increase in the woman's wage always reduces fertility when $a > 0$. In families with very low non-labor income ($a < 0$), the income effect dominates if the wage of the husband is also very low.[21]

Parameter α is related to the slope of the wage-fertility relationship. Higher α implies a more negative link between mother's education and fertility, together with a less negative link between father's education and fertility. We will come back to this result in the section on identification (Section 4.2). Finally, n_{VI} does not depend on the negotiation power $\theta(w^f, w^m)$ as there are no gender differences in preferences for children.

Regime VII. (Social sterility) When the total net income is too low to guarantee a sufficient consumption level to procreate, the couple is childless due to poverty.

Regime VIII. (Eat and procreate) When the bargaining power of the wife is too low to guarantee a sufficient consumption level allowing her to procreate in the interior regime, it can be optimal for the husband to give up some consumption in order to have children. This is the only regime in which fertility depends on the wife's bargaining power. This makes fertility subject to three effects: the usual income and substitution effects and a negotiation power effect. The latter arises because an increase in the wife's education gives her more bargaining power. As, in this regime, her consumption is fixed, the only way to obtain more utility is through increased fertility. The net effect on fertility is the result of these three forces. Notice that once w^f becomes sufficiently high, the couple will be able to have children in the interior regime (Regime VI). This is due to both the increased bargaining power of the wife allowing her to benefit more from the marriage surplus and to an income effect: as w^f increases, the total income of the couple increases allowing both spouses to consume more.

Regime IX. (Voluntary childlessness) When choosing to be childless yields the highest utility, the couple is voluntarily childless. In this case, both spouses specialize in labor market activities.

[21] Jones and Schoonbroodt (2010) provide a discussion on the possible impacts of women's wages on fertility.

Regime X. (Eat and procreate to the maximum) When it is optimal for the husband to give up some of his consumption for his wife to specialize entirely in procreation, the couple has the maximum number of children. Compared to Regime VIII, the optimal fertility rate of the couple does not depend on the negotiation power of the wife because she is not able to give birth to an additional child to increase her utility.

This regime does not exist for a single woman as it relies on the additional income provided by the husband.

Regime XI. (Maximum fertility) When the couple is sufficiently rich (either from non-labor income or from husband education) for the wife to specialize in childrearing without requiring any sacrifice from the husband, the couple has the maximum fertility.

3.3 Fertility and Consumption Choices

Conditional on being single, we can determine the optimal fertility and consumption of a woman as a function of her wage and non-labor income. Non-labor income thresholds, $\underline{a}$ and $\overline{a}$, and wage thresholds, $\mathcal{W}_0^f(a^f), \mathcal{W}_1^f(a^f)$, $\mathcal{W}_2^f(a^f)$, $\mathcal{W}_3^f(a^f)$, $\mathcal{W}_4^f(a^f)$ and $\mathcal{W}_5^f(a^f)$ are defined in Appendix B.1. We also provide and prove Proposition 1, describing the optimal behavior of a single woman endowed with (w^f, a^f). Figure 2 and the following paragraphs describe the main results of this proposition.

When the non-labor income of a single woman is too small ($a^f < \underline{a}$), she is childless no matter her wage. In this case, the wage threshold allowing her to procreate is higher than the wage for which she would choose to be voluntarily childless (once she can afford a child, the time spent with him/her becomes too expensive).

Figure 2 represents the relationship between the fertility of a single woman and her wage (education), when $a^f \geq \underline{a}$. The left panel shows a non-monotonic relationship between wage and fertility. The net non-labor income ($a^f - \mu$) is still not large enough to cover alone the minimal amount of consumption needed to procreate. For a low w^f, Regime III (get fit to procreate) prevails and an increase in the wage of the woman increases her fertility. As her consumption is fixed at level $\hat{c}$ in this regime, fertility is positively related to her wage until it is high enough to reach the interior regime (I). In the interior regime, n_{I} decreases with w^f because the substitution effect dominates ($a^f \geq \underline{a} > \mu$).

In the right panel, non-labor income is high enough to ensure a consumption $\hat{c}$ for any fertility level, even in the absence of labor income. As a consequence, Regime III disappears, and

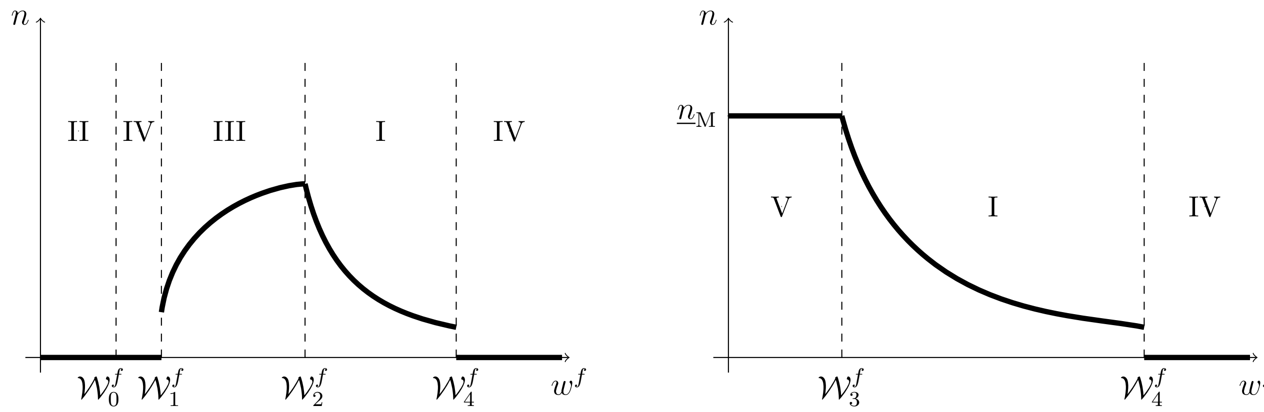


Figure 2: Fertility Conditional on Being Single when $a^f \in [\underline{a}, \overline{a}[$ and when $a^f \geq \overline{a}$

Regime V becomes possible. Regime V is a corner solution where the wage of the woman is so small that it is optimal for her to specialize in childrearing and consume her net non-labor income $a^f - \mu$.

On the whole, Figure 2 shows that uneducated single women will either be involuntarily childless or in the maximum fertility case, this will depend on their non-labor income. Conversely, highly educated women will be voluntarily childless (Regime IV) because the opportunity cost of having children (non-participation to the workforce) is high. This figure also shows another feature: the existence of a fixed cost of becoming a parent implies that the passage from childlessness to parenthood is not represented by a continuous function. The fixed cost makes it never optimal to have a very low number of children.

We now turn our attention to the couple. In Appendix B.2, we define non labor income thresholds A_0, A_1, A_2, A_3, A_4 and A_5 as well as wage thresholds $\mathcal{W}_A^f(a)$, $\mathcal{W}_B^f(a)$, $\mathcal{W}_C^f(a)$, $\mathcal{W}_{\underline{C}}^f(a)$, $\mathcal{W}_{\overline{C}}^f(a)$, $\mathcal{W}_D^f(a)$, $\mathcal{W}_E^f(a)$, $\mathcal{W}_F^f(a)$, $\mathcal{W}_G^f(a)$, $\mathcal{W}_H^f(a)$ and $\mathcal{W}_I^f(a)$, and prove Proposition 2 that describes the optimal behavior of a couple endowed with (w^f, w^m, a). Figures 3, 4 and 5 and following paragraphs describe the main results of this proposition.

As for a single woman, for very low non-labor income ($a < A_0$) and, low w^m, a couple will be childless whatever the wife's wage. The wage allowing the couple to have children is, in such a case, higher than the wage for which they would choose to be voluntarily childless (Regime IX).

For higher levels of non-labor income ($a > A_0$), having children becomes feasible and optimal for some wages. Figure 3 (left) shows that, when $a \in]A_0, A_1]$, a couple can have children in the eat and procreate regime (Regime VIII) but the interior regime (Regime VI) is never optimal. In Regime VIII, the husband voluntarily reduces his consumption in order to enable his wife to have children. As shown in Figure 3 (right), Regime VI becomes optimal for some

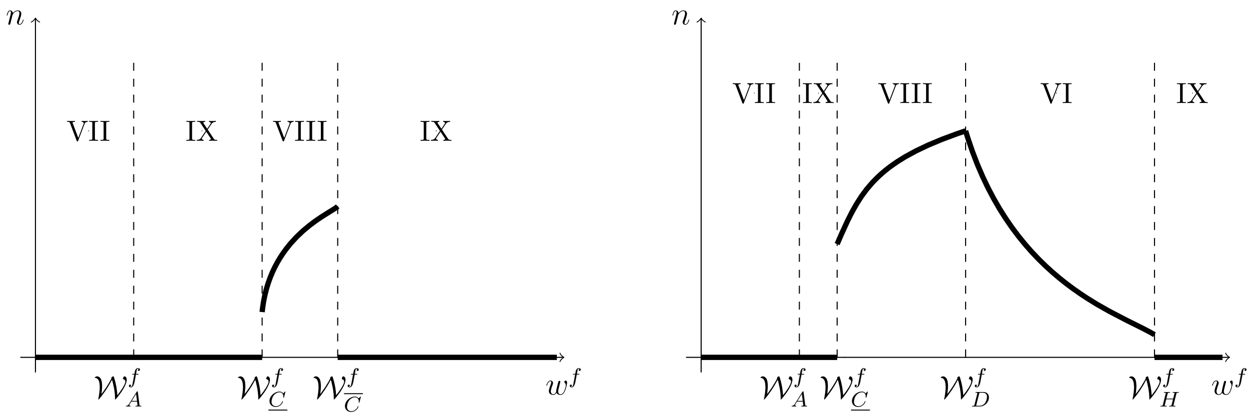


Figure 3: Fertility Conditional on Being Married when $a \in]A_0, A_1]$ and when $a \in]A_1, A_2]$

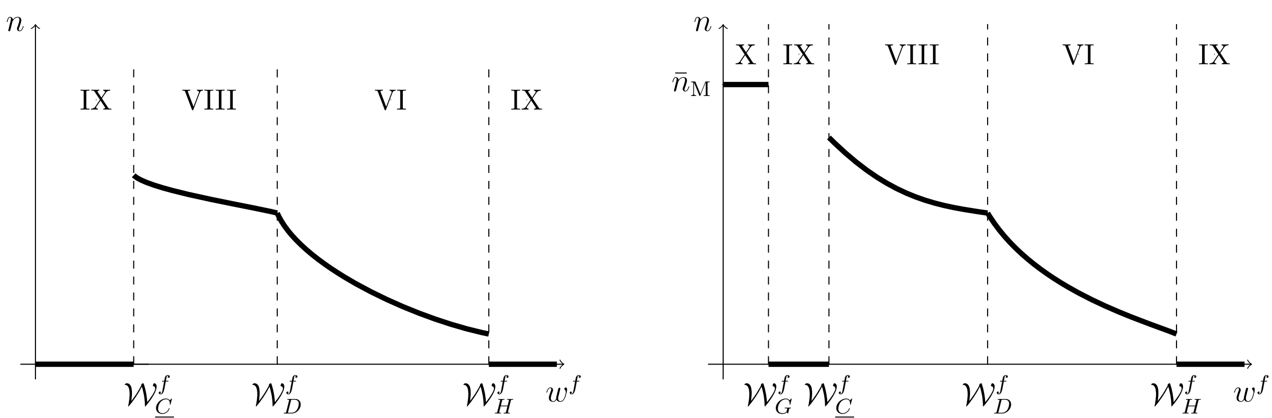


Figure 4: Fertility Conditional on Being Married when $a \in]A_2, A_3]$ and when $a \in]A_3, A_4]$

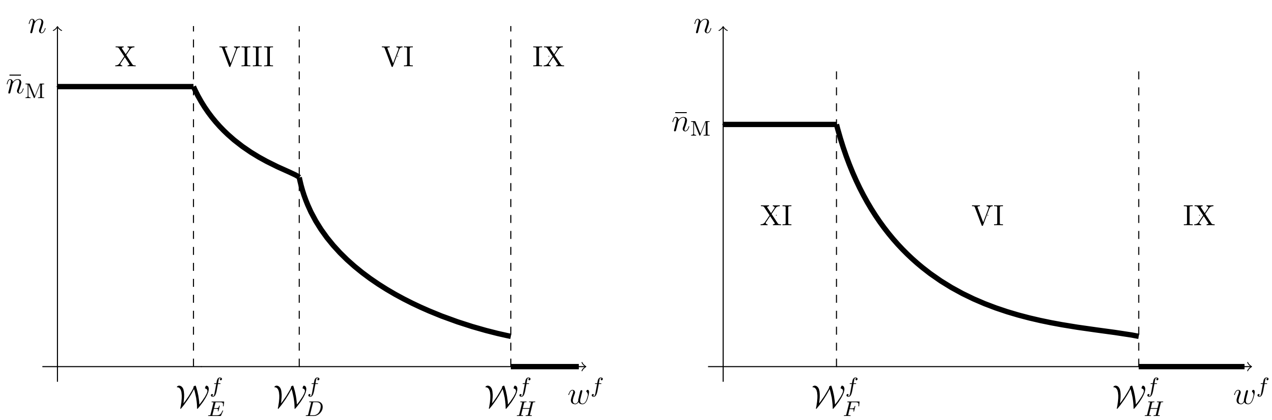


Figure 5: Fertility Conditional on Being Married when $a \in]A_4, A_5]$ and when $a > A_5$

wages and $a > A_1$. In Figure 3, the substitution effect is dominated in the eat and procreate regime and an increase in w^f increases fertility.

$\mathcal{W}_A^f$ corresponds to the wage allowing couples to procreate. For a wage greater than but close to $\mathcal{W}_A^f$, the consumption of the husband is close to zero, implying that parenthood is not optimal. This explains why Regime IX (voluntary childlessness) always precedes Regime VIII in Figure 3 and Figure 4 (left).

It can be shown that $\mathcal{W}_D^f$ decreases with a. This implies that higher levels of non-labor income move the fertility hump in Figure 3 leftwards. This means that a couple with higher non-labor income can reach the interior regime (Regime VI) for smaller w^f.[22]

In Figures 4 and 5, $a > \hat{c} - (1 - \phi(1 - \alpha)\eta)w^m$. Social sterility disappears as a couple can always ensure a consumption level of $\hat{c}$ to the wife. It also implies that fertility decreases with w^f in Regime VIII because the substitution effect dominates. When $a \in]A_3, A_4]$ and w^f is low, a couple has to choose between two extreme cases: Regime X where the wife specializes in child rearing and both spouses have a low consumption level; and Regime IX were she specializes in labor market activities and is childless. In other words, the couple has to choose between having a maximum of children but lacking of consumption goods and consuming more but lacking of children.[23] When w_f is very low, the surplus from consumption that could come from her labor market activity is also very low and the couple chooses to have the maximal number of children. When her wage becomes greater than $\mathcal{W}_G^f$, the surplus from consumption provides a higher marginal utility than the surplus from children. Then, the couple becomes voluntarily childless. Finally, for higher wages, the eat and procreate regime (Regime VIII) starts to dominate the two others.

Regimes X and XI, where the wife has the highest fertility appear in Figures 4 (right) and 5, for low wages. In Regime X, the wife consumes $\hat{c}$ and specializes in childrearing. This regime becomes feasible once $a > A_2$, but is optimal for some wages only when $a \geq A_3$ (when $a < A_3$ the consumption of the husband is too low). Once w^f becomes sufficiently high, the wife still consumes $\hat{c}$ but spends a part of her time on labor market activities. The additional income earned by the couple is allocated to the consumption of the husband. Finally, Figure 5 (right) exhibits the same fertility pattern as Figure 2 for a single woman and the same intuitions apply.

[22] Using as a proxy for wealth the fact of owning or renting the dwelling, we observe that those who rent the housing unit have larger childlessness rates, for all levels of education. This is supported by Figure 3. We can also observe that the decreasing relationship between completed fertility of mothers and education moves leftwards. This is also in line with Figures 3, 4 and 5 as $\mathcal{W}_2^f$ decreases with a.

[23] When $w^f \in [0, \mathcal{W}_{\underline{C}}^f[$, Regime VIII is always dominated by at least one of these two extreme regimes.

Poor couples will then be either childless (involuntarily or voluntarily) or in a regime where the wife has the highest fertility. In all cases, couples in which the wife is highly educated remain voluntarily childless.

In Appendix B.3 we report three-dimensional graphs which allow to see how Figures 3, 4 and 5 are related to each other when a changes. We provide graphs for three different levels of w^m, a dimension we have not exploited above to facilitate the exposition.

3.4 Marriage Decisions

We now consider the decision to accept a marriage offer from a randomly drawn person of the opposite sex. Once this potential partner has been drawn, we know the vector (w^f, a^f, w^m, a^m). This allows each individual to compare his/her utility as a single and as a married person and decide whether to accept marriage or not. In this section, we provide some elements to highlight this choice.

Let us first consider the potential bride. From Propositions in Appendices B.1 and B.2 we know the two relevant regimes she has to compare. When the potential bride has both a low wage and a low non-labor income, she could either be in Regime II or in Regime III if she remains single. If the relevant regime for the woman when single is Regime II, then she always prefers to be married with a husband allowing her to consume at least $\hat{c}$, as

$$u(c^f_{\text{II}}, 0) < \min\{u(\hat{c}, n_{\text{VIII}}), u(c^f_{\text{VII}}, n_{\text{VII}}), u(c^f_{\text{IX}}, 0), u(\hat{c}, \bar{n}_{\text{M}}), u(c^f_{\text{XI}}, \bar{n}_{\text{M}})\}.$$

She will reject a marriage offer only if she is in Regime VII if married and if $u(c^f_{\text{II}}, 0) > u(c^f_{\text{VII}}, 0)$, that is,

$$(1 - \delta^f)w^f + a^f - \mu > \theta(w^f, w^m)(w^m + w^f + a).$$

The only motive for a very poor woman to marry a man who does not provide enough income to allow her to procreate lies in the scale economies, in goods and in time, allowed by marriage. Moreover, an uneducated woman will accept to marry an uneducated man because similar education ensures that negotiation powers remain close to one half which allows the surplus from marriage to be shared equally.

If the relevant regime for a woman when single is Regime III, she always prefers to be a single mother who consumes $\hat{c}$ rather than a childless married woman who consumes less than $\hat{c}$ (i.e. $u(\hat{c}, n_{\text{III}}) > u(c^f_{\text{VII}}, 0)$). Hence, she would always reject marriage offers leading to Regime VII. Moreover, $u(\hat{c}, n_{\text{III}}) < \min\{u(\hat{c}, \bar{n}_{\text{M}}), u(c^f_{\text{XI}}, \bar{n}_{\text{M}})\}$, since in Regimes X and XI the woman would consume at least $\hat{c}$ and would have the maximum number of children. So she would always accept marriage offers leading to Regimes X and XI.

For higher w^f and a^f, Regimes I, IV or V are accessible as a single person. When a woman earns high wages, Regime IV (voluntary childlessness) does not dominate in only two circumstances: (a) when the additional income coming from the man is not high enough to incite the woman to become a mother but sufficiently close to her own income for the marriage surplus to be shared (Regime IX) and (b) when the man's wage is high enough to incite her to become a mother (Regime VI or XI).

We can conclude that, unlike highly educated women, lowly educated women will accept almost any match on the marriage market. This highlights the role of marriage as an institution protecting women against poverty, social sterility and even against living in a Malthusian like regime. Highly educated women do not need to be protected against social sterility by marriage. On the contrary, for highly educated women, being matched with highly educated men is the occasion to have children rather than being single and voluntarily childless because marriage reduces the opportunity cost of motherhood.

As being single is not only a personal choice, let us now turn to the problem of the potential husband. A single man has the following indirect utility,

$$\bar{u}^m = \ln\left((1-\delta^m)w^m + a^m - \mu\right) + \ln\nu.$$

Getting married is the only way for him to become a father but entails two costs: an opportunity cost due to the time spent with children[24] and a potential decrease of his consumption. This implies that a man will agree to marry without having children only if it increases his own consumption, which arises when the potential bride has enough education. Regimes VII and IX can then dominate singleness for the man.

The opportunity to become a father is not always sufficient to incite the man to marry: if having children decreases his consumption too sharply, he will remain single (singleness can dominate Regimes VI, VIII, X and XI). This could apply to a man with a low income, matched with a woman who would be social sterile as single: in order to have children in the "eat and procreate" regime, he would then have to give up too much of his income and consume too little.

Once again, marriage is a protection against social sterility: two partners could have children by pooling their income and their time while they would have remained childless and single

[24] In the case where $(1-\alpha)\phi n > \delta^m - \phi\eta(1-\alpha)$, his time dedicated to labor market activities is smaller when married than when single.

otherwise. Men who live in Regime VIII agree to reduce their consumption in order to enable their wives to have children. This is optimal if $\hat{c}$ is not too large and the man not too poor. Marriage can even prevent poor women becoming poor single mothers. This is the case when a highly educated man agrees to marry a poorly educated woman: even if her negotiation power is minimal ($\theta = \frac{1}{2}\underline{\theta}$), she will consume more than $\hat{c}$ because consumption increases with w^m.

The parameter α is important for marital decisions. For high values of α, the model predicts higher marriage rates for highly educated men than for highly educated women as better educated men have more reasons to marry: they can have children at a low opportunity cost. For highly educated women the incentive to marry is now low and marrying a lowly educated man would only entail costs (they would remain childless whatever the marital status). Poor women, on the contrary, will marry more often as they are less often rejected by highly educated men who can have children without reducing their consumption too much. For low values of α, the opportunity cost of becoming a father is high and nothing ensures that highly educated men marry more often than highly educated women. The parameters δ^{i}'s also affect the marital choice. In particular, a large δ^m implies that educated men are more willing to accept a marriage offer, in line with the male marriage pattern stressed in Section 2.

The model predicts a positive degree of assortativeness due to the parameter $\underline{\theta}$. A higher $\underline{\theta}$ implies that spouses' incomes are more equally shared. An individual with a much larger income than his or her potential spouse will then be more likely to reject the offer as he/she would have to give up too much of his/her own consumption.

To conclude, the model predicts lower female marriage rates for extreme social classes. Women with low levels of education are more likely to be involuntarily childless while highly educated women will probably want to remain childless. This makes women from extreme social classes less attractive to men. Marriage could however occur when wages and non-labor incomes are sufficiently close for them to share the marriage surplus.

3.5 Natural Sterility

So far we have abstracted from natural sterility. This amounts to assuming that involuntary childlessness is only due to bad lifestyle and low income. In reality, there exists a positive degree of sterility among men and women that is uncorrelated to the lifestyle. We take this into account here.

We assume that natural sterility is uniformly distributed over education categories. Let $\chi \in [0, 1]$ and $\zeta \in [0, 1]$ respectively describe the percentage of female and male who are naturally sterile. $\mathrm{E}u(c^i, n)$ denotes the expected utility of an agent of sex i and $\mathrm{E}U(c^f, c^m, n)$ the expected utility of a couple. We have that $\mathrm{E}u(c^m, n) = \overline{u}^m$ and:

$$\begin{aligned}\mathrm{E}u(c^f, n) &= \chi \cdot u((1-\delta^f)w^f + a^f - \mu, 0) + (1-\chi) \cdot u(c^f, n) \\ \mathrm{E}U(c^f, c^m, n) &= (\chi + (1-\chi)\zeta) \cdot U(c^f, c^m, 0) + (1 - \chi - (1-\chi)\zeta) \cdot U(c^f, c^m, n).\end{aligned}$$

We explicitly assume that single women are not concerned with male sterility as they can have multiple sexual partners.[25] It implies that married women are more concerned with sterility as they can be matched with a sterile partner. The marriage game now has three stages: first, people enter the marriage market ignoring their sterility status and decide whether or not to marry the partner they have been matched with; second, they discover their sterility status at no cost; and third, they decide how much to consume and, eventually, how many children to have.

As the other assumptions have not changed, we obtain that (i) when a single woman is naturally sterile, $c^f = (1-\delta^f)w^f + a^f - \mu$, and $n = 0$, (ii) when a single woman is not naturally sterile, her optimal fertility choices are described by Proposition 1, (iii) when a couple is not naturally sterile, spouses' optimal decisions are described by Proposition 2 and, finally, (iv) when a couple is naturally sterile, spouses share the net income of the household such that $n = 0$,

$$c^m = \left(1 - \theta(w^f, w^m)\right)\left(w^m + w^f + a\right) \quad \text{and} \quad c^f = \theta(w^f, w^m)\left(w^m + w^f + a\right).$$

As discovering sterility is made at no cost, choices of fecund single women and fecund couples are the same as in the benchmark model. For those who discover that they are naturally sterile, $n = 0$, implying that: (i) single women consume their net income and (ii) couples share their net income as a childless couple.

Compared to the benchmark model, we have added two categories of agents: sterile single women and sterile couples. The possibility of being naturally sterile or being matched with a naturally sterile partner has an impact on the incentive to marry. As couples face a higher risk of being sterile than single women, natural sterility tends *ceteris paribus* to reduce the overall marriage rate compared to the benchmark model. In particular, the number of marriages between highly educated men and poorly educated women are affected. Indeed, an

[25] A woman who has n different sexual partners in her life meets only infertile partners with a probability ζ^n. If natural sterility among males is 5% and a woman has only two different partners in her lifetime, she has a probability of 0.0025 of meeting only infertile partners. According to the National Survey of Family Growth (2002), the average number of lifetime sex partners for women who have always been single in the U.S. is 7.44.

incentive for highly educated men to marry a lowly educated woman lies in the possibility of becoming a father. Now, when they marry a poor woman, they have to reduce their consumption *and* to take the risk of remaining childless. Women with high education levels are less concerned by this phenomenon as they can have children on their own.

4 Identification of Parameters and Simulations

The model is now fit to match the facts provided in Table 2. There are in total fifteen parameters. We first set a priori four of these parameters (wage equation and natural sterility), based on empirical information. The eleven remaining parameters are estimated using a minimum distance estimation procedure.

4.1 A priori information

Wages are computed as follows:

$$w_e = \gamma z \exp\{\rho e\} \tag{8}$$

where e denotes the average number of years of education in each category (Table 2). We assume that the gender wage gap γ is the same across education categories and we set it equal to 0.869. We also set the Mincerian "rate of return to schooling" ρ equal to 0.092. The values of γ and ρ are estimated with Census data (see Appendix A.8.2 for details on data and regression specifications).[26] z is a normalization factor that will be used in Section 5.1 to capture the trend in total factor productivity.

Normalizing wages with respect to the wage of a man who has a doctoral degree, the minimum wage equals 0.158 for men and 0.137 for women. The main drawback of the Mincerian approach is to assume the same return to schooling for all schooling levels. In Appendix A.9, we discuss the importance of this assumption for our results. The asset of the Mincerian approach is to let income depend on two parameters only, γ and ρ, which can be the subject of counterfactual experiments. Such counterfactual exercises are provided in Subsections 5.2 and 5.3.

The two sterility parameters, χ and ζ, are fixed using information from the literature on natural sterility. The ideal population to measure sterility among couples is one where

[26]Assuming that γ increases with education, as some studies in the 1980s suggested, modifies the results only marginally. Lips (2003) shows that this "education penalty" almost disappeared during the 1990's. The estimated value for the gender wage gap is similar to the estimates of Erosa et al. (2010). The estimate of ρ is also in line with the literature (Krueger and Mikael (2001)).

marriage is associated with the desire to have children and where women marry young, do not divorce (e.g. because of sterility), are not unfaithful to their husband and live in a healthy environment. The closest to this ideal are the Hutterites. According to Tietze (1957), who studies sterility rates among this population, we should set the percentage of naturally sterile couples, $\chi + (1-\chi)\zeta$, at 2.4%. The average age at marriage in this study was 20.7 years. Two alternative measures of sterility are discussed in Appendix C.1. Our reading of the literature on the prevalence of fertility problems is that roughly half of them with a diagnosed cause are related to the man, and half to the woman.[27] We therefore set χ equal to ζ. The two restrictions lead to $\chi = \zeta = 1.21\%$.

4.2 Minimum Distance Estimates

We identify the parameters of the model using the Simulated Method of Moments (SMM). The eleven parameters, listed in Table 4, are identified by minimizing the distance between 72 empirical and simulated moments. These moments are the completed fertility of mothers and childlessness rates among both singles and married women, and the marriage rates for women and men, for the 12 education categories listed in Table 2. The objective function to minimize is given by:

$$f(p) = [d - s(p)]\,[W]\,[d - s(p)]'$$

where p is the vector of the parameters of the model, d denotes the vector of the empirical moments and s the vector of the simulated moments, depending on the parameters. W is a weighting matrix with $1/d^2$ on the diagonal, and zero elsewhere. This matrix is often used in the literature instead of the optimal weighting matrix, i.e. the inverse of the variance-covariance matrix of the empirical moments (Duffie and Singleton (1993)). When using the optimal weighting the moments computed from a larger number of observations have a lower standard errors and a higher weight. Consequently, for these moments, a higher distance between the data and the simulated moments is more heavily penalized in the objective function. In our case, using the optimal matrix lowers the weight of the observations for extreme education levels, which the most useful for identification.

To compute the vector of the simulated moments, we proceed as follows. We draw 100,000 women for each category of education. Each woman draws a potential husband from the empirical distribution of education levels among men. Each individual (man and woman) also draws a non-labor income. We assume that the non-labor income follows a log-normal

[27] This estimation is quite imprecise as definitions of sterility and infertility used in the literature lack uniformity (see Gurunath et al. (2011)).

distribution of mean κ_a and variance σ_a^2. Writing

$$\kappa_a = \ln(\bar{m}_a \bar{w}) - \frac{\sigma_a^2}{2}$$

where $\bar{w}$ denotes women's average wage, the parameter $\bar{m}_a$ can be interpreted as the average ratio of non-labor income to labor income. Then, for each category of education for women, we obtain 100,000 decisions about marriage and fertility that we can average to calculate the simulated moments.

The minimization of the objective function $f(p)$ was first run using PIKAIA and the results used as initial values in UOBYQA. PIKAIA is a genetic algorithm developed by Charbonneau (2002), it allows global extrema to be found in highly non-linear optimization problems where there exist a high number of local extrema. We used PIKAIA in a first step to identify the region in the parameter space where the global maximum lies. Once this region has been identified, we used a faster algorithm called UOBYQA (Powell (2002)) designed to find a maximum of a well-behaved problem. It was developed for optimization when first derivatives of the objective function were not available and takes account of the curvature of the objective function, by forming quadratic models by interpolation. We ran these two algorithms in FORTRAN 90. In the numerical implementation, we also assume that the number of births is an integer rather than a continuous variable. This simplification does not alter the main mechanisms at play but simplifies computations (and is also realistic).

4.3 Results

The identified parameters are listed in Table 4. Appendix C.3 explains how the standard errors are computed.[28] Assumptions 1 and 2 made in Appendices B.1 and B.2 hold under the values presented in Table 4.

Non-labor income amounts on average to 100% of labor income. This number seems quite high, unless we interpret the non-labor income as including, gifts, bequests, capital income and transfers (including social security) that are not correlated with the education level of the recipient. To have an idea of the magnitude of σ_a, we computed a Gini coefficient on women's simulated life-cycle income, $w^f + a^f$, which came 0.16. The estimated σ_a is relatively

[28]For curiosity, we have identified the parameters under the assumption that they are race specific, and that marriage markets are segmented by race in Appendix C.4. To be complete, we provide in the same appendix the identification after having removed from the sample the "disables". A discussion of whether one should include them or not in the sample is provided in Appendix A.7. Appendix C.4 also breaks down childlessness into its components for each group of individuals.

Description	Parameter	Value	Standard error
Variance of the log-normal distribution	σ_a	0.247	0.012
Average ratio of non-labor income to labor income	$\bar{m}_a$	1.001	0.012
Preference parameter	ν	9.362	0.146
Minimum consumption level to be able to procreate	$\hat{c}$	0.399	0.009
Good cost to be supported by a household	μ	0.272	0.013
Bargaining parameter	$\underline{\theta}$	0.864	0.014
Fraction of childrearing to be supported by women	α	0.524	0.005
Time cost of having children	ϕ	0.206	0.003
Fixed cost of children	η	0.114	0.006
Time cost of being single (men)	δ^m	0.256	0.015
Time cost of being single (women)	δ^f	0.077	0.013

Table 4: Identified Parameters

on the low side, but this is not surprising as some dimensions of inequality are absent from the model, such as wage dispersion for similar education levels.

To interpret the value of $\hat{c}$ and μ, remember that wages vary on a scale from 0.137 to 0.869 for women. For an average couple where both partners have 12 years of education and 2.5 children, the household cost represents 18.0% of the market consumption.This number is in line with the estimates found in Greenwood et al. (2012). A single woman with the lowest wage will need a non-labor income higher than 0.671 ($\hat{c} + \mu$, see Appendix B.1) not to be involuntarily childless.

Parameter $\underline{\theta} = 0.864$ means that the minimum negotiation power of a spouse is $\underline{\theta}/2 = 0.432$. Alternative specifications for the bargaining power function θ have been tested. Appendix C.5 shows that fixing the bargaining power to 1/2 or making it depend on relative potential incomes (hence including non labor incomes a^f and a^m) does not improve the matching with the data.

Childrearing time is shared between spouses. We estimate that men are involved for 47.6% of this time. The values for ϕ and η imply that the first child costs $\phi(1 + \eta) = 22.9\%$ of the time endowment of one parent, while the second child costs 20.6%. Following the values of α, η and ϕ, the maximum number of children that a married woman can raise is nine, while a single woman can have four children at most. This is coherent with the literature on natural fertility such as Tietze (1957) for Hutterite women, Smith (1960) for the Coco Islands' Malay women, or Henripin (1954) for the first generations in Quebec.

The interpretation of α should be broader than only active childcare spent by parents.

Quoting Folbre (2008), page 117: "Parents do more than merely devote time to their children. They work at the task of providing both active and passive care". This is also included in the parameter α but it is not in time use data. Activities such as looking for care centers or schools, or cleaning the car, in which the parent is not directly with the child also enter the amount of time parents devote to their children, which is hard to estimate from surveys or time diaries.

Our estimates of the δ^f and δ^m are such that $\delta^f < \delta^m$, implying that single women lose less time than single men because of their singleness. This can reflect the fact that women are more efficient at house keeping than men, or that single women find help more easily from others than single men. If we believe that singles have periods during which they cohabit with another person, the lower δ^f may also reflect that women benefit more from cohabitation than men. Notice finally that a positive δ^m is in line with the literature showing that married men are more productive (e.g. better health, higher wages) than single men (Korenman and Neumark (1991)).

In the theory, education (i.e. wage) is exogenous. This is generally a valid assumption as education choices precede fertility choices. However, in the case of teenage motherhood, mistimed or unwanted births can lead to endogeneity between education and fertility outcomes (see for instance Rindfuss et al (1980)). Census data neither report age at first birth nor desired fertility, preventing us from implementing a correction on the stylized facts. Furthermore, incorporating the possibility to experience unexpected pregnancies before education is completed could be done at a high cost in terms of tractability. It is worth noting, however, that the presence of women having low education because they gave birth unexpectedly could only reinforce our results. Suppose that 20 percent of the low education classes are in that case. If we were able to identify them, one should remove them from our sample and treat them separately. This would increase the level of childlessness (multiplying it by about 1.2) for those remained in the sample, as all those we withdrew had at least one child. As unexpected births decline with education, the corrected sample would display an even stronger U-shaped relationship between education and childlessness.

The simulated moments for the intensive and extensive margins of fertility that we obtain are represented in Figures 6 and 7. We reproduce the completed fertility pattern of single and married mothers (Fact 1), and the U-shaped relationship between childlessness and education for both married and single women (Fact 2).[29]

[29]One plausible reason why the simulated moments slightly underestimate fertility of lowly educated married mothers can be that we abstract from modeling unwanted births. As shown in Mosher et al (2012) and Bumpass and Westoff (1970), women who do not perfectly control their fertility are lowly educated and more often subject to experiencing both an unwanted or mistimed first birth leading to a shotgun marriage

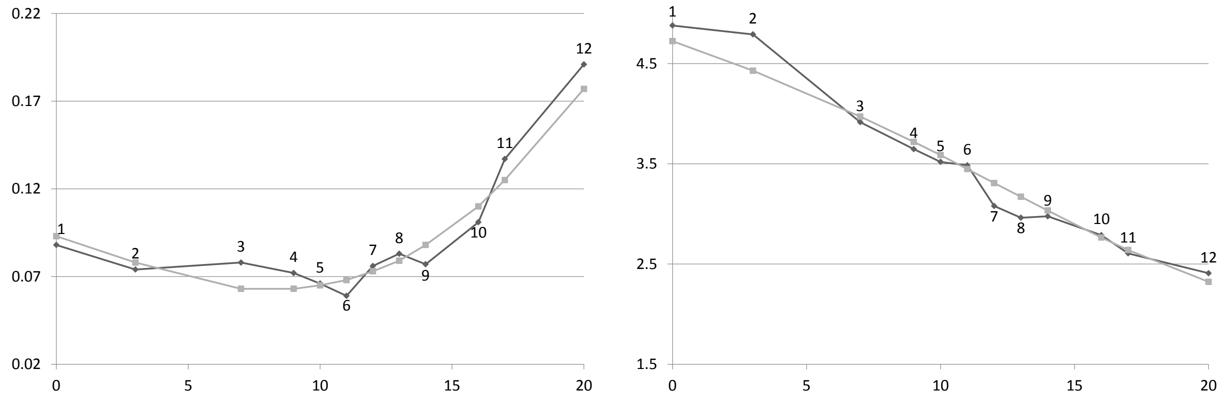


Figure 6: Childlessness Rate and Completed Fertility of Mothers, by Years of Schooling, Married Women. Data (black), Simulation (grey), Education Categories (labels)

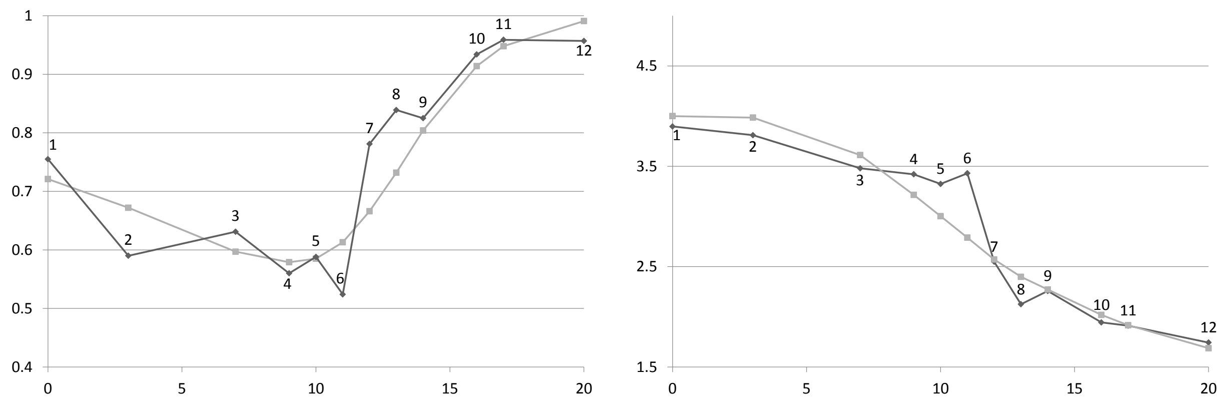


Figure 7: Childlessness Rate and Completed Fertility of Mothers, by Years of Schooling, Single Women. Data (black), Simulation (grey), Education Categories (labels)

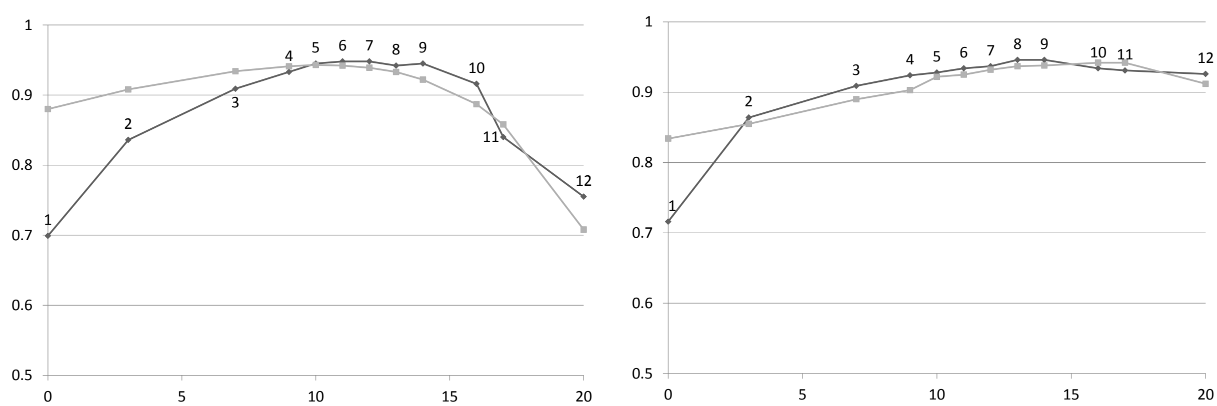
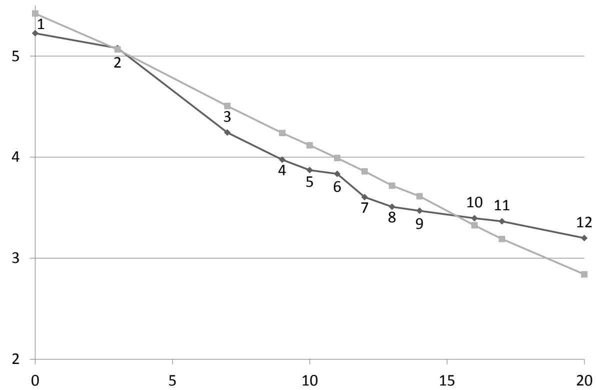


Figure 8: Marriage Rates of Women and Men, by Years of Schooling. Data (black), Simulation (grey), Education Categories (labels)



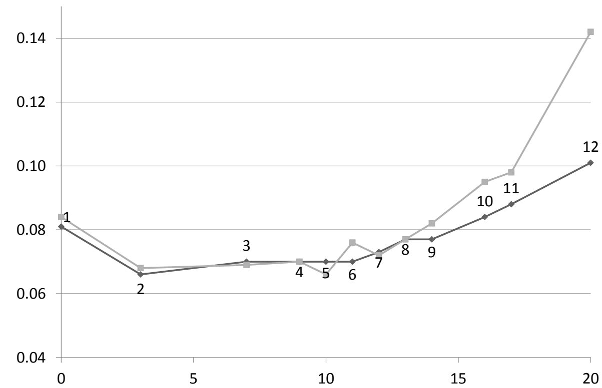


Figure 9: Completed Fertility of Married Fathers (left), Childlessness rates of Married Men (right), by Years of Schooling. Data (black), Simulation (grey), Education Categories (labels)

As for the marriage rates of men and women, Figure 8 shows that the model reproduces the hump-shaped relationship between marriage rates and education levels for women, and the positive correlation between marriage and education for men (Fact 3). For highly educated men, the high proportion of time they have to spend in rearing children (48%) implies that having children and being married is costly for them.[30] However, the 26% gain of time endowment by being married (δ^m) is enough to make them marry. The only weak aspect of the fit concerns the low education groups, for whom the theory overestimate the marriage rates by 10 to 18 percentage points.

As a test of the theory, we compare our results to empirical observations that were not used to identify the parameters. Looking at the fertility of married husbands is particularly interesting as it should allow us to assess whether our estimation of α is appropriate. Indeed, the slope of the relationship between the husband wage and fertility is determined by α (in the interior regime, see Equation (7)). Figure 9 reports the results. The theory can reproduce the negative relationship between average fertility and the education of husbands and the estimate of its slope is very good. This indicates that our estimation of the opportunity cost for men of having children is adequately measured. Figure 9 also reports the childlessness rate of married men with respect to their education. The model also captures the U-shaped relationship between both variables.

For parsimony we have included only one dimension of heterogeneity. Beyond non-labor

as well as an unwanted higher order birth (see also Akerlof et al. (1996)).

[30]Setting $\alpha = 1$ increases the marriage rates of highly educated men but the marriage rates of highly educated women are now strongly underestimated as they lose their incentive to marry and have children (we discuss this in more detail in Appendix C.6).

income, preferences are the obvious candidate for another source of heterogeneity: some people might care more about having many children than others. We accordingly introduced one new parameter: the variance of the (lognormal) distribution of ν across individuals. Estimating this parameter together with the other parameters leads to the result that this variance should be zero.[31] Considering Table 3, one sees that ν only affect fertility in the two interior regimes and in regime VIII, in addition to the conditions to be in these regimes. Introducing variability in ν increases variability of fertility in these regimes, but this does not help to match the moments better. Hence heterogeneity in preferences on top of heterogeneity in non-labor income does not bring any advantage for our analysis. Probably, if we were interested in reproducing the variability of fertility across individuals (i.e. matching the variance of fertility), heterogeneity in ν would help.

Table 5 provides the proportion of women in each regime, with respect to their education and marital status.[32] Uneducated single women are either socialy sterile (II) or have the maximum fertility (V), depending on their non-labor income (as in Figure 2). Poor married women are either socialy sterile (VII), in the eat and procreate regime (VIII), or with the maximum number of children (X and XI). This is consistent with Figures 3 to 5. Maximum fertility regimes, where poorly educated women would like to have more children but are constrained by their time endowment, concern 2.1% of American women (0.7% singles and 1.4% married). In Regime VIII, an increase in wages does not increase the consumption of the wife, who consumes $\hat{c}$. It could however increase her fertility. We estimate that 1.8% of American women are in this situation. This means that although aggregate fertility data suggests that Malthusian checks no longer prevail and Becker's model describes the negative relationship between education and fertility well, our model detects that some categories of the population are still affected by Malthusian mechanisms.

Women with the highest education are either voluntarily childless (single in IV or married in IX), or married mothers in the interior regime VI. Regime IX is the regime that we have in mind for DINKs while Regimes VI, VIII, X and XI are the corresponding regimes for DEWKs.

Finally, to dig deeper into the roots of voluntary childlessness when single, we have computed the fraction of single women in Regime IV, whose marriage offer has been rejected. In some sense, these are involuntarily childless because they are involuntarily single. It turns out that this proportion is around 0.06%. Hence, the huge majority of the (voluntarily) childless

[31] This result is robust to starting from different initial conditions.

[32] Note that Regime III (eat and procreate for single women) is not present in the simulations. This comes from the discrete choice of fertility in the quantitative analysis which does not provide a consumption level equal to $\hat{c}$.

	single				married						nat.
	I	II	IV	V	VI	VII	VIII	IX	X	XI	steril.
1	0.0	8.5	0.0	3.4	41.9	5.7	29.6	0.5	0.7	7.6	2.3
2	0.0	6.1	0.0	3.0	54.6	4.2	22.8	0.7	0.2	6.1	2.3
3	0.7	3.8	0.0	2.0	74.6	1.7	9.4	1.9	0.0	3.6	2.3
4	1.1	3.0	0.3	1.3	81.7	0.9	4.0	2.8	0.0	2.4	2.3
5	1.3	2.7	0.6	1.1	83.6	0.6	2.5	3.4	0.0	2.0	2.3
6	1.5	2.3	1.2	0.8	85.0	0.3	1.3	3.9	0.0	1.5	2.3
7	1.5	2.0	2.0	0.6	85.3	0.2	0.5	4.5	0.0	1.2	2.3
8	1.4	1.7	3.1	0.4	84.8	0.1	0.1	5.1	0.0	0.9	2.3
9	1.3	1.4	4.8	0.3	83.5	0.0	0.0	5.9	0.0	0.6	2.3
10	0.9	0.8	9.4	0.1	78.5	0.0	0.0	7.7	0.0	0.3	2.3
11	0.7	0.5	12.8	0.1	74.9	0.0	0.0	8.7	0.0	0.2	2.2
12	0.3	0.0	28.6	0.0	58.1	0.0	0.0	10.9	0.0	0.0	2.1
all	1.3	2.1	3.3	0.7	82.0	0.4	1.8	4.8	0.0	1.4	2.3

Table 5: Marital and Fertility Regimes as a Function of Women's Education in %

single women are single because they have themselves rejected a marriage offer.

Figure 10 plots the percentage of women that are childless in each category of education, distinguishing whether it is driven by the opportunity cost of raising children, by poverty or by biological reasons. Considering all education categories, social sterility concerns 2.5% of American women (2.1% singles and 0.4% married), while voluntary childlessness concerns 8.1% of American women (3.3% singles and 4.8% married). Childlessness concerns essentially either lowly educated (involuntarily childless) or highly educated women (mostly voluntarily childless), for both married and single people. This is in line with Figures 2 to 5. The percentage of involuntarily childless women decreases with education, while the percentage of voluntarily childless women increases with education. This explains the U-shaped relationship between education and childlessness (Fact 2).

The hump shaped relationship between marriage and education is related to the high childlessness rates of uneducated and highly educated single women. For a man, marrying a woman who is not fit to procreate (or would require massive help from the husband to do so) is less attractive, while marrying a woman with low incentive to have children (high opportunity cost) is hard.

Despite having assumed random matching in the marriage market, the model predicts that

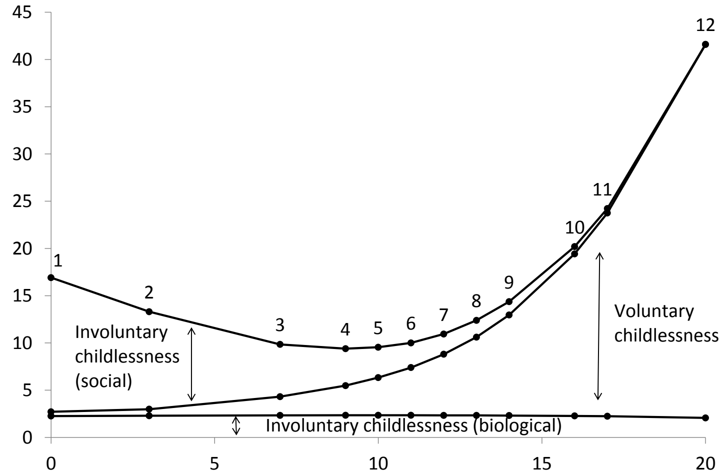


Figure 10: Childlessness Causes, per Years of Education (simulations)

individuals are more likely to marry someone with a similar level of education. This level of assortativeness is lower in the simulations than in the data because of the static nature of the model and the assumption that life only brings one chance to get married. In Appendix C.7, we provide a way to measure the degree of assortativeness in the data and the model. The model accounts for 13.8% of the variation in the assortativeness of matching. In reality, the assortativeness is higher because, first, people meet several possible matches (we would need a dynamic model to reproduce this), and second, individuals are more likely to meet others who have similar levels of education to their own.

With a model able to match the level of assortative matching, in particular among poor people, we expect a lower $\hat{c}$. In fact, 53% of women with no education marry a man with no education. This is much higher than in our model. Consequently, our estimated value of $\hat{c}$ is probably too high because we have more women with no education marrying men with higher levels of education than in reality. This means that in order to match observed childlessness, $\hat{c}$ must be higher. A way to increase the degree of endogenous assortativeness consists in modifying the bargaining rule (6) by making it less sensitive to the wage ratio. This will increase the rejection rate of matches with very different people. However, achieving more assortativeness will be at the expense of matching a reasonable marriage rate, as people only meet once in our static set-up. This is a limitation of our approach.

A complementary and exogenous way to generate the right degree of assortativeness is to assume, as in Fernández-Villaverde et al. (2010), that a fraction λ of the female population draws a possible match from her education category, while $1 - \lambda$ draws from the total population. For each chosen value of λ, we can estimate the remaining parameters and see whether changes are important.[33] We provide the results in Appendix C.7, Table 26. Our main results are robust to this change. For instance, when λ equals 0.1, the fit of the model does not change significantly although the marriage matrix becomes much more satisfying.

5 Counterfactual Experiments

Now, we use the parameters identified in Section 4 in several counterfactual experiments. The first is a historical experiment to understand the changes in childlessness and fertility for the cohorts born between 1871 and 1964. Then, we also look at how an increase in inequality and a change in the wage gap affect marriage rates, childlessness and fertility.

5.1 Historical Experiment

We have already stressed in the Introduction that childlessness and fertility are not always negatively correlated over time (left panel of Figure 1, for married women). Apart from the baby boom episode, the secular pattern of fertility is monotonically decreasing. Childlessness too, over the long-run, declined, from around 16% in the end of the nineteenth century to around 12% for the last cohorts for which completed fertility can be observed. All historical data are available in Appendix A.8. In this section we use our theory to shed light on these long term trends.

In a cross-section of households, we have seen (Figure 10) that rising education leads to a decrease in (involuntary) childlessness for low levels of education and an increase in (voluntary) childlessness for high levels of education. The question we are investigating now is whether the same reasoning could be applied to explain the secular pattern of childlessness and fertility. To do so, we will rely on two mechanisms: the rise in education as observed in the different censuses, and an overall trend in income, related to the macroeconomic trend in total factor productivity.

Before turning to the results of the simulations, let us make clear that there are two features of the data that we have no hope to capture. The first is the baby boom, for the cohorts born between 1911 to 1940. The mechanisms that could generate the baby boom are absent from

[33] In Fernández-Villaverde et al. (2010), a fraction of the population is randomly matched, the rest being matched following a Gale-Shapley algorithm, which generates perfect assortativeness.

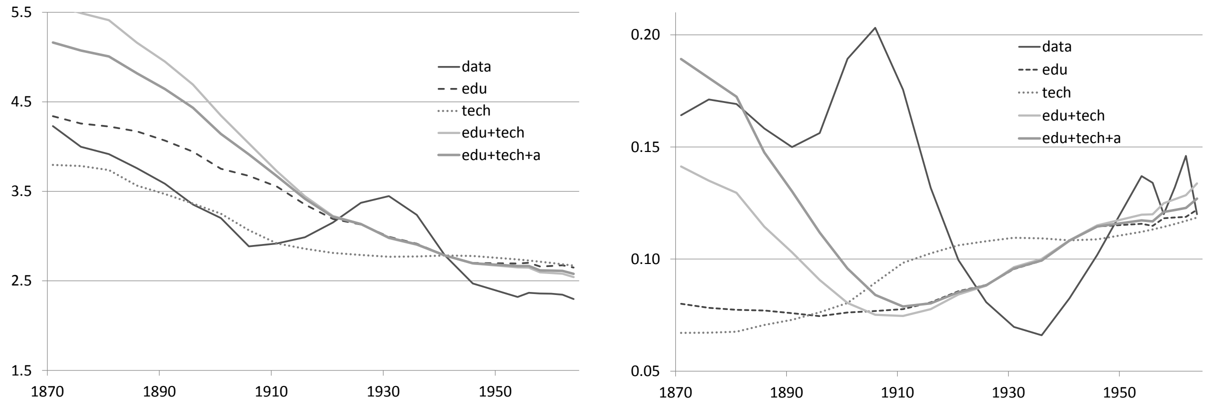


Figure 11: Completed Fertility of Mothers and Childlessness by Cohort, Married Only

our framework.[34] The second particular feature of the data is the peak in childlessness for cohorts born between 1901 and 1915. In Appendix A.8 we explain that this peak is related to a change in the Census questionnaire. The Spanish Influenza and the Great Depression may also have played a role (see Sobotka et al. (2011)).

To first address the effect of the increase in education, we keep wages fixed and rescaled so as to reproduce the average fertility of the generation born in 1941. We then simulate the model setting the shares of men and women in each education category to their historical values for each cohort, keeping everything else constant. The effects on completed fertility of mothers and childlessness are represented by the dashed lines of Figure 11. From the left panel, we observe that the rise in education leads to a monotonous decrease in fertility over time; alone it explains 87% of the drop in fertility (fertility drops from 4.23 to 2.30 in the data, and from 4.34 to 2.65 in the simulation). As for childlessness, the rise in education predicts a very slight U-shaped pattern for childlessness over time, according to the intuitions developed above. Most of the effect of education is still to increase childlessness, reflecting an increase in its voluntary components: indeed, in this counterfactual simulation, the wage is fixed at the level reproducing the fertility the 1941 cohort, which is quite high for nineteenth century standards, and this makes very few households affected by involuntary childlessness, even at the beginning of the period.

Secondly, we simulate the effect of a change in the overall wage level, following a trend in TFP. Data on TFP are taken from Chari et al. (2006) for the period 1901-2000 and from Conference Board (2013) for the period 2000-2010. We compute 30-years averages

[34]Doepke et al. (2007) suggest that part of the baby boom is caused by restrictions in the labor market for young women. Albanesi and Olivetti (2010) also suggest that improvements in maternal health contributed to the bust. The raise in technological progress in the household sector is also one major cause of the baby boom, as shown in Greenwood et al. (2006).

to represent the mean life-cycle wage (for example, the wage of the cohort born in 1871 is indexed on average TFP between 1901 and 1931). For the cohorts born in the 50s and 60s, we forecast future TFP assuming the same growth rate as the one observed in the twentieth century (average annual growth rate of 0.47%.). To isolate the effect of TFP, we keep education shares constant and equal to the levels of the cohort born in 1941. The dotted lines of Figure 11 show that the rise in TFP implies a monotonic decrease in fertility and a monotonic increase in childlessness.

We next combine the two sources of change in a single simulation. The total effect on fertility, represented by the gray line "edu+tech", is now too strong, which indicates that other exogenous elements may also have affected fertility. Interestingly, the effect on childlessness becomes strongly U-shaped over time. This is because the effect of the rise in education for the early cohorts is now evaluated at the wage relevant for each period. At the end of the nineteenth century, wages are low and many households are involuntarily childless. The rise in both education and TFP allow the subsequent generations to escape from that situation and childlessness first drops. Over time, the nature of childlessness changes, becoming more and more voluntary. Ultimately, education increases (voluntary) childlessness. This allows the simulation to capture the slight drop in childlessness over the very long run.

It is particularly interesting to notice that, in the simulation described above, the two driving forces, when taken alone, have either a monotonic increasing effect or a very slight U-shaped effect on childlessness. But once combined, their effect is to bring childlessness down through a non-monotonic path. This highlights the interest of using a structural approach, which has the potential to generate interpretable non-linear effects.

Among the forces that are missing in the above analysis, there is one we can easily incorporate into the model: the fact that part of the non-labor income a is to be indexed on growth. Indeed, a includes household production but also bequests, gifts, welfare transfers, social security etc., which should in the long-run follow TFP developments. The gray line "edu+tech+a" gives the result of a simulation where, in addition to changes in wages and education shares, one third of a is also indexed on TFP. In this simulation, households in the 19th century are less wealthy in terms of non-labor income than in the previous simulation where a was kept constant at the 1941 level. Hence, their fertility is reduced, and (involuntary) childlessness increased, getting both lines closer to data.

In sum, our theory suggests that the U-shaped relationship of childlessness rates over time is caused by a decrease in involuntary childlessness followed by an increase in voluntary childlessness. Figure 12 (left panel) confirms this interpretation, showing the breakdown of childlessness (for married women) into its components. Social causes of childlessness

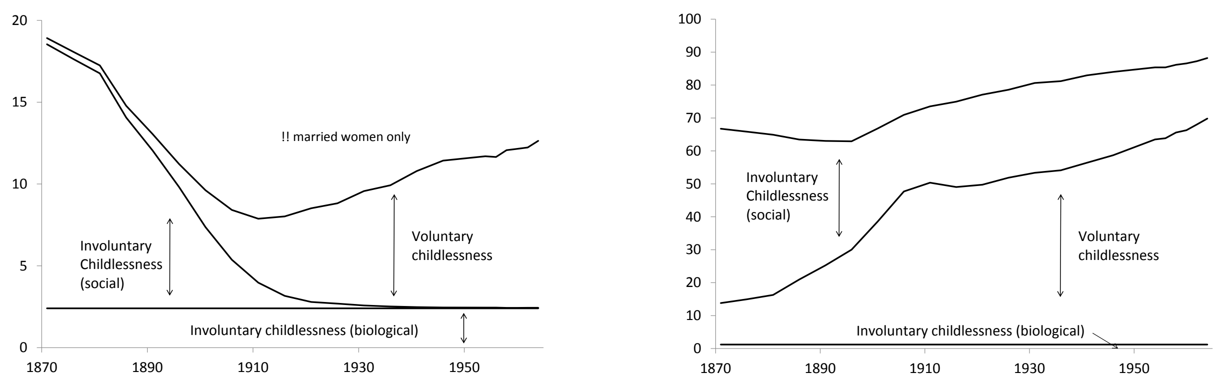


Figure 12: Detail of Simulated Childlessness by Cohort, Married (left) and Singles (right)

have now completely disappeared for married women. This is however not true for singles. Figure 12 (right panel) shows the decomposition of childlessness over time for the singles, and, although voluntary childlessness increases over time, involuntary childlessness remains important.

Historically, there were also changes on the returns to schooling and on the gender wage gap. The effect of these changes (provided in Appendix A.8) on childlessness and fertility are of second order compared to those of education and TFP. The reason is simply that changes in ρ and γ affect choices of the individuals that are in the tails of the distribution of education. As aggregate moments are mainly driven by the middle educated group, the effects of these two parameters are hidden. This is what we will analyze in the next two subsections.

5.2 Increase in Inequality

Since 1980, income inequality and skill premia have been on the rise in the U.S. To assess the effect on family patterns, we implement a rise in inequality through a change in the Mincer parameter ρ of Equation (8). We increase the return of one additional year of education from 0.092 to 0.126 (see Appendix A.8), keeping the wage of the largest category constant (Category 7); the change in the Mincer parameter thus increases the spread around the wage of high school graduates. This corresponds to an increase in the Gini coefficient computed on $w^f + a^f$ from 0.156 to 0.168.

As the lowly educated become poorer, the minimum consumption constraint binds more often and involuntary childlessness rises. For the highly educated, the opportunity cost of having children is increased by the higher skill premium (for both men and women), and more of them choose to remain childless. We then obtain a more pronounced U-shaped pattern for the higher Mincer coefficient (see Figure 13). On the whole, the proportion of socially sterile women (Regimes II and VII) increases from 2.5% to 3.3%. This suggests that economic changes that make uneducated people better-off might have a positive effect on average fertility because they can affect the intensive margin and the extensive margin of fertility in opposite directions.[35]

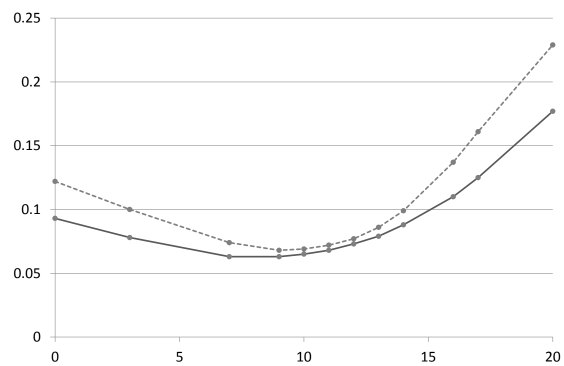


Figure 13: Childlessness Rate for Different Levels of Inequality (Mincer Coefficient) - Married. Grey Line is the Benchmark $\rho = 0.092$ and Dotted Line is the Historical Maximal $\rho = 0.126$

The rise in inequality also affects the marriage rates of the highly educated. As women in education categories 9 to 12 get richer they reject marriage offer more often. Hence, a rise in inequality is another candidate to explain the actual drop in marriage rates. The rise in inequality also increases the slope of the relationship between the fertility of mothers and education.

5.3 Changes in the Gender Wage Gap, γ

Another first order change of the last decades is the closing of the gender wage gap (Goldin (1990) or Jones et al. (2003)). To assess its effect on family patterns, we simulate the model for $\gamma = 0.755$, $\gamma = 0.869$, and $\gamma = 1$, keeping the other parameters fixed. $\gamma = 0.755$ is the lowest value provided in the historical estimates (Appendix A.8).

Figure 14 shows the effect on childlessness for single (left) and for married (right) women. As predicted by Proposition 2, there will be fewer poor women in the social sterility regime, but

[35]Such opposite effects are discussed in a historical case study by Romaniuk (1980) who shows that, during Zaire's modernization, fertility increased, partly due to a large reduction in childlessness rates between the 1930's and the 1960's.

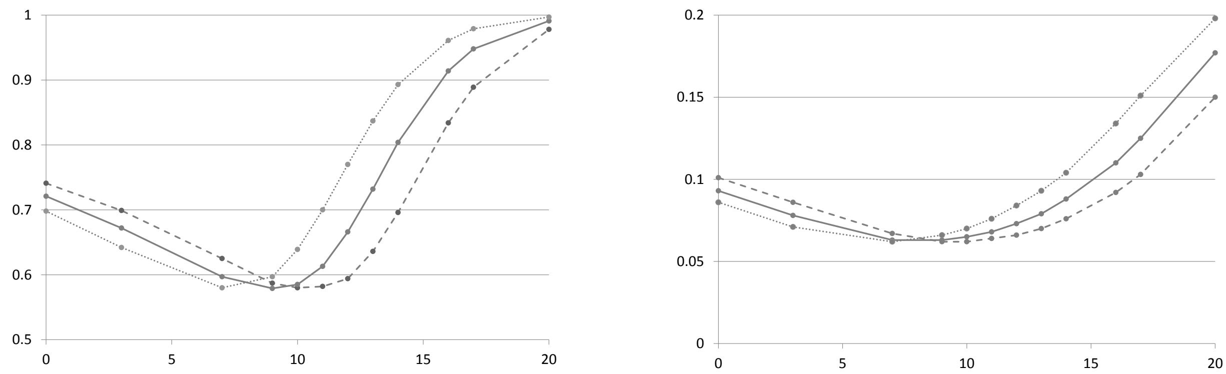


Figure 14: Childlessness Rate: $\gamma = 0.755$ (dashed), $\gamma = 0.869$ (solid), and $\gamma = 1$ (dotted) - Single (left) and Married (right)

educated women are more likely to be voluntarily childless (left panel). The same applies for married women. Closing the gender wage gap is therefore a powerful tool to fight the involuntary component of childlessness.

A change in the wage gap has the usual properties on aggregated fertility of mothers: a decrease in the wage gap decreases fertility since it increases the opportunity cost of rearing children. This is however not true for single women in the corner regime with maximum fertility. For some of these, the effect of a higher relative wage might not be large enough to exit this regime. Hence, for those who stay in this regime, fertility is not affected by the change in the wage gap. To get an idea of the magnitude of the effect of wage w^f on fertility, we compute the elasticity of total fertility to the wage gap for the largest group (education category 7, married women). When the wage gap closes from 0.869 to 1, fertility drops from 3.308 to 3.101, which gives an elasticity of -0.42.

The wage gap also has an effect on marriage rates. Poor women marry more often, while highly educated women marry less often. This effect is particularly large for Ph.Ds: if the wage gap goes from 0.869 to 1, their marriage rate drops from 0.708 to 0.585.

6 Conclusion

To analyze fertility behavior we distinguished explicitly the decision to have children from the choice of the number of children. This distinction turned out to be important, both in terms of data and theory.

Data show the following three facts. First, completed fertility decreases monotonically with education for both single and married mothers. Second, childlessness exhibits a U-shaped relationship with education for both single and married women. Third, there is a hump-shaped relationship between women marriage rates and education levels. These facts are robust for different races and age cohorts.

We have developed a model that allows us to analyze the effect of men's and women's incomes on fertility going beyond the usual distinction between income and substitution effects. Both non-labor income and wages play a complex role, shaping the incentives to marry, and affecting the allocation of resources in the couple.

The main conclusion from the theory is to identify several "regimes" and the conditions under which they prevail. Some of these regimes are new compared to the literature, and turn out to be quantitatively important. Involuntary childlessness can have natural or social causes. Social involuntary childlessness regimes appear for women with low education and low non-labor income, either single or married; we estimate that they account for 5% of American women. In the "eat and procreate" regime, the income of the woman is not high enough for her to be fit to procreate, but it is optimal for her husband to abandon part of his consumption in order to be able to produce children within the couple. This should be highlighted: although aggregate fertility data suggests that Malthusian checks do not prevail any more and Becker's model describes the relationship between education and fertility well, our model detects that some categories of the population are still affected by Malthusian mechanisms. In the voluntary childlessness regime, highly educated women do not have children because of their high opportunity cost.

Our theory also provides a framework to interpret childlessness for both single and married women, allowing for involuntary childlessness for uneducated women and voluntary childlessness for highly educated ones. Simulations show that those regimes are not "empty", and concern a significant fraction of the population. The relatively high percentage of the population in these regimes allows us to understand the U-shaped relationship between education and childlessness highlighted in the stylized facts.

Marriage interacts with childlessness in two ways; for poor women, marriage is an opportunity to get enough resources to be able to have children. Hence, marriage reduces involuntary childlessness. For highly educated women, marriage reduces the opportunity cost of having children, as husbands also help with raising the children; it therefore also reduces voluntary childlessness.

Identifying the structural parameters of the model using a simulated method of moments technique shows that the features of the data on fertility and childlessness are well captured.

On the whole, our model provides a way of understanding the relationship between fertility and education, childlessness and education, and finally, marriage and education all taken together.

To further stress the interest of our methodology which allows to breakdown childlessness into three components, we have run a historical experiment. The result from this exercise is to show that changes in education and in total factor productivity allow to replicate the overall decreasing trend in fertility for the generations born between 1871 and 1964, while childlessness first decreased and then increased. We interpret the changes in childlessness first by a decline in childlessness created by poverty (involuntary) and, after the middle of the twentieth century, by an increase in childlessness created by a rise in the opportunity cost of having children (voluntary).

Voluntary childlessness does not necessarily call for policy, while social sterility is a severe limitation of people's freedom and thus, of particular relevance as far as social welfare is concerned. Fighting the causes of social sterility would allow the set of capabilities of the poor to increase. Unfortunately, looking at the data without a structural model does not allow us to distinguish both, and evaluate their relative importance in a given society. Our structural approach allows to infer from the behavior of the agents the unobserved components of childlessness. Hence the importance of combining theory with data in order to measure the share of childlessness that is related to poverty and/or inequalities. Our analysis suggests that closing the gender wage gap is a powerful tool for limiting the proportion of childlessness that is generated by poverty.

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A Data - For Online Publication

A.1 Cohabitation

When dealing with never-married individuals we might be skeptical about whether these individuals are single or just unmarried with a partner (specially poor women who have many children). In the Census 1990, among never-married women aged 45-70, only 1.8% declared themselves as being with a partner. The percentages for mothers and childless were respectively 3.7% and 1.3%. The percentages vary, however, for different education levels: Table 6 provides the proportion of single mothers, aged 45-70, saying they are with an unmarried partner for each education level. This table shows that very few women who have not married are living with a partner. The highest percentages of cohabitation are seen for women aged 36-40, having achieved Grade 1-4, or with a doctoral degree: respectively 21.5% and 21.7% of never-married women in these education levels claim to be unmarried but have a partner.

Category	%	Category	%
1	3.8	7	3.6
2	4.8	8	2.8
3	4.0	9	3.8
4	3.6	10	3.0
5	3.8	11	2.8
6	4.0	12	8.2

Table 6: Percentage of Single Mothers, Aged 45-70, with an Unmarried Partner

According to Akerlof et al. (1996), more than 65% of white women experiencing an out-of-wedlock birth married before the first birthday of the child in 1975. Bumpass and Westoff (1970) estimate that between 1960-64 and 1985-89, around 80% of white women who gave birth to a first child out-of-wedlock married within the ten years that followed. This percentage is lower for black women (a bit larger than 50%). Without focusing on first births, Bramlett and Mosher (2002) estimate that, in 1995, 42.7% of never married women between ages 40-44 had ever cohabited. Their definition for cohabitation was being unmarried but having a sexual relationship while sharing the same usual address. They also say that the probability of transition to marriage after one year of cohabitation is 30%, after 5 years, 70%, and after 10 years, 84%. Consequently, if cohabitation lasts, marriage is very likely to follow.

Rindfuss and VandenHeuvel (1990) compared cohabitants to both married and single individuals and concluded that cohabitants' attitudes were in many respects closer to those of single than of married people. In particular, cohabitants' fertility expectations are more similar to those of single than married people. Indeed, comparing the percentage of childless individuals who expect children, cohabitants are much closer to never-married than to married respondents (among women, 11% of cohabiting, 4% of single and 40% of married people expect to have a child within two years). Consequently, although cohabitation has many of the characteristics of a marriage (sharing a dwelling unit), cohabitants also share some of the characteristics of single people (fertility expectations). This puts cohabitants in a middle position between single and married people.

A.2 Uneducated Single Mothers

Another question raised by the facts concerns the identity of those uneducated mothers who have many children. Table 7 gives some information. The column $(1-\phi n)w^f + a^f$ reports the total income of the person. Earnings are in column $(1 - \phi n)w^f$. Not surprisingly, earnings decrease with the number of children, probably because hours worked drop as the number of children rises. More interestingly, the column a^f reports the difference between total income and earnings. We observe that fertility increases with a^f, which is a prediction of our model. Now, who are these women? Very few of them report to be with an unmarried partner (see Table 6). Less than half of them are black. A majority are head of their household.

n	nobs.	$(1-\phi n)w^f + a^f$	$(1-\phi n)w^f$	a^f	% with partner	% black	% head of household
1	17,531	7,302.87	5,014.92	2,287.95	6.7%	33.5%	35.0%
2	11,797	7,889.39	5,817.37	2,072.03	9.5%	39.1%	45.3%
3	7,108	6,923.74	4,553.75	2,369.99	11.1%	39.4%	52.5%
4	4,636	6,712.25	3,998.48	2,713.77	6.4%	32.1%	56.8%
5	2,882	6,239.59	3,556.02	2,683.57	5.0%	41.8%	61.1%
6+	5,446	5,844.52	2,576.98	3,267.54	6.1%	36.4%	64.5%

Table 7: Single Mothers with no Education

A.3 NSFG surveys

The National Survey of Family Growth asks women between 15 and 44 years of age questions about their fecundity status and fertility intentions. This allows us to classify each respondent as either voluntarily or involuntarily childless. Looking at the NSFG for the years 1973 and 1976, we have examined in detail 306 childless women between 36 and 44 years old. These women belong to the cohort considered for our stylized facts in Section 2.

Figure 15 shows the relationship between involuntary and voluntary childlessness and the level of education. Notice that the U-shaped relationship between childlessness and education also holds in this dataset.

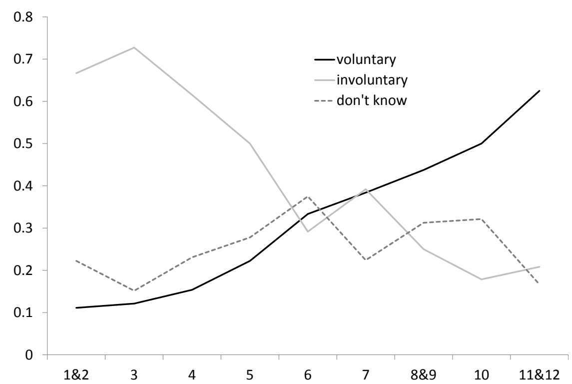


Figure 15: Involuntary and Voluntary Childlessness (in %), by Education Category

We denote a woman as voluntarily childless if: (1) she was voluntarily sterilized for contraceptive reasons, (2) she has always used contraception, (3) she has been pregnant and aborted, (4) she does not intend to have children but does not report any difficulty in becoming pregnant, or (5) she does not intend to have children but does not report any difficulty conceiving or delivering a baby for her or her husband.

We denote a woman as involuntarily childless if she would like to have children and one of the following conditions is met: (1) she has not been using birth controls for at least two years but has not become pregnant, (2) she has never used contraception and has been married for a long period, (3) she reports either a difficulty or the impossibility of having a baby, or (4) she has problems or difficulties to conceiving or delivering a baby for her or her husband (some women have had up to six miscarriages).

We assume that a woman wants a child if she says that (1) she wants to become pregnant as soon as possible, (2) she would like a baby when pregnancy is dangerous or impossible, (3) she plans to adopt if she cannot have a child of her own, (4) she has adopted and does not use contraception because she does not mind getting pregnant, (5) she wanted children before marriage and never used contraception, or (6) she talked with her doctor about increasing her chances of having a baby.

We did not know how to classify some women, either because information was missing or the information was contradictory. Here are some representative examples of these women: (1) she has never used contraception but says that she decided not to have children and has no problem becoming pregnant, (2) she used contraception, reports being able to procreate and is seeking pregnancy or intends to have a child in the next two years (she could become a mother soon), or (3) she or her husband remained sterile not for contraceptive reasons (but, for example, through accident or illness), but were still young enough to procreate.

A.4 Five-year cohorts

The stylized facts highlighted in Section 2 can also be found if we consider each 5-years cohort separately. Figures 16 and 17 show the relationship between childlessness and education and the fertility of mothers and education for married and single women for each cohort. The only major difference between cohorts is in the childlessness rate of single women, with intermediate levels of education (Figure 17): older single women were much less likely to be mothers than younger single women.

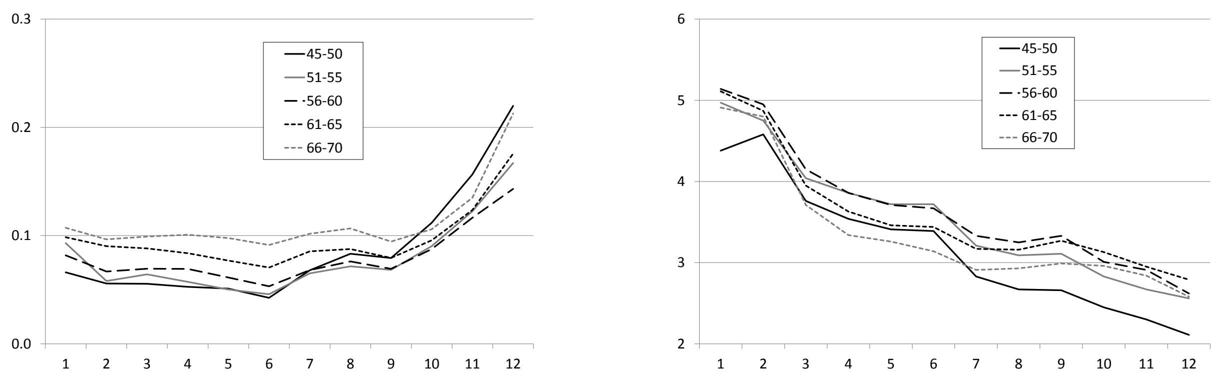


Figure 16: Childlessness Rate and Completed Fertility of Mothers, by Education Category and by Age, Married Women

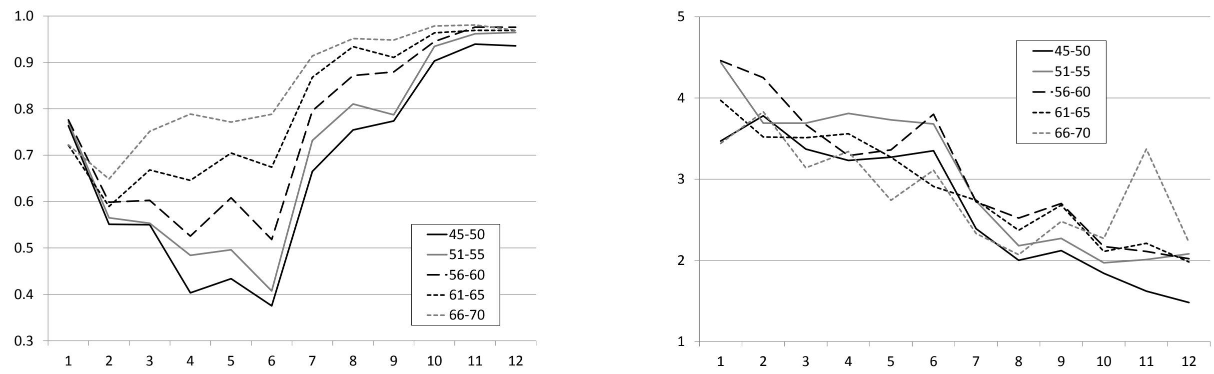


Figure 17: Childlessness Rate and Completed Fertility of Mothers, by Education Category and by Age, Single Women

A.5 Standard errors of the mean

Table 8 reports the standard errors of the mean for the completed fertility and the childlessness rates of both single and married mothers.

Education	Childlessness Rates		Completed Fertility of Mothers		Marriage Rates	
Category	Married	Single	Married	Single	Women	Men
1	0.70	1.61	7.49	22.05	0.941	0.849
2	0.55	2.35	6.30	21.54	0.717	0.524
3	0.23	1.29	2.06	11.58	0.231	0.194
4	0.32	2.27	2.62	16.76	0.297	0.286
5	0.25	2.03	1.98	15.69	0.221	0.246
6	0.25	2.29	2.09	16.26	0.231	0.271
7	0.09	0.60	0.57	06.21	0.074	0.089
8	0.15	0.83	0.86	09.38	0.126	0.114
9	0.27	1.60	1.55	16.15	0.225	0.226
10	0.23	0.62	1.07	15.56	0.199	0.144
11	0.35	0.47	1.41	22.01	0.346	0.175
12	1.47	1.34	5.08	48.75	1.398	0.403

Table 8: Standard Errors ($\times 10^{-3}$) from U.S. Census 1990

A.6 Race, Ethnicity and Place of birth

A.6.1 Race and Ethnicity

In this first part of the appendix, we split the population into five groups, Whites, Blacks, Natives, Asians and Hispanics. We constructed each group from the two variables RACE and HISPAN. In order for an individual to be considered as White, Black, Native or Asian, he or she has to be from that particular race and "not Hispanic". Table 9 gives the number of observations (unweighted) by education category for each group. Figures 18 and 19 show the childlessness rate and completed fertility of mothers, by education category, for married women and single women respectively. Figure 18 shows marriage rates for men and women for each group, by education category.

	Blacks	Whites	Natives	Asians	Hispanics
1	1,264	4,631	274	1,518	4,400
2	1,673	5,276	200	868	6,002
3	8,897	59,603	806	2,397	12,487
4	4,029	29,879	300	832	3,059
5	5,468	47,968	471	680	2,598
6	5,877	40,798	361	338	2,018
7	24,421	429,963	1,985	8,115	15,078
8	8,787	160,406	937	,2988	5,104
9	2,631	46,909	275	1,758	1,838
10	4,092	88,089	262	4,551	2,025
11	3,641	49,819	164	1,941	1,283
12	242	3,974	16	207	172
Total	71,022	967,315	6,051	26,193	56,064

Note: We do not know the race or ethnicity of 435 observations so the sum of the five groups is not equal to the sum of the observations of Table 2.

Table 9: Number of Observations by Education Category and by Race or Ethnicity

For all groups, we find a negative relationship between the fertility of married mothers and education. For single mothers, we have few observations for Asians and Natives so the relationship is not as clear. For other single mothers, the relationship holds for Blacks and Hispanics and is slightly hump-shaped for Whites. The fertility differential between single and married mothers is larger for white women than for others: married mothers have between 1.5 and 2 times more children than single mothers. In average, the differential be-

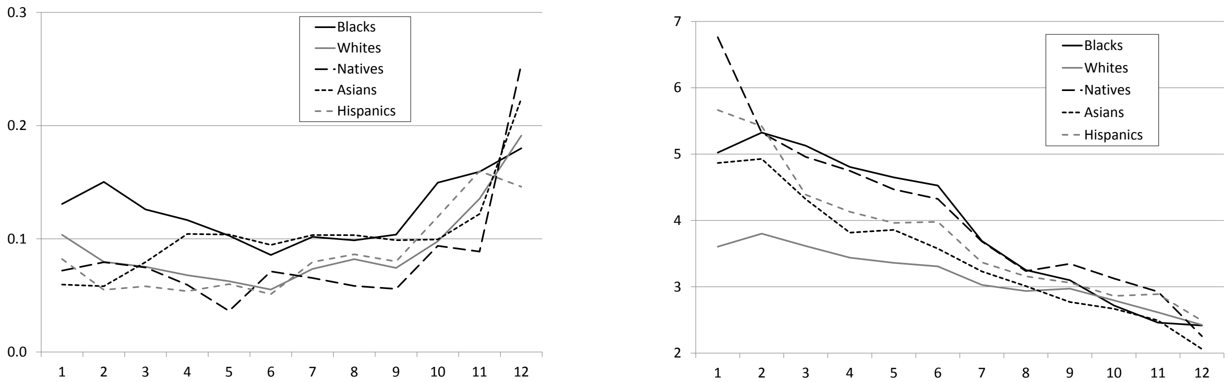


Figure 18: Childlessness Rate and Completed Fertility of Mothers, by Education Category, Married Women

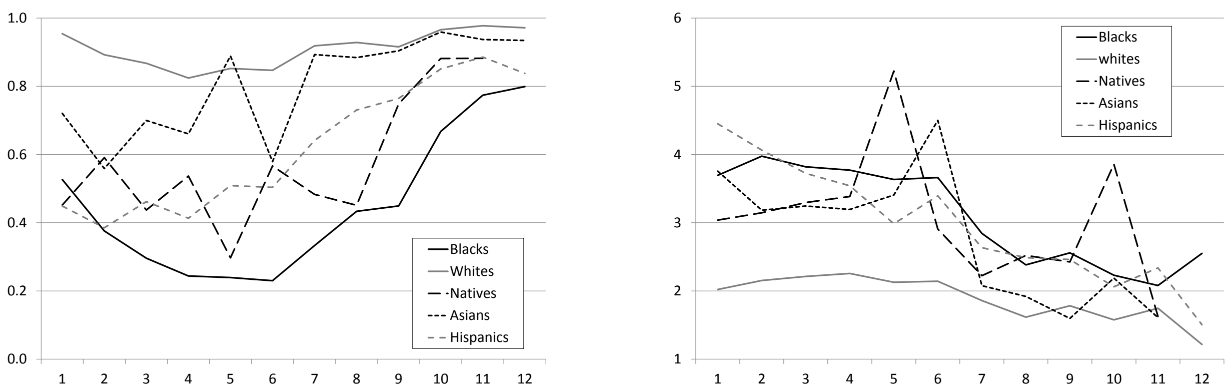


Figure 19: Childlessness Rate and Completed Fertility of Mothers, by Education Category, Single Women

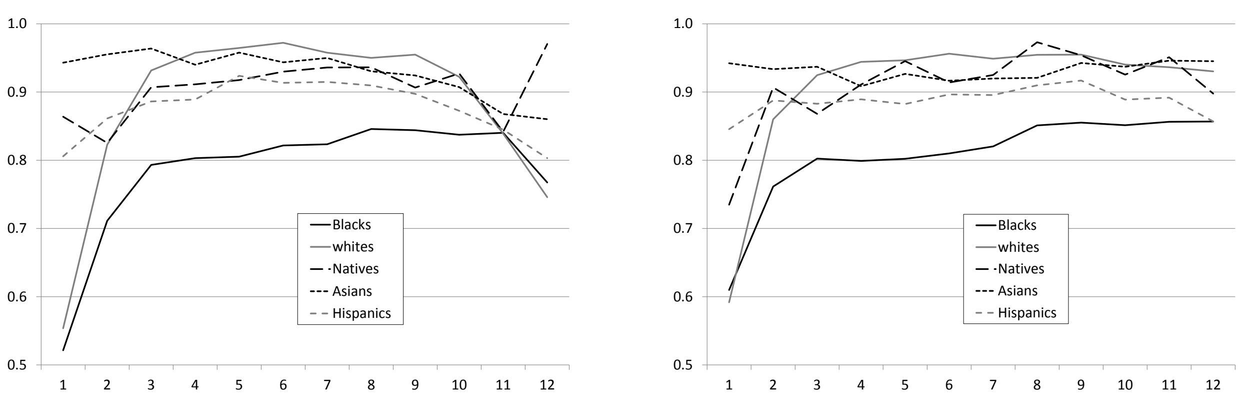


Figure 20: Marriage Rates for Female and Male, by Education Category, Single Women

tween fertility of single and fertility of married women is still of one children: single married women have in average 2.00 children while married women have 3.05. Hispanic married mothers have no more than one child more than single mothers for all education categories. For Blacks, the difference between the fertility of married and single mothers decreases as education increases. Both the U-shaped relationship between childlessness and education and the hump-shaped relationship between marriage rates and education hold in general. In particular for Asian married women, childlessness always increases with education; childlessness of single Hispanic women is flat for the first education levels and then increases. Comparing across groups, we see that Black women are in general more likely to be childless if married, less likely to be childless if single and that the U-shaped relationship (Fact 2) is more pronounced for them than for other groups. Both single and married Black mothers have more children than White mothers. Differential fertility between groups decreases with education.

A.6.2 Place of birth

Some education groups in our sample could comprise large shares of immigrants who may have made their marriage and fertility decisions in another country. It could then be the case that some of our facts do not hold when withdrawing these persons. To assess this, we divide the population into two groups: those who were born in the U.S. and those who were born abroad. The place of birth is obtained thanks to the variable "Birthplace" (BPL) in IPUMS. The percentage of persons born in the U.S. in our sample is 90,96% (so only 9,04% were born abroad). Interestingly, as Table 10 indicates, the main group is not Hispanic (Central and Southern America) but European.

Continent	%
Africa	0.009
Asia	0.220
Central America and Caribbean	0.265
Europe	0.356
Southern America	0.047
Oceania	0.038
Other north America	0.066

Table 10: Distribution of foreign born persons by continent of birth

Removing foreign born individuals from the sample results in the moments shown in Table 11. The facts highlighted in Section 2 are robust to the deletion of these observations. In

		Childlessness Rates		Completed Fertility of Mothers		Marriage Rates	
Category	Educ shares. in %	Married	Single	Married	Single	Women	Men
1	0.63	0.106	0.837	4.727	3.566	0.533	0.591
2	0.78	0.089	0.648	4.886	3.915	0.785	0.838
3	6.75	0.078	0.655	3.977	3.517	0.907	0.906
4	3.39	0.070	0.570	3.670	3.435	0.934	0.926
5	5.25	0.064	0.587	3.538	3.364	0.945	0.930
6	4.64	0.058	0.523	3.500	3.447	0.949	0.936
7	43.53	0.073	0.788	3.090	2.537	0.948	0.938
8	16.19	0.081	0.842	2.974	2.129	0.944	0.948
9	4.64	0.072	0.831	3.007	2.244	0.947	0.948
10	8.79	0.100	0.936	2.806	1.939	0.917	0.935
11	5.04	0.137	0.961	2.619	1.924	0.837	0.932
12	0.37	0.192	0.964	2.429	1.693	0.739	0.925

Note: Weights are used to compute the moments. Education shares are unweighted.

Table 11: Facts from U.S. Census 1990 without foreign born persons

particular, we see that the relationship between childlessness rates and education for single women is even more U-shaped than in the benchmark case which includes foreign born persons.

Table 12 shows the facts for the individuals that were born abroad only. We can observe that the marriage rate of males is no longer hump-shaped but rather high and flat. Fertility of mothers still decreases with education. We also find that childlessness rates are monotonously increasing with education. We suspect that, this absence of U-shaped relationship between childlessness and education among foreign born persons, is due to a selection effect. Indeed, the persons who are included in the Census are those who succeeded in emigrating to the U.S.. They are probably in better health and have higher non-labor income than the persons without education born in the U.S.. Then, they are less concerned with involuntary childlessness.

A.7 Disability

The U-shaped relationship between childlessness and education is highly affected when we consider the data without individuals who have any lasting physical or mental health condi-

Category	Educ shares in %	Childlessness Rates		Completed Fertility of Mothers		Marriage Rates	
		Married	Single	Married	Single	Women	Men
1	5.6	0.077	0.430	4.976	4.270	0.875	0.912
2	5.9	0.058	0.441	4.693	3.643	0.898	0.933
3	14.9	0.078	0.517	3.670	3.356	0.919	0.935
4	3.3	0.094	0.479	3.420	3.309	0.925	0.927
5	3.4	0.095	0.546	3.126	3.095	0.933	0.926
6	1.8	0.105	0.698	2.949	2.640	0.942	0.937
7	32.7	0.116	0.803	2.796	2.093	0.928	0.939
8	11.9	0.111	0.795	2.724	2.318	0.932	0.950
9	5.8	0.110	0.917	2.617	1.979	0.909	0.944
10	8.9	0.100	0.936	2.806	1.939	0.917	0.935
11	5.1	0.140	0.931	2.487	1.822	0.870	0.943
12	0.8	0.186	0.905	2.317	1.892	0.836	0.940

Note: Weights are used to compute the facts. Education shares are unweighted.

Table 12: Facts from U.S. Census 1990 including only foreign born persons - Total number of observations (unweighted) 97135

tion that prevents or causes difficulty working, living independently or taking care of their own personal needs (respectively, variables DISABWRK, DIFFMOB and DIFFCARE of IPUMS). Note that these variables say nothing about the ability to reproduce.[36] For married women the relationship between childlessness and education is not affected, but the relationship for single women becomes flat for the first education levels and only increases from Grade 11 on.

We cannot distinguish in the 1990 Census those who are born with a disability from those who have acquired one later in life. Among the first group of people, are those who were born disable, did not receive education because their handicap, and then did neither married, nor had children. This group biases our estimates of involuntary childlessness upwards. However, this bias is expected to be small, as the persons born with a severe handicap in 1920-45 had a low chance to survive to 45 years and beyond and are therefore absent from our sample. For example, a person affected by the Down Syndrome, which is one of the most common disabilities, life expectancy was below 18 in the thirties (while it is nowadays around 50 years). In the second group, we have those who became disabled because of poverty. Either

[36] See `http://www.census.gov/hhes/www/disability/sipp/disab02/ds02t2.html` for information about disability types and their proportions in the U.S.

because they worked or lived in an unhealthy or dangerous environment or because they had a bad medical treatment, or none, and remained single and childless. These two factors are likely to affect their health aged 45-70. This leads to an endogeneity problem in the relationship between disability and education: adults who lived in the worst conditions (the ones with the lowest education) are the most likely to suffer health problems when older. In other words, the lowly educated are more likely to have health problems when old than the highly educated. In this group, disabled women are, *de facto*, lowly educated and the constraint $c^f > \hat{c}$ reflects their incapacity to have children when they do not have a husband investing in them. Our position is that we should keep this group in the sample.

A.8 Historical Data

In the historical exercise we also use Census data, taken from IPUMS. Table 13 lists the 22 cohorts that were considered as well as the Census from which they were constructed. As the fertility of single women was not asked before 1970, the data is only for ever-married women.

Two remarks before analyzing completed fertility over time should be noticed. First, there was a change on how the number of children ever born was reported. First, for the 1940 Census, there was only one case for the number of children that a woman had had while in later Censuses a case for "none" was added to the questionnaire - see Figure 21. This could then lead many childless women in cohorts 1971-1900 to be counted as not respondents instead of as childless and hence explain partly the increase in childlessness rates for the following cohorts. This was already pointed out in Grabill and Glick (1959).

A second aspect concerns the high childlessness rates for the cohorts of women born at the beginning of last century. These women have been touched by the 1918 Spanish Influenza. Pregnant women were particularly vulnerable to the Spanish Influenza. If they were affected by the flu, they had high chances to either die or have a miscarriage (Bloom-Feshbach et al. (2011)).

Morgan (1991) also shows childlessness data using vital registration estimates. His estimates for the cohorts that were born at the beginning of the 1900s are lower than the average rates from the Census (see Figure 1 in Morgan (1991). Our estimates of childlessness rates are close to those of Figure A2 in Jones and Tertilt (2008).

Cohort	Age	Census	Completed Fertility of Mothers	Childlessness Rate
1871-1875	65-69	1940	4.228	0.164
1876-1880	60-64	1940	3.998	0.171
1881-1885	55-59	1940	3.915	0.169
1886-1890	50-54	1940	3.758	0.158
1891-1895	45-49	1940	3.585	0.150
1896-1900	40-44	1940	3.353	0.156
1901-1905	45-49	1950	3.202	0.189
1906-1910	40-44	1950	2.885	0.203
1911-1915	45-49	1960	2.917	0.176
1916-1920	40-44	1960	2.987	0.132
1921-1925	45-49	1970	3.152	0.100
1926-1930	40-44	1970	3.372	0.081
1931-1935	45-49	1980	3.448	0.070
1936-1940	40-44	1980	3.237	0.066
1941-1945	45-49	1990	2.786	0.082
1946-1950	40-44	1990	2.471	0.102
1954-1958	40-44	1998	2.320	0.137
1956-1960	40-44	2000	2.367	0.134
1958-1962	40-44	2002	2.359	0.120
1960-1964	40-44	2004	2.357	0.132
1962-1966	40-44	2006	2.345	0.146
1964-1968	40-44	2008	2.297	0.120

Note: Data for cohorts born after 1954 are taken from Tables available at the Census webpage.

Table 13: Cohorts for the Historical Exercise and Completed Fertility of Mothers and Childlessness Rates for Married Women of Each Cohort

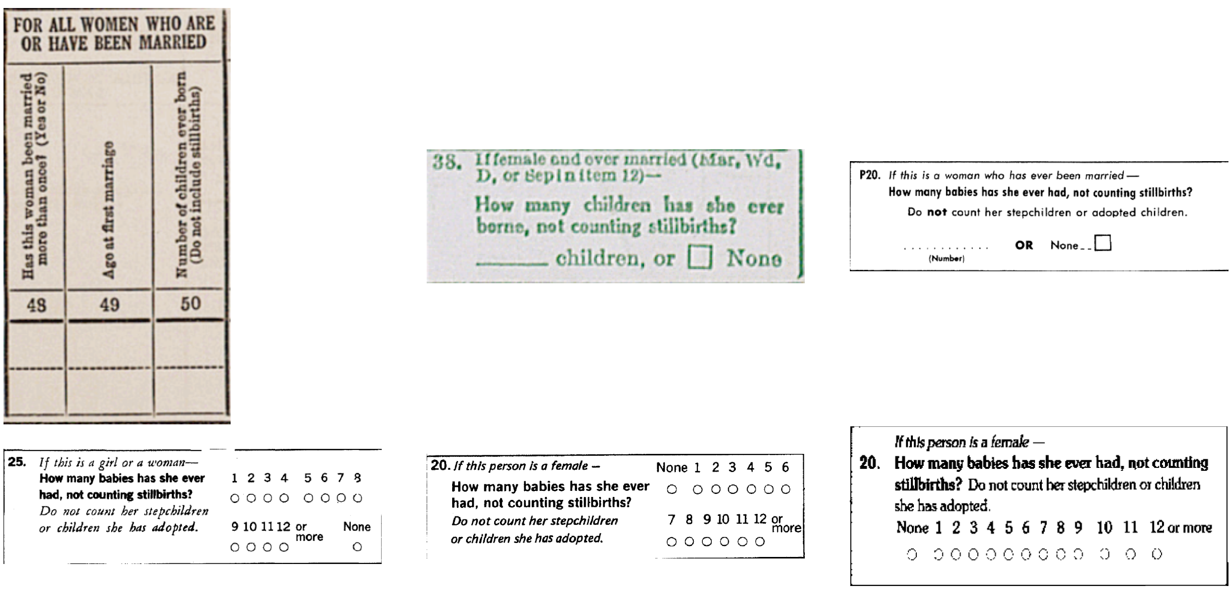


Figure 21: Question on Children Ever Born in 1940, 50, 60, 70, 80 and 90 censuses

A.8.1 Changes in Education

To compute changes in education, we divided the sample of each cohort into the 12 categories of education. For cohorts generated from Censuses before 1900, we cannot distinguish between categories 11 and 12 so they were merged. The codes (from the variable EDUCD) that were used for each Census are provided in Table 14. Results are provided in Tables 15 and 16.

	Education Level	Census			
		1980-1940	1990	2000-2006	2008
1	No school	2	2	2	2
2	Grade 1-4	14-17	13	10	14-17
3	Grade 5-8	22, 23, 25, 26	20	21, 24	22, 23, 25, 26
4	Grade 9	30	30	30	30
5	Grade 10	40	40	40	40
6	Grade 11	50	50	50	50
7	Grade 12	60	61, 62	61, 62	61, 63, 64
8	1 year of college	65, 70	71	65, 71	65, 71
9	2 years of college	80	82-83	81	81
10	Bachelor degree	90, 100	101	101	101
11	Master degree	110-113	114, 115	114, 115	114, 115
12	doctoral degree		116	116	116

Note: Data for years 2010-2008 is taken from the American Community Survey (ACS). As Census 1998 is not available in IPUMS, we used Census 2000 for the education shares of cohort 1958-1958.

Table 14: Codes for Education Shares for Each Cohort (from EDUCD variable)

Cohort	Age	Census	1	2	3	4	5	6	7	8	9	10	11	12
1871-1875	65-69	1940	0.062	0.120	0.559	0.036	0.038	0.017	0.097	0.013	0.022	0.030	0.005	
1876-1880	60-64	1940	0.054	0.112	0.539	0.044	0.044	0.022	0.111	0.016	0.019	0.032	0.006	
1881-1885	55-59	1940	0.050	0.108	0.530	0.053	0.049	0.021	0.116	0.014	0.021	0.032	0.007	
1886-1890	50-54	1940	0.048	0.106	0.512	0.055	0.051	0.024	0.123	0.016	0.024	0.034	0.008	
1891-1895	45-49	1940	0.041	0.093	0.490	0.057	0.061	0.027	0.134	0.019	0.029	0.039	0.008	
1890-1900	40-44	1940	0.026	0.079	0.472	0.061	0.069	0.032	0.155	0.021	0.033	0.043	0.009	
1901-1905	45-49	1950	0.013	0.070	0.406	0.061	0.072	0.041	0.187	0.036	0.040	0.055	0.019	
1906-1910	40-44	1950	0.008	0.050	0.347	0.065	0.081	0.053	0.230	0.039	0.040	0.064	0.021	
1911-1915	45-49	1960	0.009	0.033	0.306	0.073	0.086	0.056	0.271	0.044	0.038	0.061	0.021	
1916-1920	40-44	1960	0.006	0.027	0.233	0.069	0.089	0.063	0.347	0.047	0.037	0.063	0.019	
1921-1925	45-49	1970	0.007	0.021	0.162	0.058	0.079	0.080	0.406	0.053	0.042	0.065	0.026	
1926-1930	40-44	1970	0.007	0.016	0.132	0.059	0.083	0.075	0.414	0.058	0.046	0.080	0.029	
1931-1935	45-49	1980	0.007	0.015	0.097	0.046	0.066	0.058	0.435	0.075	0.057	0.094	0.049	
1936-1940	40-44	1980	0.006	0.012	0.075	0.044	0.061	0.053	0.437	0.086	0.061	0.108	0.059	
1941-1945	45-49	1990	0.009	0.008	0.040	0.024	0.037	0.034	0.388	0.191	0.066	0.121	0.075	0.007
1946-1946	40-44	1990	0.008	0.007	0.030	0.018	0.025	0.024	0.344	0.212	0.079	0.154	0.092	0.007
1954-1958	42-46	2000	0.011	0.005	0.026	0.014	0.018	0.018	0.305	0.226	0.091	0.184	0.094	0.007
1956-1960	40-44	2000	0.011	0.005	0.026	0.014	0.018	0.019	0.305	0.228	0.093	0.188	0.086	0.007
1958-1962	40-44	2002	0.006	0.007	0.025	0.013	0.018	0.020	0.311	0.076	0.232	0.197	0.086	0.008
1960-1964	40-44	2004	0.006	0.006	0.028	0.013	0.019	0.021	0.318	0.154	0.106	0.224	0.097	0.008
1962-1966	40-44	2006	0.007	0.006	0.029	0.016	0.018	0.019	0.307	0.148	0.110	0.228	0.103	0.009
1964-1968	40-44	2008	0.010	0.006	0.029	0.016	0.017	0.020	0.252	0.168	0.110	0.243	0.116	0.011

Note: Education categories 11 and 12 were merged before 1970.

Table 15: Proportion of Women in Each Education Category

Cohort	Age	Census	1	2	3	4	5	6	7	8	9	10	11	12
1871-1875	65-69	1940	0.078	0.163	0.550	0.029	0.030	0.012	0.064	0.013	0.016	0.033	0.014	
1876-1880	60-64	1940	0.063	0.152	0.545	0.033	0.036	0.013	0.079	0.013	0.016	0.033	0.017	
1881-1885	55-59	1940	0.059	0.145	0.539	0.040	0.041	0.018	0.080	0.013	0.016	0.036	0.015	
1886-1890	50-54	1940	0.054	0.136	0.526	0.043	0.048	0.019	0.085	0.014	0.019	0.041	0.015	
1891-1895	45-49	1940	0.041	0.118	0.515	0.051	0.055	0.023	0.100	0.015	0.022	0.042	0.017	
1890-1900	40-44	1940	0.027	0.097	0.499	0.056	0.066	0.030	0.114	0.018	0.024	0.048	0.019	
1901-1905	45-49	1950	0.014	0.086	0.445	0.061	0.070	0.043	0.135	0.027	0.028	0.059	0.031	
1906-1910	40-44	1950	0.014	0.086	0.445	0.061	0.070	0.043	0.135	0.027	0.028	0.059	0.031	
1911-1915	45-49	1960	0.012	0.071	0.378	0.063	0.081	0.054	0.172	0.033	0.036	0.063	0.038	
1916-1920	40-44	1960	0.008	0.043	0.256	0.067	0.082	0.065	0.269	0.048	0.042	0.071	0.049	
1921-1925	45-49	1970	0.007	0.031	0.186	0.059	0.078	0.070	0.294	0.052	0.054	0.096	0.072	
1926-1930	40-44	1970	0.007	0.026	0.171	0.064	0.082	0.062	0.289	0.054	0.053	0.109	0.084	
1931-1935	45-49	1980	0.006	0.023	0.125	0.043	0.053	0.044	0.326	0.064	0.070	0.128	0.118	
1936-1940	40-44	1980	0.006	0.015	0.093	0.042	0.050	0.041	0.343	0.075	0.076	0.129	0.130	
1941-1945	45-49	1990	0.010	0.010	0.050	0.026	0.034	0.028	0.301	0.192	0.058	0.150	0.117	0.024
1946-1946	40-44	1990	0.010	0.008	0.035	0.018	0.024	0.021	0.260	0.214	0.078	0.191	0.125	0.018
1954-1958	42-46	2000	0.012	0.005	0.030	0.016	0.021	0.022	0.306	0.219	0.072	0.179	0.102	0.015
1956-1960	40-44	2000	0.012	0.005	0.030	0.017	0.022	0.024	0.318	0.216	0.071	0.177	0.095	0.013
1958-1962	40-44	2002	0.007	0.008	0.030	0.016	0.022	0.025	0.332	0.062	0.204	0.189	0.092	0.014
1960-1964	40-44	2004	0.007	0.006	0.033	0.016	0.023	0.027	0.337	0.142	0.082	0.209	0.101	0.017
1962-1966	40-44	2006	0.009	0.008	0.036	0.019	0.023	0.025	0.344	0.134	0.081	0.202	0.102	0.016
1964-1968	40-44	2008	0.012	0.008	0.036	0.019	0.022	0.027	0.286	0.159	0.082	0.220	0.110	0.017

Note: Education categories 11 and 12 were merged before 1970.

Table 16: Proportion of Men in Each Education Category

A.8.2 Changes in Wages

For each cohort we run the following linear regression for single individuals with a positive income wage (INCWAGE variable in the Census):

$$\ln(\text{INCWAGE}) = \text{constant} + \ln(\gamma)i + \rho e \quad (9)$$

where $i = 0$ if the individual is a man and $i = 1$ if she is a woman. e are the years of schooling that we give to each category of education (Table 2). The estimated parameters $\ln(\gamma)$ and ρ denote the natural logarithm of the gender wage gap and the mincer coefficient respectively. Table 17 provides the estimated values for γ and ρ with the respective standard errors (SE) for each cohort. The values of the estimates that we use in Section 4.1 are those of the cohort 1941-1945 (in bold).

Cohort	γ	SE	ρ	SE	Cohort	γ	SE	ρ	SE
1871-1875	0.988	(0.0076)	0.092	(0.0009)	1926-1930	0.755	(0.0020)	0.073	(0.0003)
1876-1880	0.928	(0.0053)	0.093	(0.0007)	1931-1935	0.816	(0.0025)	0.080	(0.0004)
1881-1885	0.927	(0.0041)	0.085	(0.0005)	1936-1940	0.787	(0.0022)	0.086	(0.0003)
1886-1890	0.891	(0.0033)	0.083	(0.0004)	1941-1945	**0.869**	(0.0019)	**0.092**	(0.0003)
1891-1895	0.859	(0.0028)	0.072	(0.0004)	1946-1950	0.904	(0.0013)	0.094	(0.0002)
1896-1900	0.805	(0.0023)	0.090	(0.0003)	1954-1958	0.893	(0.0011)	0.111	(0.0002)
1901-1905	0.787	(0.0025)	0.057	(0.0003)	1956-1960	0.877	(0.0015)	0.112	(0.0003)
1906-1910	0.843	(0.0021)	0.046	(0.0003)	1958-1962	0.843	(0.0010)	0.119	(0.0002)
1911-1915	0.767	(0.0023)	0.076	(0.0003)	1960-1964	0.841	(0.0011)	0.126	(0.0002)
1916-1920	0.772	(0.0019)	0.073	(0.0003)	1962-1966	0.848	(0.0009)	0.125	(0.0002)
1921-1925	0.789	(0.0022)	0.066	(0.0003)	1964-1968	0.836	(0.0009)	0.112	(0.0002)

Note:As Census 1998 is not available in IPUMS, we used Census 2000 to construct cohort 1954-1958.

Table 17: Estimates for the Gender Wage Gap and Returns to Schooling for Cohorts born between 1971-1968

A.9 Returns to schooling

Individuals in the extreme education levels are relatively few. This could imply that Mincer equations linking economic returns to schooling works well for the middle of the distribution of wages, but is less adequate for the tails of the distribution. As the tails are important to reproduce the U-shaped childlessness pattern, we looked at this question in detail.

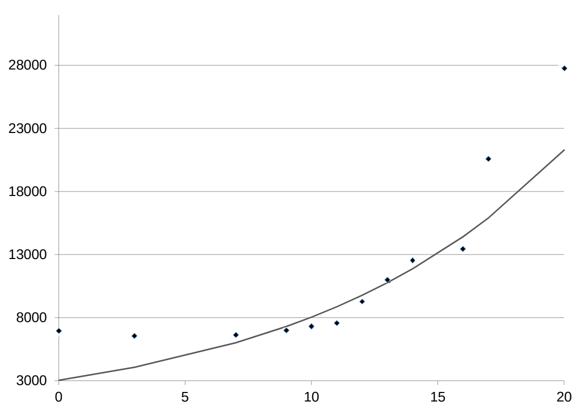
We estimate a Mincer equation by regressing the natural logarithm of the variable "wage and salary income (INCWAGE)" on a constant plus the years of education for our sample of women who are still working (UHRSWORK > 0). Results are provided in Table 18. We see that the regression coefficients are significant at the 1% level and close to 0.1.

	Married		Single		All
Dependent Variable	All	Childless	All	Childless	
Years of Education	0.097	0.113	0.111	0.109	0.100
	(0.000137)	(0.000451)	(0.000310)	(0.000365)	(0.000126)
Constant	8.019	7.961	8.157	8.210	8.005

Note: The dependent variable is the logarithm of the variable "wage and salary income". We only show results for women in our sample for whom the variable "usual hours worked per week" was positive.

Table 18: Univariate Estimates of the Returns to Schooling

Figure 22 plots the estimates of the variable "wage and salary income" together with the mean observations for different years of education, for married and single women respectively (childless and mothers). In both cases, the fit for highly educated women is good, so we do not expect biases for the right tail of the wage distribution. For low skilled women results are mixed. For married women, the exponential function clearly underestimate the earnings of the low education women. For singles, however, the fit remains good.



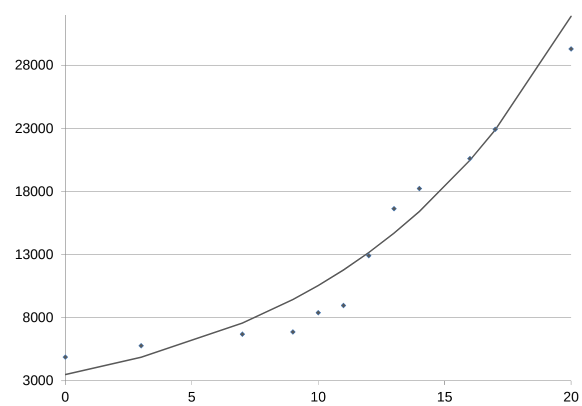


Figure 22: Earnings as a Function of Education. Married (left), Single (right)

As most of the involuntary childless women are single women, our result on the estimate of involuntary childlessness is robust to alternative specifications of the wage function that would track the observed wages even better. For married women, on the contrary, our specification is too pessimistic concerning their income, and we may overestimate the proportions of the women in the involuntary childless regimes, and eat and procreate regimes.

As fertility decisions affect the number of hours worked and consequently, earnings, we remove this endogeneity by reestimating the Mincer equation on a subsample, including only the childless women. Results are presented in Figure 23. We find the same result as above, that the exponential relationship between years of schooling and wages underestimates the wages for low educated married women, but does fine for single women.

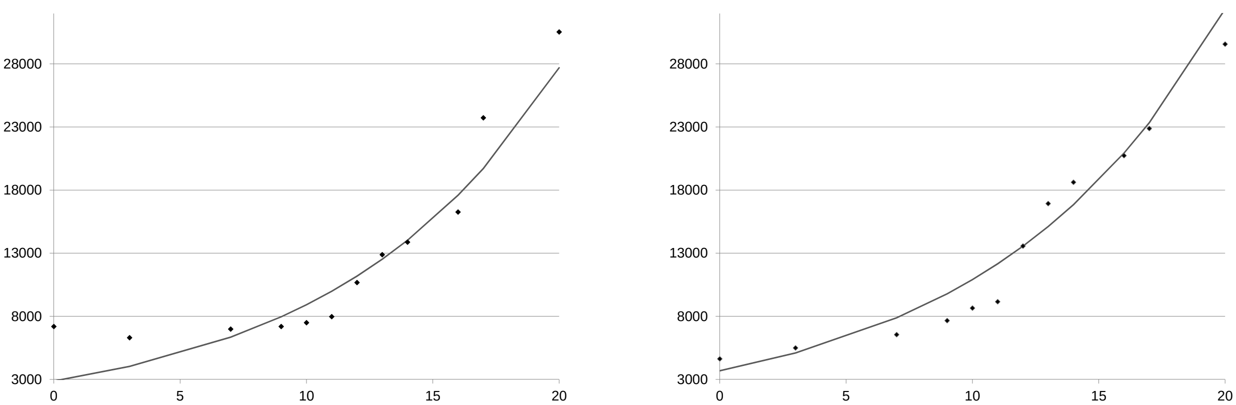


Figure 23: Earnings as a Function of Education among Childless. Married (left), Single (right)

B Theory

B.1 Optimal Decisions of Single Women

In this Appendix, we first provide definitions for wage and non-labor income thresholds. These will be useful to establish a complete description of the choices made by single women as a function of their wage and their non-labor income. We then provide Proposition 1 and prove it. Before turning to definitions, we impose the following assumptions on the parameters, they ensures that the problem is not degenerated in the sense that only corner solutions prevail:

Assumption 1

$$1-\delta^f > \phi(\nu-\eta)(1-\hat{c}), \quad \eta \geq \nu + \frac{\sqrt{1-\delta^f}(\sqrt{1-\delta^f}-2\sqrt{\phi\nu})}{\phi}$$

Definition 1 (Wage thresholds for singles)

$$\mathcal{W}_0^f(a^f) = \frac{\hat{c}+\mu-a^f}{1-\phi\eta-\delta^f}, \quad \mathcal{W}_2^f(a^f) = \frac{2\hat{c}+\mu-a^f}{1-\delta^f+\phi(\nu-\eta)},$$

$$\mathcal{W}_3^f(a^f) = \frac{a^f-\mu}{1-\delta^f+\phi(\nu-\eta)}, \quad \mathcal{W}_5^f(a^f) = \frac{a^f-\mu}{\phi(\nu+\eta)+\delta^f-1},$$

$\mathcal{W}_1^f(a^f)$ is the smallest root, in w^f, of the quadratic equation $u(\hat{c}, n_{III}) = u(c_{IV}, 0)$:

$$\left((1-\delta^f)w^f + a^f - \mu\right)\nu = \hat{c}\left(\frac{w^f(1-\delta^f-\phi\eta)+a^f-\mu-\hat{c}}{\phi w^f}+\nu\right), \tag{10}$$

$\mathcal{W}_4^f(a^f)$ is the highest root of the quadratic equation in w^f:

$$\frac{\left(w^f(1-\delta^f+\phi(\nu-\eta))+a^f-\mu\right)^2}{4\phi w^f} = \nu((1-\delta^f)w^f+a^f-\mu).$$

$\mathcal{W}_0^f(a^f)$ is the value of w^f above which a single woman can have children and consume at least $\hat{c}$. $\mathcal{W}_2^f(a^f)$ is the value of w^f that solves $n_{\text{III}} = n_{\text{I}}$. $\mathcal{W}_3^f(a^f)$ is the value of w^f such that $n_{\text{I}} = \underline{n}_{\text{M}}$. $\mathcal{W}_4^f(a^f)$ is either the lowest or the unique value of w^f such that $u(c_{\text{IV}}^f, 0) = u(c_{\text{I}}^f, n_{\text{I}})$. $\mathcal{W}_5^f(a^f)$ is the value of w^f that solves $n_{\text{I}} = 0$.

Definition 2 (Non-labor income thresholds for singles)

$$\underline{a} = \hat{c}\left(\frac{\phi(\nu+\eta) - 1 + \delta^f}{\phi\nu}\right) + \mu \quad , \quad \overline{a} = \hat{c} + \mu > \underline{a}$$

$\underline{a}$ is the value of a under which a single woman is always childless. $\overline{a}$ is the value of a^f such that $\hat{c} = c^f_{\text{V}}$.

Proposition 1 (Fertility of singles) *Under Assumption 1, the optimal choice of a single woman with non-labor income a^f and wage w^f is given by:*

1. *when $a^f < \underline{a}$:*
   - $\forall w^f < \mathcal{W}^f_0(a^f)$, $c^f = c^f_{II}$, $n = 0$,
   - $\forall w^f \geq \mathcal{W}^f_0(a^f)$, $c^f = c^f_{IV}$, $n = 0$.
2. *when $a^f \in [\underline{a}, \overline{a}[$:*
   - $\forall w^f < \mathcal{W}^f_0(a^f)$, $c^f = c^f_{II}$, $n = 0$,
   - $\forall w^f \in [\mathcal{W}^f_0(a^f), \mathcal{W}^f_1(a^f)[$, $c^f = c^f_{IV}$, $n = 0$,
   - $\forall w^f \in [\mathcal{W}^f_1(a^f), \mathcal{W}^f_2(a^f)[$, $c^f = \hat{c}$, $n = n_{III}$,
   - $\forall w^f \in [\mathcal{W}^f_2(a^f), \mathcal{W}^f_4(a^f)]$, $c^f = c^f_I$, $n = n_I$,
   - $\forall w^f > \mathcal{W}^f_4(a^f)$, $c^f = c^f_{IV}$, $n = 0$.
3. *when $a^f \geq \overline{a}$:*
   - $\forall w^f < \mathcal{W}^f_3(a^f)$, $c^f = c^f_V$, $n = \underline{n}_M$,
   - $\forall w^f \in [\mathcal{W}^f_3(a^f), \mathcal{W}^f_4(a^f)]$, $c^f = c^f_I$, $n = n_I$,
   - $\forall w^f > \mathcal{W}^f_4(a^f)$, $c^f = c^f_{IV}$, $n = 0$.

Proof. To prove Proposition 1, we need to show that, under Assumption 1:

1. when $a^f < \underline{a}$, $\mathcal{W}^f_5(a^f) < \mathcal{W}^f_2(a^f) < \mathcal{W}^f_0(a^f)$ and $\mathcal{W}^f_0(a^f)$ is greater than the highest root of Equation (10);
2. when $a^f \in [\underline{a}, \overline{a}[$, $\mathcal{W}^f_0(a^f) < \mathcal{W}^f_1(a^f) < \mathcal{W}^f_2(a^f) < \mathcal{W}^f_4(a^f) < \mathcal{W}^f_5(a^f)$;
3. when $a^f \geq \overline{a}$, $\mathcal{W}^f_3(a^f) < \mathcal{W}^f_4(a^f) < \mathcal{W}^f_5(a^f)$.

Proof and Implications of 1.

Straightforward computations indicate that when $a^f < \underline{a}$, $\mathcal{W}_5^f(a^f) < \mathcal{W}_2^f(a^f) < \mathcal{W}_0^f(a^f)$. When $w^f \leq \mathcal{W}_0^f(a^f)$, becoming a mother is not feasible either in the interior regime or in the get fit and procreate one.

Once $w^f > \mathcal{W}_0^f(a^f)$, a single woman can have children in Regime III. Comparing utility in Regime IV with utility in Regime III, $u(c^f_{\text{IV}}, 0) \leq u(\hat{c}, n_{\text{III}})$ if and only if,

$$\left((1-\delta^f)w^f + a^f - \mu\right)\nu \leq \hat{c}\left(\frac{w^f(1-\delta^f-\phi\eta) + a^f - \mu - \hat{c}}{\phi w^f} + \nu\right). \tag{11}$$

The left-hand side (hereafter LHS) is linear and increasing in w^f and the right-hand side (hereafter RHS) is increasing and concave in w^f (since $a^f - \mu < \hat{c}$), so Equation (11) holding with equality (i.e. Equation (10)) has at most two solutions. At $\mathcal{W}_0^f(a^f)$, the LHS of Equation (11) is higher than the RHS, implying that $u(c^f_{\text{IV}}, 0) > u(\hat{c}, n_{\text{III}})$. This implies that $\mathcal{W}_0^f(a^f)$ is either smaller than the lowest root or greater than the highest root of Equation (10). Under Assumption 1, at $\mathcal{W}_2^f(a^f)$, $u(\hat{c}, n_{\text{III}}) \geq u(c^f_{\text{IV}}, 0)$ when $a < \overline{a}$, meaning that $\mathcal{W}_2^f(a^f)$ is in between the roots of Equation (10). As $\mathcal{W}_0^f(a^f) > \mathcal{W}_2^f(a^f)$, $\mathcal{W}_0^f(a^f)$ is above the highest root of Equation (10). We can then conclude that once the woman can have children in the interior regime, choosing to live in this regime is not optimal (so we do not have to compare other indirect utilities to $u(c^f_{\text{I}}, n_{\text{I}})$).

This means that when $a^f < \underline{a}$, women are either involuntarily childless, when $w^f \leq \mathcal{W}_0^f(a^f)$, or voluntarily childless, when $w^f > \mathcal{W}_0^f(a^f)$: once women become able to procreate ($w^f > \mathcal{W}_0^f(a^f)$), becoming a mother is not optimal as the opportunity cost of children is too high.

Proof and Implications of 2.

- First, we show that, for $a^f \in [\underline{a}, \overline{a}[$, $\mathcal{W}_0^f(a^f) < \mathcal{W}_1^f(a^f) < \mathcal{W}_2^f(a^f)$.

  Considering Equation (11), when $w^f = \mathcal{W}_2^f(a^f)$, we have

$$\text{LHS}(\mathcal{W}_2^f(a^f)) = \frac{2(1-\delta^f)\hat{c} + (a^f-\mu)\phi(\nu-\eta)}{1-\delta^f+\phi(\nu-\eta)}\nu \quad \text{and}$$
$$\text{RHS}(\mathcal{W}_2^f(a^f)) = \frac{\hat{c}^2}{2\hat{c}-a^f+\mu}\frac{1-\delta^f+\phi(\nu-\eta)}{\phi}.$$

  When $a^f \in [\underline{a}, \overline{a}[$, $\text{RHS}(\mathcal{W}_2^f(a^f)) > \text{LHS}(\mathcal{W}_2^f(a^f))$ is satisfied under Assumption 1. This ensures that $\mathcal{W}_2^f(a^f)$ is in between the two roots that solve Equation (10). As $\mathcal{W}_1^f(a^f)$ is the smallest root of Equation (10) (Definition 1), $\mathcal{W}_1^f(a^f) < \mathcal{W}_2^f(a^f)$.

$\mathcal{W}_0^f(a^f) < \mathcal{W}_2^f(a^f)$ when $a^f > \underline{a}$ and the Inequality (11) is not satisfied for $\mathcal{W}_0^f(a^f)$. So $\mathcal{W}_0^f(a^f) < \mathcal{W}_1^f(a^f) < \mathcal{W}_2^f(a^f)$.

We can then conclude that, under Assumption 1, $\forall w^f \in [\mathcal{W}_0^f(a^f), \mathcal{W}_1^f(a^f)[$, $u(c_{\text{IV}}^f, 0) > u(\hat{c}, n_{\text{III}})$ and that $\forall w^f \in [\mathcal{W}_1^f(a^f), \mathcal{W}_2^f(a^f)[$, $u(\hat{c}, n_{\text{III}}) \geq u(c_{\text{IV}}^f, 0)$.

- Second, we show that when $a \in [\underline{a}, \overline{a}[$, $\mathcal{W}_2^f(a^f) < \mathcal{W}_4^f(a^f) < \mathcal{W}_5^f(a^f)$.

  At $\mathcal{W}_2^f(a^f)$, $u(\hat{c}, n_{\text{III}}) = u(c_{\text{I}}^f, n_{\text{I}})$ and Regime I is feasible. So, $\forall w^f > \mathcal{W}_2^f(a^f)$, $u(c_{\text{I}}^f, n_{\text{I}}) > u(\hat{c}, n_{\text{III}})$. At $\mathcal{W}_2^f(a^f)$, $u(\hat{c}, n_{\text{III}}) \geq u(c_{\text{IV}}^f, 0)$. So, we can conclude that $u(c_{\text{I}}^f, n_{\text{I}}) \geq u(c_{\text{IV}}^f, 0)$ at $\mathcal{W}_2^f(a^f)$.

  Regime I exists for $w^f \in [\mathcal{W}_2^f(a^f), \mathcal{W}_5^f(a^f)[$. For all $w^f > \mathcal{W}_5^f(a^f)$, $u(c_{\text{I}}^f, n_{\text{I}})$ is not defined since $\partial n_{\text{I}}/\partial w^f < 0$, so n_{I} would be negative. Then, for $w^f > \mathcal{W}_5^f(a^f)$, Regime IV prevails. Let us compare the utility in Regime IV with the utility in Regime I when $w^f \leq \mathcal{W}_5^f(a^f)$. $u(c_{\text{IV}}^f, 0) \geq u(c_{\text{I}}^f, n_{\text{I}})$ if and only if,

$$\ln \frac{(1-\delta^f)w^f + a^f - \mu}{w^f(1-\delta^f+\phi(\nu-\eta)) + a^f - \mu} + \ln \nu \geq \ln \frac{w^f(1-\delta^f+\phi(\nu-\eta)) + a^f - \mu}{\phi w^f} - 2\ln 2. \tag{12}$$

  Considering Equation (12), at $w^f = 0$, the LHS is equal to $\ln \nu$ and the RHS goes to $+\infty$. The limits of the LHS and the RHS at $+\infty$ are respectively,

$$\ln \frac{1}{1+\phi(\nu-\eta)} + \ln \nu \quad \text{and} \quad \ln \frac{1+\phi(\nu-\eta)}{\phi} - 2\ln 2.$$

  This implies that the RHS is above the LHS for low values of w^f but as we cannot rank the two limits, the RHS can be above or below the LHS for large values of w^f. As both sides of the inequality are strictly decreasing and convex with w^f, the LHS can be equal to the RHS for either one or two values of w^f. At $w^f = \mathcal{W}_5^f(a^f)$, the LHS is strictly larger than the RHS, implying that $u(c_{\text{IV}}^f, 0) > u(c_{\text{I}}^f, n_{\text{I}})$. This implies that $\mathcal{W}_5^f(a^f)$ is either in between the roots of Equation (12) holding at equality or at the right of the only root. Since $u(c_{\text{I}}^f, n_{\text{I}})$ is not defined for $w^f \geq \mathcal{W}_5^f(a^f)$, the relevant root of LHS=RHS, $\mathcal{W}_4^f(a^f)$, is strictly lower than $\mathcal{W}_5^f(a^f)$. This proves that $u(c_{\text{IV}}^f, 0) < u(c_{\text{I}}^f, n_{\text{I}})$ for $w^f < \mathcal{W}_4^f(a^f)$, and that $u(c_{\text{IV}}^f, 0) \geq u(c_{\text{I}}^f, n_{\text{I}})$ for $w^f \in [\mathcal{W}_4^f(a^f), \mathcal{W}_5^f](a^f)$. Intuitively, because there is a fixed cost to becoming a parent, the optimal fertility is not continuous in w^f.

  We showed that $\forall w^f < \mathcal{W}_4^f(a^f)$, $u(c_{\text{I}}^f, n_{\text{I}}) > u(c_{\text{IV}}^f, 0)$ and that at $\mathcal{W}_2^f(a^f)$, Regime I exists and $u(c_{\text{I}}^f, n_{\text{I}}) > u(c_{\text{IV}}^f, 0)$. This implies that $\mathcal{W}_2^f(a^f) < \mathcal{W}_4^f(a^f)$.

Regime V is not reachable when $a^f < \overline{a}$ since the consumption in Regime V is equal to $a^f - \mu$, which does not allow the woman to reach the minimal consumption level allowing her to procreate.

We have proved that, when $a \in [\underline{a}, \overline{a}[$, $\mathcal{W}_0^f(a^f) < \mathcal{W}_1^f(a^f) < \mathcal{W}_2^f(a^f) < \mathcal{W}_4^f(a^f) < \mathcal{W}_5^f(a^f)$ under Assumption 1. We can then conclude that when $a \in [\underline{a}, \overline{a}[$,

- $\forall w^f < \mathcal{W}_0^f(a^f)$, $c^f = c_{\mathrm{II}}^f$, $n = 0$,
- $\forall w^f \in [\mathcal{W}_0^f(a^f), \mathcal{W}_1^f(a^f)[$, $u(c_{\mathrm{IV}}^f, 0) > u(\hat{c}, n_{\mathrm{III}})$ and $c^f = c_{\mathrm{IV}}^f$, $n = 0$,
- $\forall w^f \in [\mathcal{W}_1^f(a^f), \mathcal{W}_2^f(a^f)[$, $u(\hat{c}, n_{\mathrm{III}}) \geq u(c_{\mathrm{IV}}^f, 0)$ and $c^f = \hat{c}$, $n = n_{\mathrm{III}}$,
- $\forall w^f \in [\mathcal{W}_2^f(a^f), \mathcal{W}_4^f(a^f)[$, $u(c_{\mathrm{I}}^f, n_{\mathrm{I}}) > u(c_{\mathrm{IV}}^f, 0)$ and $c^f = c_{\mathrm{I}}^f$, $n = n_{\mathrm{I}}$,
- $\forall w^f \geq \mathcal{W}_4^f(a^f)$, $c^f = c_{\mathrm{IV}}^f$, $n = 0$.

Proof and Implications of 3.

We are now in the case $a^f \geq \overline{a}$, where Regime V is feasible for all $w^f \geq 0$. Furthermore, Regimes II and III no longer exist since even with a wage equal to zero, a woman can consume more than $\hat{c}$. We then just need to compare the utilities in Regimes I, IV and V.

We can check that $n_{\mathrm{I}} > \underline{n}_{\mathrm{M}} \iff w^f < \mathcal{W}_3^f(a^f)$. $\forall w^f < \mathcal{W}_3^f(a^f)$, $u(c_{\mathrm{I}}^f, n_{\mathrm{I}})$ is not defined as n_{I} would be above the maximum possible.

Let us now show that $\mathcal{W}_3^f(a^f) < \mathcal{W}_4^f(a^f) < \mathcal{W}_5^f(a^f)$. We already know that both the LHS and the RHS of Equation (12) are strictly decreasing and convex in w^f. We can check that at $\mathcal{W}_3^f(a^f)$ the LHS is lower than the RHS. It follows from Definition 1 that $\mathcal{W}_3^f(a^f) < \mathcal{W}_5^f(a^f)$. Since at $\mathcal{W}_4^f(a^f)$ the LHS is equal to the RHS, then $\mathcal{W}_3^f(a^f) < \mathcal{W}_4^f(a^f) < \mathcal{W}_5^f(a^f)$.

At $\mathcal{W}_3^f(a^f)$, we can show that $u(c_{\mathrm{V}}^f, \underline{n}_{\mathrm{M}}) = u(c_{\mathrm{I}}^f, n_{\mathrm{I}}) > u(c_{\mathrm{IV}}^f, 0)$. As $u(c_{\mathrm{I}}^f, n_{\mathrm{I}})$ is increasing in w^f and $u(c_{\mathrm{V}}^f, \underline{n}_{\mathrm{M}})$ is unaffected by w^f, we have that $u(c_{\mathrm{I}}^f, n_{\mathrm{I}}) \geq u(c_{\mathrm{V}}^f, \underline{n}_{\mathrm{M}})$ for $w^f \in [\mathcal{W}_3^f(a^f), \mathcal{W}_5^f(a^f)]$. As $u(c_{\mathrm{IV}}^f, 0)$ is increasing in w^f and $u(c_{\mathrm{V}}^f, \underline{n}_{\mathrm{M}})$ is unaffected by w^f, we also have that $u(c_{\mathrm{V}}^f, \underline{n}_{\mathrm{M}}) > u(c_{\mathrm{IV}}^f, 0)$ $\forall w^f < \mathcal{W}_3^f(a^f)$.

Given that $\mathcal{W}_3^f(a^f) < \mathcal{W}_4^f(a^f) < \mathcal{W}_5^f(a^f)$, we can conclude that:

- when $w^f < \mathcal{W}_3^f(a^f)$, $c^f = c_{\mathrm{V}}$ and $n = \underline{n}_{\mathrm{M}}$,
- when $w^f \in [\mathcal{W}_3^f(a^f), \mathcal{W}_4^f(a^f)]$, $u(c_{\mathrm{IV}}^f, 0) \geq u(c_{\mathrm{I}}^f, n_{\mathrm{I}}) > u(c_{\mathrm{V}}^f, \underline{n}_{\mathrm{M}})$, which implies that $c^f = c_{\mathrm{I}}^f$ and $n = n_{\mathrm{I}}$,
- when $w^f > \mathcal{W}_4^f(a^f)$, $u(c_{\mathrm{IV}}^f, 0) > u(c_{\mathrm{V}}^f, \underline{n}_{\mathrm{M}})$ and $c^f = c_{\mathrm{V}}^f$ and $n = 0$.

■

B.2 Optimal Decisions of Couples

This Appendix describes the optimal decisions of couples and shows that for some values of the model's parameters, the relationship between fertility and the wage of women is rightly described in Section 3. Notice that all the cases described in this section exist for $w^m > 0$ but sufficiently small to ensure that all the thresholds on a are positive and for $\theta(w^f, w^m)$ fixed to $\widehat{\theta}$. At the end of this subsection, we discuss the cases where w^m becomes high and θ is not fixed but depends on w^f and w^m. Definitions for wage and non-labor income thresholds are given in Definitions 3 and 4 respectively. These thresholds are used to describe the optimal decisions of couples as a function of wages and non-labor incomes, formally stated in Proposition 2. Results are valid under Assumption 2.

Definition 3 (Wage thresholds for couples)

$$
\begin{aligned}
\mathcal{W}^f_A(a, w^m) &= \frac{\hat{c} - a - (1 - \phi(1-\alpha)\eta)w^m}{1 - \alpha\phi\eta},\\
\mathcal{W}^f_B(a, w^m) &= \frac{\hat{c} - a + \left(\phi(1-\alpha)((1-\widehat{\theta})\nu - \eta) - 1\right) w^m}{1 - \alpha\phi(\eta + \nu(1-\widehat{\theta}))},\\
\mathcal{W}^f_C(a, w^m) &= \frac{\Xi_1 + \left(\Xi_1^2 + 4\widehat{\theta}\alpha(1 + \phi\alpha(\nu - \eta))\Xi_2\right)^{\frac{1}{2}}}{2\widehat{\theta}\alpha(1 + \alpha\phi(\nu - \eta))},\\
\mathcal{W}^f_D(a, w^m) &= \frac{\frac{2\hat{c}}{\widehat{\theta}} - a - (1 + \phi(1-\alpha)(\nu - \eta))\, w^m}{1 + \alpha\phi(\nu - \eta)},\\
\mathcal{W}^f_E(a, w^m) &= \frac{a - \hat{c} + \left(1 - (1-\widehat{\theta})\phi(1-\alpha)(\nu - \eta) - \frac{1-\alpha}{\alpha}(2 - \widehat{\theta})\right) w^m}{(1 - \widehat{\theta})\left(1 + \alpha\phi(\nu - \eta)\right)},\\
\mathcal{W}^f_F(a, w^m) &= \frac{a + \left(\frac{2-3\alpha}{\alpha} + \phi(1-\alpha)(\nu - \eta))\right) w^m}{1 + \alpha\phi(\nu - \eta)},\\
\mathcal{W}^f_G(a, w^m) &= \left(\frac{\hat{c}}{\widehat{\theta}}\right)^{\widehat{\theta}} \left(\frac{\frac{2\alpha-1}{\alpha}w^m + a - \hat{c}}{1 - \widehat{\theta}}\right)^{1-\widehat{\theta}} \frac{1 + \alpha\phi(\nu - \eta)}{\alpha\phi\nu} - a - w^m\\
\mathcal{W}^f_I(a, w^m) &= \frac{1 - \phi\alpha\eta}{\phi\alpha\nu} a + \left(\frac{(2\alpha - 1)(1 + \phi\alpha(\nu - \eta)}{\alpha^2\phi\nu} - 1\right) w^m,\\
\mathcal{W}^f_J(a, w^m) &= \frac{\Xi_3}{[1 + \alpha\phi(\nu - \eta)^2] - 4\phi\alpha\nu},\\
\mathcal{W}^f_{\underline{J}}(a, w^m) &= \frac{-\Xi_3 + \left(\Xi_3^2 - 4\left([1 + \alpha\phi(\nu - \eta)^2] - 4\phi\alpha\nu\right)\Xi_4\right)^{\frac{1}{2}}}{2\left([1 + \alpha\phi(\nu - \eta)^2] - 4\phi\alpha\nu\right)},\\
\mathcal{W}^f_{\overline{J}}(a, w^m) &= \frac{-\Xi_3 - \left(\Xi_3^2 - 4\left([1 + \alpha\phi(\nu - \eta)^2] - 4\phi\alpha\nu\right)\Xi_4\right)^{\frac{1}{2}}}{2\left([1 + \alpha\phi(\nu - \eta)^2] - 4\phi\alpha\nu\right)}
\end{aligned}
$$

$$
\begin{aligned}
\textit{where} \quad \Xi_1 &\equiv (1-\widehat{\theta})(1+\phi\alpha(\nu-\eta))(w^m+\alpha a) - 2\alpha(a-\hat{c}+(1+\phi(1-\alpha)(\nu-\eta))w^m), \\
\Xi_2 &\equiv (2-\widehat{\theta})(1+\phi\alpha(\nu-\eta))(1-\alpha)w^m(a+w^m) \\
&\qquad\qquad -(a+\alpha w^m)(a-\hat{c}+(1+\phi(1-\alpha)(\nu-\eta))w^m), \\
\Xi_3 &\equiv 2\left(w^m[1-\phi(\nu+\eta)+\phi\alpha(1-\alpha)(\nu-\eta)^2]+a[1-\phi\alpha(\nu+\eta)]\right), \\
\textit{and} \quad \Xi_4 &\equiv (a+(1+\phi(1-\alpha)[\nu-\eta])w^m)^2 - 4\phi\nu(1-\alpha)(w^m+a)w^m.
\end{aligned}
$$

We finally denote as $\mathcal{W}^f_{\underline{C}}(a,w^m) \leq \mathcal{W}^f_{\overline{C}}(a,w^m)$ *the two potential roots of the equation*

$$U(\hat{c}, c^m_{VIII}, n_{VIII}) = U(c^f_{IX}, c^m_{IX}, 0)$$

$\mathcal{W}^f_A(a,w^m)$ is such that the couple can afford positive fertility ($n \to 0$, $c^f = \hat{c}$ and $c^m = 0$). $\mathcal{W}^f_B(a,w^m)$ is such that $n_{\text{VIII}} = 0$. $\mathcal{W}^f_C(a,w^m)$ is the value of w^f that maximizes $U(c^m_{\text{VIII}}, c^f_{\text{VIII}}, n_{\text{VIII}}) - U(c^m_{\text{IX}}, c^f_{\text{IX}}, 0)$. $\mathcal{W}^f_D(a,w^m)$ is such that $n_{\text{VIII}} = n_{\text{VI}}$. $\mathcal{W}^f_E(a,w^m)$ is such that $n_{\text{VIII}} = \bar{n}_{\text{M}}$. $\mathcal{W}^f_F$ is such that $n_{\text{VI}} = \bar{n}_{\text{M}}$. $\mathcal{W}^f_G$ is the unique root of $U(c^f_{\text{X}}, \hat{c}, \bar{n}_{\text{M}}) - U(c^f_{\text{IX}}, c^m_{\text{IX}}, 0) = 0$. $\mathcal{W}^f_I(a,w^m)$ is such that $U(c^m_{\text{XI}}, c^f_{\text{XI}}, \bar{n}_{\text{M}}) = U(c^m_{\text{IX}}, c^f_{\text{IX}}, 0)$. Finally, $\mathcal{W}^f_J(a,w^m)$ is the unique maximum of $U(c^m_{\text{VI}}, c^f_{\text{VI}}, n_{\text{VI}}) - U(c^m_{\text{IX}}, c^f_{\text{IX}}, 0)$ while $\mathcal{W}^f_{\underline{J}}(a,w^m)$ and $\mathcal{W}^f_{\overline{J}}(a,w^m)$ are the lowest and the highest roots of the equation $U(c^m_{\text{VI}}, c^f_{\text{VI}}, n_{\text{VI}}) = U(c^m_{\text{IX}}, c^f_{\text{IX}}, 0)$ respectively.

For ease of notation, we will denote $\mathcal{W}^f_\kappa(a,w^m)$ (for $\kappa = A, B, \underline{C}, C, \overline{C}, D, E, F, G, I, \underline{J}, J, \overline{J}$) as $\mathcal{W}^f_\kappa$ and use the complete notation only when necessary.

Definition 4 (Non-labor income thresholds for couples)
A_0 is the value of a that solves $U(\hat{c}, c^m_{VIII}, n_{VIII}) = U(c^f_{IX}, c^m_{IX}, 0)$ when $w^f = \mathcal{W}^f_C$.
A_1 is the value of a that solves $U(c^f_{VI}, c^m_{VI}, n_{VII}) = U(c^f_{IX}, c^m_{IX}, 0)$ when $w^f = \mathcal{W}^f_D$.
$A_2 = \hat{c} - (1-\phi(1-\alpha)\eta)w^m$ is the value of a above which a couple can have children even if the wife has no labor income (or has the maximal number of children).
A_3 is the lowest value of a such that $\mathcal{W}^f_G = 0$.
A_4 is the value of a such that $\mathcal{W}^f_G = \mathcal{W}^f_E$.
$A_5 = \frac{2\hat{c}}{\underline{\theta}} - \frac{2\alpha-1}{\alpha}w^m$. Above A_5, even when her wage is zero and her negotiation power is minimal, a woman can consume more than $\hat{c}$ even in the interior regime.

Assumption 2 *Parameters α, ϕ, ν, η, $\underline{\theta}$, and $\hat{c}$ satisfy:*

- $(1+\alpha\phi(\nu-\eta))^2 - 4\phi\alpha\nu < 0$, *this condition ensures that the problem is not degenerated, in the sense that the interior regime is chosen for some values of w^f and a,*

- *when* $w^f = \mathcal{W}^f_D$,
$$\frac{\partial U(\hat{c}, c^m_{VIII}, n_{VIII})}{\partial w^f} = \frac{\partial U(c^f_{VI}, c^m_{VI}, n_{VI})}{\partial w^f} - \varepsilon$$
with $\varepsilon \to 0^+$, *this assumption excludes a degenerated case where, over a range of* w^f, *couples first choose the "Eat and Procreate Regime" then "Voluntary Childlessness" and then the "Interior Solution".*

- *when* $w^f = \mathcal{W}^f_C$,
$$\frac{\partial \left[U(\hat{c}, c^m_{VIII}, n_{VIII}) - U(c^f_{IX}, c^m_{IX}, n_{IX})\right]}{\partial a} > 0, \forall a < A_4.$$
This assumption ensures that the eat and procreate regime will be chosen for some triplets of (w^f, w^m, a).

- $\mathcal{W}^f_G(A_2, w^m) < \mathcal{W}^f_E(A_2, w^m)$; *furthermore,* A_5 *lies in-between the two roots of equation* $\mathcal{W}^f_G = \mathcal{W}^f_E$ *solved with respect to* a.

- $\forall a < A_4$; $\mathcal{W}^f_J < \mathcal{W}^f_D$. *Thus assumption ensures that for high wages, regime IX is preferred to regime VI.*

Proposition 2 *Under Assumption 2, the optimal choice of a couple with non-labor income* a *and wages* w^f *and* w^m *is given by:*

1. *When* $a \leq A_0$:
   - *if* $w^f < \mathcal{W}^f_A$, $c^f = c^f_{VII}$, $c^m = c^m_{VII}$, $n = 0$
   - *if* $w^f \geq \mathcal{W}^f_A$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$
2. *When* $a \in]A_0, A_1]$:
   - *if* $w^f < \mathcal{W}^f_A$, $c^f = c^f_{VII}$, $c^m = c^m_{VII}$, $n = 0$
   - *if* $w^f \in [\mathcal{W}^f_A, \mathcal{W}^f_{\underline{C}}[$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$
   - *if* $w^f \in [\mathcal{W}^f_{\underline{C}}, \mathcal{W}^f_{\overline{C}}]$, $c^f = c^f_{VIII}$, $c^m = c^m_{VIII}$, $n = n_{VIII}$
   - *if* $w^f > \mathcal{W}^f_{\overline{C}}$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$
3. *When* $a \in]A_1, A_2]$:
   - *if* $w^f < \mathcal{W}^f_A$, $c^f = c^f_{VII}$, $c^m = c^m_{VII}$, $n = 0$

- *if* $w^f \in [\mathcal{W}^f_A, \mathcal{W}^f_{\underline{C}}[$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$
- *if* $w^f \in [\mathcal{W}^f_{\underline{C}}, \mathcal{W}^f_D[$, $c^f = c^f_{VIII}$, $c^m = c^m_{VIII}$, $n = n_{VIII}$
- *if* $w^f \in [\mathcal{W}^f_D, \mathcal{W}^f_{\bar{J}}]$, $c^f = c^f_{VI}$, $c^m = c^m_{VI}$, $n = n_{VI}$
- *if* $w^f > \mathcal{W}^f_{\bar{J}}$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$

4. *When* $a \in]A_2, A_3]$*:*

- *if* $w^f < \mathcal{W}^f_{\underline{C}}$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$
- *if* $w^f \in [\mathcal{W}^f_{\underline{C}}, \mathcal{W}^f_D[$, $c^f = c^f_{VIII}$, $c^m = c^m_{VIII}$, $n = n_{VIII}$
- *if* $w^f \in [\mathcal{W}^f_D, \mathcal{W}^f_{\bar{J}}]$, $c^f = c^f_{VI}$, $c^m = c^m_{VI}$, $n = n_{VI}$
- *if* $w^f > \mathcal{W}^f_{\bar{J}}$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$

5. *When* $a \in]A_3, A_4]$*:*

- *if* $w^f \leq \mathcal{W}^f_G$, $c^f = c^f_X$, $c^m = c^m_X$, $n = \bar{n}_M$
- *if* $w^f \in]\mathcal{W}^f_G, \max\{\mathcal{W}^f_E, \mathcal{W}^f_{\underline{C}}\}[$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$
- *if* $w^f \in [\max\{\mathcal{W}^f_E, \mathcal{W}^f_{\underline{C}}\}, \mathcal{W}^f_D[$, $c^f = c^f_{VIII}$, $c^m = c^m_{VIII}$, $n = n_{VIII}$
- *if* $w^f \in [\mathcal{W}^f_D, \mathcal{W}^f_{\bar{J}}]$, $c^f = c^f_{VI}$, $c^m = c^m_{VI}$, $n = n_{VI}$
- *if* $w^f > \mathcal{W}^f_{\bar{J}}$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$

6. *When* $a \in]A_4, A_5]$*:*

- *if* $w^f \leq \mathcal{W}^f_E$, $c^f = c^f_X$, $c^m = c^m_X$, $n = \bar{n}_M$
- *if* $w^f \in]\mathcal{W}^f_E, \mathcal{W}^f_D[$, $c^f = c^f_{VIII}$, $c^m = c^m_{VIII}$, $n = n_{VIII}$
- *if* $w^f \in [\mathcal{W}^f_D, \mathcal{W}^f_{\bar{J}}]$, $c^f = c^f_{VI}$, $c^m = c^m_{VI}$, $n = n_{VI}$
- *if* $w^f > \mathcal{W}^f_{\bar{J}}$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$

7. *When* $a > A_5$*:*

- *if* $w^f \leq \mathcal{W}^f_F$, $c^f = c^f_{XI}$, $c^m = c^m_{XI}$, $n = \bar{n}_M$
- *if* $w^f \in]\mathcal{W}^f_F, \mathcal{W}^f_{\bar{J}}]$, $c^f = c^f_{VI}$, $c^m = c^m_{VI}$, $n = n_{VI}$
- *if* $w^f > \mathcal{W}^f_{\bar{J}}$, $c^f = c^f_{IX}$, $c^m = c^m_{IX}$, $n = 0$

Proof. We use a two-step strategy. First, we highlight some general properties of the model. Second, we prove each part of the proposition. Notice that all the cases exist only when w^m is sufficiently close to zero. Successive increases of w^m will make cases of poverty disappear.

For instance, one can easily check that for high values of w^m, $\mathcal{W}_A^f$, $\mathcal{W}_B^f$, A_2 and A_5 become negative making the study of fertility behaviors of couples much easier.

Step 1: Some General Properties of the Model

In what follows, we assume w^m small enough to ensure that $A_0 > 0$.

- *Regimes VII and IX are "continuous" in the sense that $U(c_{VII}^f, c_{VII}^m, 0) = U(c_{IX}^f, c_{IX}^m, 0)$, $\forall w^f > 0$.* A couple is able to have children once its total income is greater than $\hat{c}$ which is satisfied when $w^f > \mathcal{W}_A^f$. This implies that $\forall w^f < \mathcal{W}_A^f$, childlessness is involuntary, while it is voluntary otherwise.

- *Regimes VIII and VI are "continuous".* Regime VIII exists as long as $n_{\text{VIII}} \geq 0$ which is satisfied $\forall w^f \geq \mathcal{W}_B^f$. Regime VI exists once $c_{\text{VI}}^f \geq \hat{c}$ which is satisfied $\forall w^f \geq \mathcal{W}_D^f$. We can verify that when $w^f = \mathcal{W}_D^f$, $c_{\text{VI}}^f = \hat{c}$ and $n_{\text{VI}} = n_{\text{VIII}}$, implying that $U(\hat{c}, c_{\text{VIII}}^m, n_{\text{VIII}}) = U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}})$. It also implies that Regime VIII is defined $\forall w^f \in [\mathcal{W}_B^f, \mathcal{W}_D^f]$ while Regime VI is defined $\forall w^f \geq \mathcal{W}_D^f$ ($\lim_{w^f \to \infty} n_{\text{VI}} > 0$).

- Equation $U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}}) = U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$ can be written as

$$\left([1 + \alpha\phi(\nu - \eta)^2] - 4\phi\alpha\nu\right)(w^f)^2 - \Xi_3 w^f + \Xi_4 = 0. \tag{13}$$

  As $[1 + \alpha\phi(\nu - \eta)^2] - 4\phi\alpha\nu < 0$, Equation (13) has, at most, two positive roots and admits a unique maximum at $w_f = \mathcal{W}_J^f$, then $\mathcal{W}_J^f > 0$ if $\Xi_3 > 0$.

- *When $a^f \leq A_0$:*

  i. $\forall w^f > 0$, $U(c_{\text{VII}}^f, c_{\text{VII}}^m, 0) = U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0) > U(\hat{c}, c_{\text{VIII}}^m, n_{\text{VIII}})$ *and,*

  ii. *when* $w^f = \mathcal{W}_D^f$, $U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}}) = U(\hat{c}, c_{\text{VIII}}^m, n_{\text{VIII}}) < U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$.

  $U(\hat{c}, c_{\text{VIII}}^m, n_{\text{VIII}}) - U(c_{\text{IX}}^m, c_{\text{IX}}^f, 0)$ is positive when,

$$\begin{aligned}&(1 + \phi\alpha(\nu - \eta))w^f + (1 + \phi(1 - \alpha)(\nu - \eta))w^m + a - \hat{c} \geq \\ &\qquad\left(\frac{\widehat{\theta}}{\hat{c}}\right)^{\frac{\widehat{\theta}}{2-\widehat{\theta}}}(2 - \widehat{\theta})\left[\nu\phi\alpha(w^f + w^m + a)(\alpha w^f + (1 - \alpha)w^m)\right]^{\frac{1}{2-\widehat{\theta}}}\end{aligned} \tag{14}$$

  Since the LHS is linearly increasing with respect to w^f and the RHS is increasing and convex with w^f, we know that $U(\hat{c}, c_{\text{VIII}}^m, n_{\text{VIII}}) - U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$ admits a unique maximum, denoted $\mathcal{W}_C$, and at most two positive roots, denoted $\mathcal{W}_{\underline{C}}^f < \mathcal{W}_{\overline{C}}^f$. Furthermore, we can check that Inequality (14) is not satisfied when $w^f = 0$ and $a < \hat{c} - (1 + \phi(1 - \alpha)(\nu - \eta)w^m)$.

Under Assumption 2, when $w^f = \mathcal{W}_C^f$, $U(\hat{c}, c_{\text{VIII}}^m, n_{\text{VIII}}) - U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$ is increasing in a, for $a < A_4$. This means that the maximum value of $U(\hat{c}, c_{\text{VIII}}^m, n_{\text{VIII}}) - U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$ increases monotonically with a. It implies that there is a value, $a = A_0$, where A_0 solves

$$\begin{aligned}&(1 + \phi\alpha(\nu - \eta))\mathcal{W}_C^f(A_0, w^m) + (1 + \phi(1 - \alpha)(\nu - \eta))w^m + A_0 - \hat{c} = \\ &\left(\frac{\widehat{\theta}}{\hat{c}}\right)^{\frac{\widehat{\theta}}{2-\widehat{\theta}}}(2 - \widehat{\theta})\left[\nu\phi\alpha(\mathcal{W}_C^f(A_0, w^m) + w^m + A_0)(\alpha\mathcal{W}_C^f(A_0, w^m) + (1 - \alpha)w^m)\right]^{\frac{1}{2-\widehat{\theta}}}.\end{aligned} \tag{15}$$

This means that when $a^f \leq A_0$, $U(c_{\text{VII}}^f, c_{\text{VII}}^m, 0) = U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0) > U(\hat{c}, c_{\text{VIII}}^m, n_{\text{VIII}})$, $\forall w^f \geq 0$. As $w^f = \mathcal{W}_D^f$ implies that $U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}}) = U(c_{\text{VIII}}^f, c_{\text{VIII}}^m, n_{\text{VIII}})$, we also find that $U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}}) < U(c_{\text{IX}}^f, c_{\text{IX}}^m, n_{\text{IX}})$ for $w^f = \mathcal{W}_D^f$ and $a \leq A_0$.

- *$U(c_{VI}^f, c_{VI}^m, n_{VI}) < U(c_{IX}^f, c_{IX}^m, 0)$, $\forall w^f > \mathcal{W}_D^f$ and $a \leq A_1$.* As already shown, Equation (13) is quadratic with respect to w^f. It admits a unique maximum for $w^f = \mathcal{W}_J^f < \mathcal{W}_D^f$ when $a \leq A_1$. Then $U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}}) - U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$ is decreasing $\forall w^f > \mathcal{W}_D^f$. Since $\forall a \leq A_1$ and $w^f = \mathcal{W}_D^f$, $U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0) > U(c_{\text{VIII}}^f, c_{\text{VIII}}^m, n_{\text{VIII}}) = U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}})$, then $\forall a \leq A_1$ and $\forall w^f > \mathcal{W}_D^f$, $U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}}) < U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$.

- *Regime X is never preferred to Regime IX when $a < A_3$.* This is due to $\mathcal{W}_G^f < 0$ for $a < A_3$.

- *There exists a unique $A_4 > 0$ such that $\forall a \in [A_3, A_4]$, $\mathcal{W}_G^f \leq \mathcal{W}_E^f$ while $\forall a \in]A_4, A_5]$, $\mathcal{W}_G^f > \mathcal{W}_E^f$.* Equation $\mathcal{W}_G^f = \mathcal{W}_E^f$ can be written as

$$\begin{aligned}&\left(\frac{\hat{c}}{\widehat{\theta}}\right)^{\widehat{\theta}}\left(\frac{\frac{2\alpha-1}{\alpha}w^m + a - \hat{c}}{1 - \widehat{\theta}}\right)^{1-\widehat{\theta}}\frac{1 + \alpha\phi(\nu - \eta)}{\alpha\phi\nu} \\ &\qquad = a + w^m - \frac{a - \hat{c} + \left(1 - (1 - \widehat{\theta})\phi(1 - \alpha)(\nu - \eta) - \frac{1-\alpha}{\alpha}(2 - \widehat{\theta})\right)w^m}{(1 - \widehat{\theta})\,(1 + \alpha\phi(\nu - \eta))},\end{aligned} \tag{16}$$

where the LHS is increasing and concave with respect to a while the RHS is linearly increasing with a. It implies that the equation has at most two roots and as the LHS is only defined for $a > \hat{c} - \frac{2\alpha-1}{\alpha}w^m$, which is positive for sufficiently low values of w^m, these roots are strictly positive. Furthermore, under Assumption 2, $\mathcal{W}_G^f(A_2, w^m) <$

$\mathcal{W}_E^f(A_2, w^m)$ and A_5 lies in between the two roots of Equation (16). It directly implies that there exists a unique A_4 such that $\mathcal{W}_G^f < \mathcal{W}_E^f$ when $a \in [A_3, A_4[$ and $\mathcal{W}_G^f \geq \mathcal{W}_E^f$ $\forall a \in [A_4, A_5]$.

Step 2: Proof of Each Part of Proposition 2

We still assume w^m low enough for A_0 to be positive. This will be relaxed afterwards.

Step 2.1: Cases where $a < A_2$

Regimes X and XI are not reachable because $c_{\text{X}}^m < 0$ and $c_{\text{XI}}^f < \hat{c}$ for $a^f < A_2$. So we only need to compare the utilities $U_{\text{VII}}(a, w^f)$, $U_{\text{VIII}}(a, w^f)$, $U_{\text{VI}}(a, w^f)$ and $U_{\text{IX}}(a, w^f)$ to see the outcome of a marriage.

Case 1: $a < A_0$: Here, we show that, as in the first case of Proposition 1, once agents with very low non-labor income are able to be parents, the opportunity cost of childrearing is too high for them to be willing to. $\mathcal{W}_A^f$, defined in Definition 3, solves:

$$(1 - \phi\alpha\eta)\mathcal{W}_A^f + 1 - \phi(1-\alpha)\eta)w^m + a = \hat{c}.$$

We can check that $\mathcal{W}_A^f > 0 \iff a < A_2$ and that $A_0 < A_2$. For $w^f < \mathcal{W}_A^f$, the consumption of a woman who wants to have children would be lower than $\hat{c}$ even if her husband does not consume. Consequently, for $w^f < \mathcal{W}_A^f$, the couple is involuntarily childless. For $w^f \geq \mathcal{W}_A^f$, the couple can decide whether to have children or to be voluntarily childless.

When $a < A_0$, $U(c_{\text{VIII}}^f, c_{\text{VIII}}^m, n_{\text{VIII}}) < U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0), \forall w^f$. By continuity we know that $U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}})$ is lower than $U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$ for $w^f = \mathcal{W}_D^f$. As shown in Step 1 of the proof, when $a < A_1$, $U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}}) - U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$ decreases with w^f which directly implies that $\forall w^f > \mathcal{W}_D^f$, $U(c_{\text{VI}}^f, c_{\text{VI}}^m, n_{\text{VI}}) < U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$.

We can then conclude that if $w^f < \mathcal{W}_A^f$, $c^f = c_{\text{VII}}^f$, $c^m = c_{\text{VII}}^m$, $n = 0$, while if $w^f \geq \mathcal{W}_A^f$, $c^f = c_{\text{IX}}^f$, $c^m = c_{\text{IX}}^m$, $n = 0$.

Case 2: $a \in [A_0, A_1[$: In this case, we will show that Malthusian mechanisms, where fertility is related positively to female's wages, can be at play among some poor couples.

As $A_1 < A_2$, $\mathcal{W}_A^f > 0$ and couples are involuntarily childless when $w^f < \mathcal{W}_A^f$. Furthermore,

$$\lim_{w^f \to \mathcal{W}_A^f} U(c_{\text{VIII}}^f, c_{\text{VIII}}^m, n_{\text{VIII}}) = -\infty$$

because $c_{\text{VIII}}^m \to 0$ when $w^f \to \mathcal{W}_A^f$. $U(c_{\text{VIII}}^f, c_{\text{VIII}}^m, n_{\text{VIII}})$ is then lower than $U(c_{\text{IX}}^f, c_{\text{IX}}^m, 0)$ in the neighborhood of $\mathcal{W}_A^f$.

When $a > A_0$, the difference $U(c^f_{\text{VIII}}, c^m_{\text{VIII}}, n_{\text{VIII}}) - U(c^f_{\text{IX}}, c^m_{\text{IX}}, 0)$ admits two positive roots, denoted $\mathcal{W}^f_{\underline{C}}$ and $\mathcal{W}^f_{\overline{C}}$, and is positive between them. As $a < A_1$, when $w^f \geq \mathcal{W}^f_D$, $U(c^f_{\text{VIII}}, c^m_{\text{VIII}}, n_{\text{VIII}}) = U(c^f_{\text{VI}}, c^m_{\text{VI}}, n_{\text{VI}}) < U(c^f_{\text{IX}}, c^m_{\text{IX}}, 0)$. This means that once the interior regime can be reached by the couple, having children is no longer optimal.

We conclude that for $a \in]A_0, A_1]$, couples are in Regime VII for $w^f < \mathcal{W}^f_A$, in Regime IX for $w^f \in]\mathcal{W}^f_A, \mathcal{W}^f_{\underline{C}}]$, in Regime VIII for $w^f \in]\mathcal{W}^f_{\underline{C}}, \mathcal{W}^f_{\overline{C}}]$ and in Regime IX again for $w^f > \mathcal{W}^f_{\overline{C}}$.

<u>**Case 3:** $a \in]A_1, A_2]$</u>: In this case, we can show that for intermediary values of non-labor incomes, a couple can either be in a Malthusian or a modern fertility pattern, where fertility decreases with the mothers' wage.

As $a \leq A_2$, $\mathcal{W}^f_A \geq 0$ and couples are involuntarily childless for $w^f < \mathcal{W}^f_A$. Using the same reasoning as for the previous case,

$$\lim_{w^f \to \mathcal{W}^f_A} U(c^f_{\text{VIII}}, c^m_{\text{VIII}}, n_{\text{VIII}}) = -\infty$$

and $U(c^f_{\text{VIII}}, c^m_{\text{VIII}}, n_{\text{VIII}}) < U(c^f_{\text{IX}}, c^m_{\text{IX}}, 0)$ in the neighborhood of $\mathcal{W}^f_A$.

Since $a > A_1$, $U(c^f_{\text{VI}}, c^m_{\text{VI}}, n_{\text{VI}})$ is higher than $U(c^f_{\text{IX}}, c^m_{\text{IX}}, 0)$ for $w^f = \mathcal{W}^f_D$. $\mathcal{W}^f_D$ then lies in-between the roots of Equation (13), denoted by $\mathcal{W}^f_{\underline{J}}$ and $\mathcal{W}^f_{\overline{J}}$. Furthermore, as $U(c^f_{\text{VIII}}, c^m_{\text{VIII}}, n_{\text{VIII}}) = U(c^f_{\text{VI}}, c^m_{\text{VI}}, n_{\text{VI}})$ for $w^f = \mathcal{W}^f_D$, we know that $\mathcal{W}^f_D$ also lies in between $\mathcal{W}^f_{\underline{C}}$ and $\mathcal{W}^f_{\overline{C}}$.

We conclude that for $a \in]A_1, A_2]$, couples are in Regime VII when $w^f < \mathcal{W}^f_A$, in Regime IX when $w^f \in]\mathcal{W}^f_A, \mathcal{W}^f_{\underline{C}}]$, in Regime VIII when $w^f \in]\mathcal{W}^f_{\underline{C}}, \mathcal{W}^f_D]$, in Regime VI when $w^f \in]\mathcal{W}^f_D, \mathcal{W}^f_{\overline{J}}]$ and in Regime IX again when $w^f > \mathcal{W}^f_{\overline{J}}$.

Step 2.2: Cases where $a \geq A_2 = \hat{c}$

For $a > A_2$, Regime VII no longer exists: a couple can procreate for any $w^f \geq 0$. Regimes X and XI become reachable.

<u>**Case 4:** $a \in]A_2, A_3[$</u>:

We will show that in this case, having the maximum fertility is not yet optimal.

Here, $\mathcal{W}^f_G < 0$. As long as $w^f < \mathcal{W}^f_D$, only Regimes IX, VIII and X can be reached. We then have to compare Regimes VIII and IX for $w^f < \mathcal{W}^f_D$. As $a > A_0$, $U(c^f_{\text{VIII}}, c^m_{\text{VIII}}, n_{\text{VIII}}) \geq U(c^f_{\text{IX}}, c^m_{\text{IX}}, 0)$ for $w^f \in [\mathcal{W}^f_{\underline{C}}, \mathcal{W}^f_{\overline{C}}]$. This implies that Regime IX prevails $\forall w^f \in [0, \mathcal{W}^f_{\underline{C}}[$.

As shown above, if $a > A_1$, then $\mathcal{W}^f_D \in [\min\{\mathcal{W}^f_{\underline{C}}, \mathcal{W}^f_{\underline{J}}\}, \max\{\mathcal{W}^f_{\overline{C}}, \mathcal{W}^f_{\overline{J}}\}]$. This implies that Regime VIII prevails $\forall w^f \in [\mathcal{W}^f_{\underline{C}}, \mathcal{W}^f_D[$ while Regime VI prevails $\forall w^f \in [\mathcal{W}^f_D, \mathcal{W}^f_{\overline{J}}[$. Finally, Regime IX prevails when $w^f \geq \mathcal{W}^f_{\overline{J}}$.

__**Case 5:** $a \in]A_3, A_4[$__:

For these values of a, having the maximum number of children is feasible only in the "eat and procreate a maximum" Regime. This behavior will be optimal only for very low values of w^f.

In line with previous cases and Step 1, under Assumption 2, we know that, for $a \in]A_3, A_4[$ $\mathcal{W}^f_G < \max\{\mathcal{W}^f_{\underline{C}}, \mathcal{W}^f_E\} < \mathcal{W}^f_D < \mathcal{W}^f_{\overline{J}}$. We can conclude that Regime X prevails when $w^f < \mathcal{W}^f_G$ while Regime IX prevails when $w^f \in]\mathcal{W}^f_G, \max\{\mathcal{W}^f_{\underline{C}}, \mathcal{W}^f_E\}[$. When $w^f \in [\max\{\mathcal{W}^f_{\underline{C}}, \mathcal{W}^f_E\}, \mathcal{W}^f_D[$, Regime VIII is chosen by the couple while Regime VI is preferred to others when $w^f \in [\mathcal{W}^f_D, \mathcal{W}^f_{\overline{J}}]$. Finally, for $w^f > \mathcal{W}^f_{\overline{J}}$, Regime IX prevails again.

__**Case 6:** $a \in]A_4, A_5[$__: The only difference between Case 6 and Case 5 lies in the fact that for $a \in [A_4, A_5[$, $\mathcal{W}^f_G > \mathcal{W}^f_E$. Therefore, we have that $\mathcal{W}^f_E < \mathcal{W}^f_{\underline{C}} < \mathcal{W}^f_D < \mathcal{W}^f_{\overline{J}}$. The results of Case 6 directly follow.

__**Case 7:** $a > A_5$__: When $a > A_5$, the eat and procreate regimes no longer exist. Indeed, a woman with a wage equal to zero (or equivalently, having the maximum number of children) would consume more than $\hat{c}$. This implies that only Regimes VI, IX and XI potentially exist. Under Assumption 2, $\mathcal{W}^f_F < \mathcal{W}^f_I < \mathcal{W}^f_{\overline{J}}$. One can easily check that $\mathcal{W}^f_F(a, 0) < \mathcal{W}^f_I(a, 0)$; intuitively, higher w^m will reinforce this inequality as higher w^m means a smaller opportunity cost for the wife to fully specialize in labor market activities.

For $w^f < \mathcal{W}^f_F$, $U(c^f_{\text{XI}}, c^m_{\text{XI}}, n_{\text{XI}}) > U(c^f_{\text{IX}}, c^m_{\text{IX}}, n_{\text{IX}})$. By continuity, we know that for $w^f = \mathcal{W}^f_F$, $U(c^f_{\text{VI}}, c^m_{\text{VI}}, n_{\text{VI}}) > U(c^f_{\text{IX}}, c^m_{\text{IX}}, n_{\text{IX}})$ (which is why we know that $\mathcal{W}^f_F < \mathcal{W}^f_{\overline{J}}$). This implies that for $w^f \in [\mathcal{W}^f_F, \mathcal{W}^f_{\overline{J}}]$, Regime VI prevails, while for $w^f > \mathcal{W}^f_{\overline{J}}$, Regime IX prevails.

■

As explained at the beginning of the proof, all the seven cases we have studied exist for $\theta(w^f, w^m)$ fixed to $\widehat{\theta}$ and when $w^m > 0$ but sufficiently small to ensure that all the thresholds on a are positive. Keeping $\theta(w^f, w^m) = \widehat{\theta}$, successive increases of w^m will make some cases disappear. For instance, if a couple enjoy a $w^m > \frac{\hat{c}}{1-\phi\eta(1-\alpha)}$, A_2 becomes negative and this couple will never be concerned with involuntary childlessness. This is due to a simple income effect: the higher the income of the husband, the more the couple is protected against situations of childlessness or constrained fertility due to poverty. In the extreme case where $w^m \geq \frac{2\alpha}{(2\alpha-1)\underline{\theta}}\hat{c}$, only case 7 exists and regimes of fertility with Malthusian properties disappear.

Obviously, the present proof does not necessarily hold when the assumption on $\theta(w^f, w^m)$ fixed is relaxed. Indeed, equations involving θ (like Equations (14) to (16)) would not behave in the same way (for instance, their left or right hand sides would not be strictly concave or convex). This could induce the existence of more thresholds on a and w^f. Nevertheless, intuitively, we can say that for $\underline{\theta} \to 1$, our results hold as θ would vary only weakly around one half.

We propose in the next subsection to verify if the results of our proposition hold for the values of parameters provided in Table 4. We can see that despite the estimated value of $\underline{\theta}$ is not close to one, our proposition still holds. In a weak sense, this allows us to affirm that there exists a non empty set of parameters of the model for which our proposition holds even if θ is not constant.

B.3 Fertility as a function of w^f, a and w^m (married)

Figure 24 allows to see how the various 2d plots of the main text for married women are slices for a given level of a of a 3d picture. The parameters are set equal to those estimated in Subsection 4.2. Three different levels of w^m are shown.

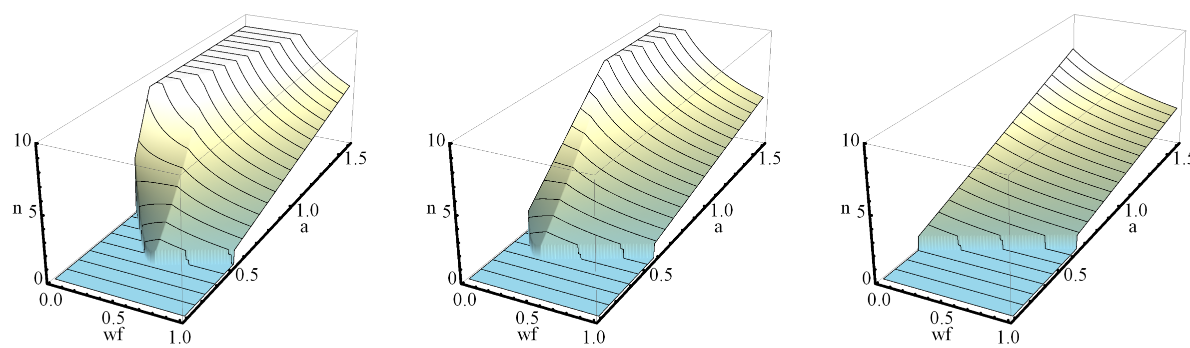


Figure 24: Fertility for different levels of mother's education (w^f) and non-labor income (a). Father with no school (left), Grade 12 (middle), Doctoral degree (right)

C Simulation and Robustness

C.1 Alternative Measures of Sterility

In a previous version of the paper (see Baudin et al. (2012)), we followed an alternative where natural sterility equals 3.7%. This number is obtained from Leridon (2008) who uses the Henry database, which consists of data for rural France from the 18th century, when fertility control was ineffective. Restricting the sample to couples where the husband and the wife were still living together at age 50, 3.7 % of women who married at age 20-24 remained childless. The main problem from this study is that living conditions and bad nutrition at that time might positively influence childlessness.

An alternative to that of the Hutterites and the historical Frenchmen is to take the value of the country that has the lowest childlessness rate for a given education level. We looked at childlessness rates in 31 developing countries where the Census included the variable "children surviving" (data was taken from IPUMS International). We only considered developing countries as they are more likely to endorse a culture where the role of the woman is a procreation one. The 31 censuses considered are: Bolivia (2001), Brazil (200), Chile (2002), Colombia (2005), Costa Rica (2000), Ecuador (2001), Jamaica (2001), Mexico (2012), Nicaragua (2005), Panama (2000), Peru (2007), Uruguay (1996), Venezuela (2001), Ghana (2000), Guinea (1996), Kenya (1999), Morocco (2004), Mali (1998), Malawi (2008), Rwanda (2002), Senegal (2002), Sierra Leone (2004), Tanzania (2002), Uganda (2002), South Africa (2001), Indonesia (1995), Cambodia (2008), Nepal (2001), Thailand (2000), Vietnam (2009) and Palestine (West Bank and Gaza) (1997). Among these countries, Vietnamese married women with 9 years of schooling have the lowest rate of childlessness, 1.26% (2,626 married women were childless among 208,761 married women). However, this number might underestimate natural sterility. A first reason for this is that it may be affected by divorce rates caused by fecundity problems. Another possible argument is that sterile vietnamese couples adopt orphans and then answer to the number of children they have ever born as if these children were biologically theirs.

C.2 Non-labor income

In the main text, we assume that non-labor income does not depend on the education of individuals. Here, we relax this hypothesis with the following expression for a^i:

$$a^i = \beta w^i + \tilde{a}^i \tag{17}$$

where β describes the marginal impact of education on non-labor income (bequests and ones education might have common determinants, such as the education of the parents, see Hertz et al. (2007)). $\tilde{a}^i$ is the part of non labor income that is uncorrelated to education and differs for each individual: each adult draws a different $\tilde{a}^i$ from a log-normal distribution, whose mean and variance will be estimated as before. We then estimate the same model adding one parameter to the estimation, β. Results are provided in Table 19. We see that the estimates are similar comparing both specifications and that the estimated value for β is zero. Hence, the three facts exposed in Section 2 do not allow to identify an effect of education on income other than the one coming through the labor income.

Parameter	Value with $\beta = 0$	Value with $\beta > 0$
σ_a	0.247	0.249
$\bar{m}_a$	1.001	1.003
ν	9.362	9.438
$\hat{c}$	0.399	0.400
μ	0.272	0.270
$\underline{\theta}$	0.864	0.862
α	0.524	0.527
ϕ	0.206	0.206
η	0.114	0.110
δ^m	0.256	0.261
δ^f	0.077	0.081
β		0.000

Table 19: Identified Parameters with $\beta > 0$

C.3 Computation of the Standard Errors

Two methods are used in the literature to obtain the standard errors of parameters estimated by the simulated method of moments: bootstrapping and the delta method. The static nature of the model makes the bootstrapping method preferable to the delta method which is usually faster but tends to underestimate the standard errors. We first draw 200 random new samples with replacement from the original data. Each new bootstrap sample is of equal size of the original one (1,127,080 observations) but the frequency of each observation changes. For each of these new datasets we generate the 72 moments and estimate the corresponding parameters. We then compute the standard errors of these estimators. By doing so, the uncertainty surrounding the estimated parameters comes exclusively from the uncertainty around the estimated moments. The results are reported in Column (2) of Table 20, and in Table 4 of the main text.

	Model only (1)		Data only (2)		Both (3)	
	mean	s.e.	mean	s.e.	mean	s.e.
σ_a	0.259	0.010	0.265	0.012	0.254	0.010
$\bar{m}_a$	1.005	0.010	1.005	0.012	1.006	0.010
ν	9.315	0.132	9.241	0.146	9.306	0.134
$\hat{c}$	0.406	0.008	0.404	0.009	0.403	0.007
μ	0.256	0.015	0.257	0.013	0.262	0.011
$\underline{\theta}$	0.878	0.014	0.883	0.014	0.873	0.013
α	0.110	0.005	0.108	0.006	0.111	0.007
ϕ	0.524	0.004	0.527	0.005	0.524	0.005
η	0.209	0.003	0.210	0.003	0.209	0.003
δ^m	0.275	0.018	0.276	0.015	0.268	0.014
δ^f	0.095	0.016	0.096	0.013	0.088	0.013

Table 20: Mean and Standard Errors of Parameters

We expected the uncertainty coming from the randomness of the artificial population used to simulate the model to be minimal when drawing a large enough number of individuals (typically, it is $100,000$ per education category). To check this expectation we also provide in Column (1) of Table 20 the parameters when the only uncertainty comes from the model: that is when we estimate the parameters 200 times using the same empirical moments but drawing different households from the distribution. Column (3) of Table 20 presents the parameters when we combine both uncertainties, using the empirical moments from the bootstrap samples and drawing different households from the distribution. The difference between Columns (2) and (3) is very small.

C.4 Identified Parameters for Subsamples

Identification by Race or Ethnicity. In the main text we have assumed an homogeneous marriage market. Here we assume instead that there are fragmented markets for each race separately (see Appendix A.6.1 for how the groups are constructed). We therefore reestimate the parameters for each race independently. The results are provided in Table 21.

Some parameters show remarkable constancy across races, such as the mean of the non labor income m_a, the rearing cost ϕ and η, the bargaining power parameter $\underline{\theta}$. There are strong

Parameter	All	Blacks	Whites	Natives	Asians	Hispanics
σ_a	0.247	0.552	0.222	0.290	0.226	0.395
$\bar{m}_a$	1.001	1.072	0.964	1.032	0.955	1.111
ν	9.362	7.986	8.447	9.330	9.422	8.932
$\hat{c}$	0.399	0.358	0.406	0.344	0.279	0.280
μ	0.272	0.359	0.294	0.299	0.289	0.256
$\underline{\theta}$	0.864	1.000	0.852	0.875	0.754	0.830
α	0.524	0.660	0.540	0.606	0.503	0.589
ϕ	0.206	0.201	0.229	0.191	0.202	0.214
η	0.114	0.074	0.082	0.092	0.119	0.085
δ^m	0.256	0.349	0.121	0.294	0.192	0.251
δ^f	0.077	0.079	0.065	0.190	0.060	0.119
% voluntary (IV+X)	8.1%	6.1%	7.3%	3.4%	10.4%	5.8%
% social (II+VII)	2.4%	5.1%	5.8%	1.8%	2.3%	3.4%
% natural	2.3%	2.3%	2.3%	2.4%	2.3%	2.3%

Table 21: Identified Parameters by Race or Ethnicity

variations in the δ^i's across races. Black single men loose 34.9% of their time endowment compared to married black men, while this loss equals 25.6% in the whole population. Asian single men only loose 19.2%. Black fathers are characterized by a lower involvement in childrearing. Non-labor income is less dispersed for Whites and more dispersed for Blacks and Hispanics.

The breakdown of childlessness into its components shows that blacks are relatively more affected by involuntary childlessness (social), while Asians are more into voluntary childlessness than all the other groups.

Identification Removing Disabled. Since 83.5% of single childless women with no schooling are disabled, we also identify the parameters of the model after removing the disabled from the data. The results are shown in Table 22. We can check that the parameter $\hat{c}$ still plays a role and that its value is not significantly different from its benchmark estimate. The estimated social sterility drops with respect to the benchmark.

Parameter	"All"	Without "disabled"
σ_a	0.247	0.237
$\bar{m}_a$	1.001	1.025
ν	9.362	9.342
$\hat{c}$	0.399	0.390
μ	0.272	0.265
$\underline{\theta}$	0.864	0.802*
α	0.524	0.511*
ϕ	0.206	0.210
η	0.114	0.115
δ^m	0.256	0.248
δ^f	0.077	0.085
% voluntary (IV+X)	8.1%	8.0%
% social (II+VII)	2.4%	2.1%
% natural	2.3%	2.3%

* indicates a significant difference from "All".

Table 22: Identified Parameters Without the "Disabled"

C.5 Alternative Specification for θ

In this appendix, we provide results under two alternative specifications for the bargaining power θ. The first fixes the bargaining power to 1/2, independently of relative education levels. The second makes bargaining depend on relative potential incomes, hence including non labor incomes a^f and a^m.

First, assume $\underline{\theta} = 1$ (equal negotiation power of spouses). Table 23 reports the implied parameter values. The objective $f(p)$ and the R^2 deteriorate. The cost of marriage drastically increases for a highly educated individual matched with someone with a low education: he/she will have to give up a large part of his/her income to ensure equal consumption in the household. As a result, highly educated will reject unions with lowly educated more often and the percentage of married people in the extreme education categories decreases. As heterogamous unions become less valuable, marriage rates also decrease on average. But this is partly compensated by a rise in the parameter δ^m. Finally, as poor women are less often married, they are also less protected against social sterility when $\underline{\theta} = 1$. Then the estimates of both types of childlessness increase.

Second, we include non-labor incomes into the bargaining power of each spouse as follows:

$$\theta(w^f, w^m, a^f, a^m) = \frac{1}{2}\underline{\theta} + (1-\underline{\theta})\frac{w^f + a^f}{w^f + w^m + a^f + a^m}$$

This specification leads to the results shown in Table 23. We see that most of the parameters are significantly different but that the match of the model with the data is still very good. Some moments are better matched with one specification of θ while others with the other.

C.6 Changes in the Time Allocation Parameter α

The model assumes that the share that mothers (α) and fathers ($1-\alpha$) devote to childrearing is constant across education groups and over time. Bianchi et al. (2004) show that the ratio of married mothers' to married fathers' time in child care declined between the mid-1960s and the late 1990s, which could be an indication that either social norms changed for all education categories, or that the increase in the education of women makes it optimal for couples to have the father more involved in the raising of children.

To have some insights on the role of α we carried out the following two experiments. First, fixing $\alpha = 1$ instead of identifying it from the cross sectional data, and re-estimating the rest of the parameters, we find that the quality of the match is lower: the model becomes unable to reproduce (a) a reasonable marriage rate (especially for highly educated women who have lost their incentive to marry), (b) childlessness rates for highly educated married women (for whom the cost of raising children becomes extremely high so they have more incentives to be voluntarily childless), (c) the gap between the fertility of married mothers and that of single mothers, who now face the same opportunity cost. Hence, allowing $\alpha < 1$ is important to generate the nice features of the model.

Second, consider that couples set α optimally under the constraint $\alpha \in [1/2, 1]$. A full-fledged model with proper bargaining on α would be the topic of another paper; however, a simple benchmark would be to assume the following rule:

$$\alpha = \begin{cases} 1 & \text{if } w^f < w^m \\ 1/2 & \text{if } w^f \geq w^m. \end{cases}$$

With this specification the marriage rates are reasonable, the U-shaped relationship between childlessness and education of married women is preserved, but the model fails in reproducing the high fertility of poorly educated married mothers (by about one child for the two lowest

Parameter	$\theta(w^f, w^m)$	$\theta = 1/2$	$\theta(w^f, w^m, a^f, a^m)$
$f(p)$	0.426	0.599	0.449
R^2			
childlessness single	0.847	0.652	0.733
childlessness married	0.934	0.872	0.952
fertility single	0.903	0.913	0.912
fertility married	0.960	0.954	0.950
marriage female	0.453	0.315	0.441
marriage male	0.659	0.732	0.791
σ_a	0.247	0.273*	0.254
$\bar{m}_a$	1.001	1.002	1.046*
ν	9.362	9.202	9.447
$\hat{c}$	0.399	0.393	0.437*
μ	0.272	0.276	0.246*
$\underline{\theta}$	0.864	1.000	0.955*
α	0.524	0.523	0.518
ϕ	0.206	0.206	0.214
η	0.114	0.112	0.100*
δ^m	0.256	0.304*	0.347*
δ^f	0.077	0.073*	0.121*
% voluntary (IV+X)	8.1%	8.6%	7.7%
% social (II+VII)	2.4%	2.7%	2.2%
% natural	2.3%	2.3%	2.3%

* indicates a significant difference from the benchmark $\theta(w^f, w^m)$.

Table 23: Identified Parameters for Alternative Specifications of the Bargaining Power

education categories), as poor married mothers face almost the same incentives as poor single mothers. Otherwise, the simple *ad-hoc* rule above does rather well, which indicates that bargaining over α could be a promising extension. Finally, we also made α depend on the education of the mother, independently of the education of the father: $\alpha = 1 - w^f/2$. The properties of this simulation are similar to those in the previous case.

C.7 Assortative Matching

The number of marriages by education category from the data are given in Table 24. This table is constructed in the following way: first, we drop from the data all the individuals who are not married (MARST > 1) or who do not have an identified partner in the Census (SPLOC= 0). Then, we sort observations, first, by their serial number, corresponding to the household, and then by their sex, so that the man of the household comes before his wife in the data. We then generate a variable saying that the husband has a corresponding wife after him (the serial number for both has to be the same). The last step is to generate a variable with the education of the husband and another variable with the education of the wife. The ratio

$$\frac{\pi(i,j)}{\sum_j \pi(i,j)}$$

gives the proportion of men of type j having married a woman of type i, where $\pi(i,j)$ is the number of couples in which the woman has an education level i and the man an education level j. Dividing this number by the proportion of women of type i in the total population,

$$\frac{\sum_i \pi(i,j)}{\sum_{i,j} \pi(i,j)},$$

we obtain the data in Table 25. Each cell gives, for each married man, his increased chances marrying a woman in a given category of education, compared with a purely random matching framework. If there was no assortativeness, all the cells would equal 1. The first cell means that a man of category 1 has 57 times more chances of marrying a woman of education category 1 than in the case of pure random matching. We are able to compute similar statistics with the simulated data. These are provided at the bottom of Table 25. We see that the model generates some assortative matching endogenously, although lower than in the data. Regressing each cell in the top part of the table on the corresponding cell in the bottom part leads to an R^2 of 0.138 meaning that we are able to account for 13.8% of the variations in the assortativeness matching.

To assess the influence of having abstracted from exogenous assortative matching on the estimated parameters, we conducted the following exercise. A share, λ, of the population draws a potential spouse from his/her education category. The remaining share $1-\lambda$ draws his/her partner randomly from the whole population. For different values of λ, we reestimate the parameters, minimizing the same objective as before. Table 26 presents the results. The case $\lambda = 0$ refers to the benchmark case of Table 4. The parameters which seems to be the most sensitive to changing λ are α and $\underline{\theta}$: these are significantly different from the benchmark for all the different given values of λ.

Table 26 also provides the fit of the model with respect to each of the moments used for estimating the parameters. These R^2 are computed as follows:

$$R^2 = 1 - \frac{\sum_{k=1}^{12} (d_k - s_k)^2}{\sum_{k=1}^{12} (d_k - \overline{d}_k)^2}$$

where d_k and s_k are respectively the empirical and simulated moments for each education category k. Comparing the different R^2, we see that the simulated moments are quite insensitive to the choice of λ. For childlessness rates and fertility of single women, the best specification is the benchmark, for the fertility of married women, it is $\lambda = 0.2$ or $\lambda = 0.3$, and for marriage rates, $\lambda = 0.4$.

The value of λ that minimizes the objective function $f(p)$ is 0.1. For $\lambda = 0.1$, we obtain the marriage matrix presented in Table 27. Regressing now each cell in the top part of Table 25 on the corresponding cell of Table 27 leads to an R^2 of 0.784. We conclude that introducing some exogenous assortative matching, captured by $\lambda > 0$, allows us to get much closer to the observed assortativeness, without modifying the estimated parameters or the ability of the model to reproduce the targeted empirical moments very much.

	1	2	3	4	5	6	7	8	9	10	11	12
1	71,179	13,086	17,869	3,842	2,971	1,751	13,915	4,122	1,630	1,431	988	332
2	14,465	68,397	53,512	7,801	5,114	3,638	18,566	5,054	1,398	1,543	778	104
3	26,113	98,004	472,453	79,705	72,697	43,995	229,332	58,307	14,805	14,369	7,674	1,004
4	6,114	23,070	135,612	88,745	53,731	34,170	151,668	38,420	7,745	8,634	3,924	560
5	6,353	22,146	159,403	69,339	132,115	62,352	282,405	75,275	16,301	18,667	7,760	1,026
6	4,745	15,159	125,973	58,541	83,086	94,623	256,444	73,902	14,284	17,530	7,061	1,066
7	23,125	49,898	565,508	264,819	380,715	300,185	3,455,022	1,261,526	283,977	595,232	251,575	28,998
8	4,840	8,097	92,315	48,220	71,447	57,596	672,543	837,035	159,921	525,637	300,613	44,805
9	1,337	2,726	23,806	12,216	17,571	15,488	186,588	169,093	98,724	188,244	125,071	16,869
10	1,245	2,095	17,462	8,040	12,546	11,090	170,890	209,703	56,760	548,646	445,586	86,506
11	916	1,519	11,343	5,403	8,345	7,326	96,804	105,051	28,313	180,051	292,392	87,314
12	146	63	575	231	374	234	3,409	3,883	1,541	9,598	17,766	22,918

Table 24: Marriages per Education Category. Men in Columns, Women in Rows

	1	2	3	4	5	6	7	8	9	10	11	12
Data												
1	57.233	5.553	1.377	0.767	0.456	0.357	0.324	0.187	0.307	0.088	0.087	0.147
2	8.584	21.421	3.043	1.149	0.580	0.548	0.319	0.169	0.194	0.070	0.051	0.034
3	2.499	4.950	4.332	1.893	1.329	1.069	0.636	0.315	0.332	0.105	0.081	0.053
4	1.185	2.359	2.518	4.268	1.989	1.681	0.852	0.421	0.352	0.127	0.084	0.060
5	0.797	1.466	1.916	2.159	3.166	1.986	1.027	0.534	0.479	0.178	0.107	0.071
6	0.675	1.138	1.717	2.067	2.258	3.418	1.058	0.594	0.476	0.190	0.110	0.084
7	0.332	0.378	0.777	0.943	1.043	1.093	1.437	1.023	0.955	0.650	0.397	0.229
8	0.184	0.162	0.335	0.454	0.517	0.554	0.739	1.794	1.421	1.517	1.253	0.936
9	0.167	0.180	0.285	0.378	0.419	0.491	0.675	1.192	2.886	1.788	1.715	1.160
10	0.085	0.075	0.114	0.136	0.163	0.192	0.338	0.808	0.906	2.846	3.337	3.248
11	0.119	0.104	0.141	0.174	0.207	0.241	0.364	0.770	0.861	1.779	4.170	6.242
12	0.257	0.059	0.097	0.101	0.126	0.105	0.174	0.387	0.636	1.287	3.441	22.248
Simulation												
1	1.123	1.117	1.056	1.036	1.025	1.010	1.008	0.997	0.992	0.960	0.946	0.831
2	1.126	1.115	1.058	1.039	1.028	1.016	1.009	0.996	0.992	0.955	0.942	0.852
3	1.128	1.112	1.060	1.037	1.024	1.015	1.005	0.991	0.984	0.962	0.951	0.896
4	1.115	1.095	1.047	1.027	1.018	1.012	1.002	0.993	0.985	0.970	0.964	0.931
5	1.092	1.078	1.037	1.025	1.015	1.011	1.001	0.994	0.987	0.977	0.972	0.945
6	1.072	1.053	1.025	1.018	1.010	1.010	1.001	0.996	0.988	0.984	0.980	0.967
7	1.023	1.019	1.012	1.010	1.004	1.004	1.000	0.998	0.993	0.993	0.991	0.993
8	0.984	0.988	0.995	0.997	1.001	1.000	1.000	1.001	1.000	1.003	1.003	1.014
9	0.933	0.948	0.975	0.985	0.992	0.990	1.000	1.004	1.007	1.015	1.018	1.037
10	0.822	0.853	0.920	0.942	0.967	0.975	0.996	1.015	1.030	1.049	1.061	1.101
11	0.770	0.794	0.883	0.916	0.953	0.962	0.993	1.019	1.047	1.073	1.091	1.142
12	0.594	0.627	0.723	0.795	0.846	0.901	0.966	1.030	1.106	1.198	1.255	1.383

Table 25: Assortative Matching in the Data and Generated by the Model with $\lambda = 0$, Men's Education in Columns, Women's in Rows

λ	0	0.1	0.2	0.3	0.4	0.5
$f(p)$	0.426	0.421	0.457	0.479	0.523	0.606
R^2						
childlessness single	0.847	0.837	0.843	0.805	0.777	0.757
childlessness married	0.934	0.931	0.925	0.924	0.918	0.913
fertility single	0.903	0.897	0.875	0.882	0.886	0.875
fertility married	0.960	0.970	0.973	0.973	0.940	0.871
marriage female	0.453	0.476	0.455	0.555	0.567	0.498
marriage male	0.659	0.676	0.655	0.605	0.694	0.422
σ_a	0.247	0.252	0.252	0.284*	0.281*	0.284*
$\bar{m}_a$	1.001	1.002	0.999	0.990*	1.003	1.037
ν	9.362	9.074*	9.300	8.633*	8.557*	8.374*
$\hat{c}$	0.399	0.386	0.360*	0.336*	0.332*	0.308*
μ	0.272	0.253	0.264	0.252	0.257	0.284
$\underline{\theta}$	0.864	0.908*	0.898*	0.920*	0.990*	0.950*
α	0.524	0.504*	0.500*	0.503*	0.500*	0.500*
ϕ	0.206	0.213	0.208	0.218*	0.221*	0.224*
η	0.114	0.120*	0.109	0.120*	0.116	0.109
δ^m	0.256	0.287*	0.262	0.254	0.278	0.218*
δ^f	0.077	0.090	0.064	0.070	0.076	0.048*

* indicates a significant difference from the case $\lambda = 0$.

Table 26: Identified Parameters for λ between 0 and 0.5

	1	2	3	4	5	6	7	8	9	10	11	12
1	10.123	1.02	0.964	0.943	0.927	0.913	0.91	0.895	0.897	0.84	0.831	0.708
2	1.026	7.223	0.97	0.939	0.928	0.914	0.907	0.892	0.894	0.842	0.841	0.73
3	1.021	1.009	2.087	0.932	0.919	0.909	0.903	0.889	0.894	0.858	0.856	0.774
4	1.002	0.988	0.947	3.727	0.912	0.908	0.902	0.892	0.896	0.869	0.872	0.811
5	0.985	0.968	0.935	0.921	3.068	0.909	0.901	0.894	0.898	0.875	0.879	0.826
6	0.951	0.938	0.925	0.912	0.902	3.722	0.901	0.896	0.899	0.884	0.888	0.847
7	0.908	0.912	0.909	0.905	0.897	0.902	1.218	0.898	0.904	0.892	0.898	0.874
8	0.863	0.874	0.891	0.893	0.893	0.895	0.899	1.495	0.911	0.901	0.91	0.893
9	0.819	0.833	0.869	0.88	0.886	0.884	0.897	0.902	3.331	0.913	0.925	0.913
10	0.717	0.733	0.809	0.827	0.856	0.864	0.89	0.909	0.934	1.791	0.963	0.97
11	0.643	0.677	0.771	0.799	0.838	0.85	0.882	0.909	0.952	0.964	2.198	1.004
12	0.461	0.506	0.597	0.662	0.721	0.765	0.829	0.896	0.98	1.061	1.123	8.754

Table 27: Simulated Marriage Matrix for $\lambda = 0.1$, Men's Education in Columns, Women's in Rows