Childcare and Commitment within Households

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Abstract

Parental time with children increases with the education of both the mother and the father. As the education of parents increases, the gap between childcare supplied by mothers relative to that supplied by fathers decreases. A two steps semi-cooperative marital decision model is proposed to explain these two facts. First, parents collectively choose the amount of labor to supply and, in a second step, each of them chooses the amount of childcare as the outcome of a Cournot game. This framework gives rise to indeterminacy of the equilibrium and four selection criteria are proposed: one of a machist society, one of a feminist society, one of a random equilibrium and a last one that estimates the degree of social gender bias towards men. The semi-cooperative theoretical frameworks with the random selection criterion and the criterion that estimates the bias towards men provide the best match with the data.

Keywords: Childcare, Education, Commitment, Semi-Cooperative Model.
JEL Classification Numbers: J11; J13; J16; O11.

1 Introduction

Can spouses commit to allocate a given amount of their time to childcare? The answer suggested by this paper is that it does not seem to be happening. This raises the question of what type of marital decision process determines childcare.

The question is relevant for three reasons, at least. First, time spent with children by parents affects the education of children and hence, fosters human capital accumulation (Lundborg et al. (2012)). Childcare arrangements also provides an additional incentive for two individuals to marry other than marital bliss and economies of scale in sharing the cost of forming and maintaining a household, already present in the literature (Greenwood et al. (2012) and Greenwood et al. (2003)). Finally, a more equal distribution of childcare within households could decrease the opportunity cost of raising children faced by highly educated women, leading to an increase in fertility rates (Baudin et al. (2012)).

This study focuses on the following facts: (i) the amount of time parents devote to their children increases with their education as well as that of their partner, and (ii), across households where both partners have the same level of education, the time spent in childrearing by women relative to men decreases as the education of partners increases. These facts have already been noticed over time by Bianchi et al. (2004). They find that between 1985 and the end of 1990, the time that parents spent in childcare activities increased and that the ratio of time spent in childcare by married mothers relative to married fathers decreased. Ramey and Ramey (2010) support that one major cause for the increase of time spent by parents on childcare since the mid-1990s is the higher competition for college admissions. The focus of this paper is on how does the education of parents affect childcare.[1]

To address what type of marital decision process is coherent with the facts, I compare the respective merits of two theoretical models. The first is a collective decision model *à la* Chiappori (1988) where households maximize a weighted sum of individual utilities leading to efficient choices in the sense of Pareto. The implicit assumption of this framework is that households can credibly commit to respect their decisions. This makes sense relative to labor decisions where contracts are usually signed with an employer but no such a thing exists for childcare duties. Marriage contracts do not have any clause on the amount of childcare that each spouse should provide. Moreover, the lack of monitoring from the partner

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[1]Childcare is different from other non market-work activities. For example, time spent in housework activities by women decrease with their education and that of their husband. This could mean that other non market-work activities have a closer market substitute than childcare (Guryan et al. (2008)). The distinction between non market work activities and leisure is that leisure does not have a market substitute .

incites deviations from any previous agreement relative to childcare. The second theoretical model, proposed in this paper to explain the facts relating education to childcare, is a semi-cooperative model that assumes that labor supplies are cooperatively chosen in a first step and that the provision of childcare is decided later as the outcome of a Cournot game between partners.[2]

Non-cooperative marital decision models have been used to study saving decisions (Anderson and Baland (2002)) or to assess how to break the income pooling result that is implied by the unitary and collective models (Doepke and Tertilt (2011)). Cigno (2012) also proposes a non-cooperative framework to explain childcare decisions when there is no credible compensation promise. Meier and Rainer (2012) also use a non-cooperative framework to analyze the optimal family taxation scheme. In particular to family choices, d'Aspremont and Dos Santos Ferreira (2013) argue that semi-cooperation is plausible when there is joint contribution to more than one public good. Cigno (2012) and d'Aspremont and Dos Santos Ferreira (2013) do not try however to match the theory with the data, which is the concern of this paper. To my knowledge, this is the first paper to provide an empirical application of a semi-cooperative model.

The underlying assumptions of the theory are the following ones. Consumption of material goods is entirely public within households, while consumption of leisure is entirely private, as in Guvenen and Rendall (2012). As in de la Croix and Doepke (2003), individuals care about the quality of their children and child quality is also a public good for the household. Child quality depends on the amount of time invested by parents, their education and a random component which differs between households and can be interpreted as innate ability. Men and women are substitutes in producing child quality.[3]

As has already been said, the amount of childcare supplied by each spouse, in the semi-cooperative model, is decided non-cooperatively. This implies that the positive externality of childcare on the utility of the couple is not internalized by the partners. This results in an inefficient under-provision of care.[4] This framework also implies that, given the education of parents, there will be multiple equilibria. This happens because, in the first stage, when choosing labor supplies, couples believe on a future arrangement with respect to childcare. In the second stage, this arrangement will be a Nash equilibrium. Different beliefs then lead

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[2]This type of setup has already been applied to firm level decisions concerning R&D (see d'Aspremont and Jacquemin (1988)).

[3]Appendix C provides a discussion about the substitutability vs. the complementarity of parents in producing child quality and analyzes the case where parents are complements. I abstract from schooling decisions. See de la Croix and Doepke (2004) on this issue.

[4]This result is already present in Cigno (2012). Inefficient investment in childcare might also be caused by the absence of insurance markets and the presence of borrowing constraint (Aiyagari et al. (2002)).

to different "family types" in terms of time allocation. Differences in beliefs could come from different social norms and indeterminacy then reflects other sources of heterogeneity than the education of spouses.[5]

In order to assess what is the theoretical framework that is most compatible with the facts, the deep parameters are estimated using the Simulated Method of Moments. The identified parameters are then used to compare each model with the facts. The collective choice model is unable to reproduce that childcare supplied by men increases with the education of their wives. The reason is that the presence of a gender wage gap makes it efficient for the couple to choose an arrangement where the man specializes in the supply of labor and the woman in the supply of childcare.

In the semi-cooperative setup, the multiplicity of equilibria allows couples to be in different types of family arrangements, in particular where the man also supplies childcare. Four selection criteria for choosing among the equilibria are compared. The first is a random selection criteria, where each possible equilibrium has the same probability of being selected. The second chooses the equilibrium that maximizes the utility of the male partner. The third is the symmetric case, the one that maximizes the utility of the female partner. These last two could be the expected outcomes from a very machist or feminist society. The last criterion estimates the degree of machism in the society. The criteria allowing the semi-cooperative framework to replicate the facts are the first and the last. In particular, they can replicate that female childcare relative to male decreases as the education of partners increases. This is due to a composition effect: as the education of partners increases, there are more households where both partners work and the man supplies childcare. This generates at the macro level a shrink in the gap between the time supplied by men and women for highly educated couples. The semi-cooperative framework with random selection criterion is then used to estimate the inefficiency in terms of childcare. The result is that children would gain in average 70 minutes of childcare if their parents cooperated. Another exercise shows that, compared to the collective framework, semi-cooperative households' decisions relative to childcare are less sensitive to changes in the gender wage gap.

The rest of the paper has the following outline. Section 2 exposes the facts. Section 3 describes both the collective and semi-cooperative models. The parameters of each of these theoretical frameworks are estimated in Section 4. Section 5 uses the identified parameters to simulate each model and compare it to the facts and provides counterfactual exercises. Section 6 provides a conclusion.

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[5]One equilibrium is the "separate spheres" equilibrium where each spouse specialized in the production of one public good (Lundberg and Pollak (1993)). This can, for instance, reflect traditional social norms.

2 Facts

The data is taken from the American Time Use Survey (ATUS) and the Current Population Survey for the years 2003 to 2011. The two datasets are merged using the ATUS-X project. The sample considered consists in men and women between ages 25 to 55, who live with their spouse, have at least one child under 18 years of age in the household, with no other adult living in the household and for which information on both partner's education is available. This results in 36,144 observations, 19,314 women and 16,830 men (each observation has a weight given by the variable WT06). Each individual is assigned to one out of seven categories of education, depending on his or her education level. Table 1 shows the number of observation and the average number of years of schooling, e, by education category ("Nb.").

The relationship between the amount of childcare per child, his (her) education and the education of her (his) partner is described by the following descriptive model:

$$t_i = \beta_1 e_i + \beta_2 e_{-i} + \beta_3 X_{i,-i} + \epsilon_i \tag{1}$$

where t_i is the number of minutes per day that individual i spends caring for and helping household children divided by the number of children in the household. This includes activities related to children's education and health. e_i is the education category of this person, e_{-i} is the education of the spouse or unmarried partner. $X_{i,-i}$ is a set of controls related to the person or the partner. It includes dummies for the age group of the mother (< 25, 25-29, 30-34, 35-39, 40-44, 45-49, 50-55)[6], the age group of the youngest child (0, 1-2, 3-5, 6-12, 13-18) as well as the race and Hispanic origin. ϵ_i captures omitted factors affecting t_i.

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[6]The age group < 25 for the age of the mother is only relevant when the respondent is a man, as I only look at respondents between 25-55 years old.

Nb.	e	Education Level	Observations
1	5	No Education to Grade 8	1,245
2	10	Grades 9 to 12, no diploma	1,763
3	12	High School Diploma, no college	8,524
4	13	Some College but no degree	5,879
5	14	Associate Degree, Occupational/Vocational or Academic Program	3,853
6	16	Bachelor's Degree	9,641
7	18	Master's Degree, Professional School and Doctorate Degree	5,239

Table 1: Number of (unweighted) observations by education category.

Education, Women	Education, Men						
	1	2	3	4	5	6	7
1	12.685	20.727	26.152				
2	9.878	17.969	22.236	30.711			
3	12.317	21.315	26.324	32.127	32.611	34.297	33.481
4		23.515	27.288	31.852	31.933	34.403	33.580
5		27.291	33.838	38.702	39.781	40.730	38.402
6		33.031	35.508	40.451	41.743	42.571	41.335
7			41.350	47.871	47.926	50.160	48.535

Education, Women	Education, Men						
	1	2	3	4	5	6	7
1	34.521	28.390	35.226				
2	37.341	32.675	40.920	45.990			
3	42.271	40.954	48.958	53.620	54.609	61.847	62.650
4		39.961	51.183	55.531	54.919	63.699	65.015
5		47.602	52.476	55.417	54.277	62.486	67.226
6		50.335	61.398	65.126	64.452	74.237	76.463
7			66.999	69.528	69.018	77.223	79.714

Table 2: Top: Minutes per day spent in childcare by men per child as a function of his education and of his wife or female partner's. Down: Minutes per day spent in childcare by women per child as a function of her education and of her husband or male partner's.

Table 2 shows the predicted values, from a linear regression, of t_i with respect to the education of i and the education of the spouse or unmarried partner.[7] The estimates are computed for white individuals where the youngest child is between 3 and 5 years old and the mother's age is between 35-39 years. Estimates for the cases with less than 40 observations are not considered. We can highlight the following facts.

(1) The time spent caring and helping children increases with education.

For both men and women, the amount of time spent in childcare activities increases with their education. Women who have an education level lower than grade 8 provide, in average, between 34.5 and 42.3 minutes of childcare per day, depending on their husband's education. Women with the highest levels of education supply between 62.7 and 79.7 minutes per day of childcare activities. The difference for men is between 9.9 and 12.7 for those with the lowest education and between 33.5 and 48.5 for those with the highest.

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[7]The estimates obtained from a Poisson and a Tobit model are similar to the ones shown.

(2) The time spent caring and helping children increases with the education of the partner.

The amount of childcare also increases with the education of the partner. A woman with a high school diploma coupled with a very uneducated man supplies in average 35.2 minutes of childcare per day while a woman with the same education coupled with a highly educated man supplies 67.0 minutes per day of childcare. A man with a high school diploma supplies in average 26.2 minutes per day if his partner has low education while he supplies 41.4 minutes per day with a highly educated woman.

(3) For households where the partners have the same education, the amount of childcare done by women relative to men decreases as the education of both increases.

The ATUS data has information about the time allocation of only one member in the household. We can however compare the amount of time supplied by women relative to men in the same type of household in terms of education of the partners. Within couples where both partners are in category 1 in terms of education, the amount of childcare supplied by men relative to that supplied by women is 36.7%. For couples where both are in category 7, the ratio of what men do compared to women is 60.9%. This implies that, eventhough the gap in childcare between genders is always present, it shrinks as the level of education of both partners increases.[8]

Next Section provides the theoretical frameworks that are used to explain childcare choices within households.

3 The Model

3.1 Setup

The economy is populated by couples composed by a man and a woman, respectively denoted by $i = m, f$. Individuals differ in their education level, e_i. Each individual has one unit of time that is divided between leisure, l_i, work, L_i, and childcare. Childcare has two components, an exogenous component related to the minimal time that children require

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[8]Facts *(1)* to *(3)* are robust if we only consider married couples and if we look at the overall childcare individuals supply instead of the childcare per child (see Appendix A). I abstract from the possibility of buying childcare services from the market. All types of childcare (basic, educational, recreational, and travel related) increase with the education of the parents (Guryan et al. (2008)). This supports the idea that nannies are poor substitutes to the time of highly educated parents (see also Hallberg and Klevmarken (2002)). See Hazan and Zoabi (2012) on this marketization hypothesis and how it affects fertility rates.

their parents, $\bar{t}_i$, and an endogenous component related to activities that enhance their human capital, t_i.[9] The time constraint for an individual i is

$$1 \geq l_i + L_i + \left(t_i + \bar{t}_i\right) n \tag{2}$$

where n is the number of children in the household, assumed exogenous.

Time spent by parents in providing education for their children translates into child quality, q, as follows:

$$q = t_f e_f^\alpha + t_m e_m^\alpha + \bar{q} \tag{3}$$

where $\alpha > 0$ represents the returns for one more year of parent education in terms of child quality.[10] This specification assumes that men and women are substitutes in the efficient time that each of them supplies for the production of the quality of the children. Child quality also depends on the innate ability of children, $\bar{q}$, which is drawn from a distribution $\mathcal{F}\left(\bar{q}_{me}, \bar{q}_{se}\right)$.[11]

Within couples, resources are used to buy public goods for the household, denoted by c. The budget constraint of a couple is,

$$c = w_f e_f L_f + w_m e_m L_m \tag{4}$$

where w_f and w_m are the respective wages of men and women for a given education level.

Assuming that men and women have identical preferences, the individual's utility is:

$$u_i = \ln c + \mu \ln l_i + \gamma \ln\left(qn\right) \tag{5}$$

where $\mu > 0$ and $\gamma > 0$ are respectively preference parameters for leisure and total quality of children.

In the following two subsections I will expose the two theoretical frameworks that will be compared under the assumptions that were just exposed.

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[9]The fixed amount of time that a child needs from their parents includes childbearing for women, but also includes all the activities that are not related to providing child quality. Setting $\bar{t}_i = 0$ do not change the theoretical results, but allowing for $\bar{t}_i > 0$, helps in having the right levels of childrearing in Section 4.

[10]Lundborg et al. (2012) show that parental education has a positive effect on cognitive skills and health status of sons.

[11]See Aiyagari et al. (2002) for a formulation where innate ability of children depends on that of the parents.

3.2 Collective Decision Process

In the cooperative setup, households make joint decisions concerning consumption, labor supplies, childcare and leisure. Denoting by $\theta \in (0,1)$ the marital bargaining power of the woman, couples maximize a weighted sum of individual utilities:[12]

$$U = \ln c + \theta \mu \ln l_f + (1 - \theta)\mu \ln l_m + \gamma \ln(qn) \tag{6}$$

subject to (2), (3), (4), $t_i \geq 0$ and $L_i \geq 0$. From the properties of the individual utility function (5), neither consumption nor leisure can be nil. Denoting $A_f = 1 - \bar{t}_f n$ and $A_m = 1 - \bar{t}_m n$, the Kuhn-Tucker (KT) conditions are,

$$a + \frac{\gamma e_f^\alpha}{t_f e_f^\alpha + t_m e_m^\alpha + \bar{q}} - \frac{\theta \mu n}{A_f - L_f - t_f n} = 0,$$

$$b + \frac{\gamma e_m^\alpha}{t_f e_f^\alpha + t_m e_m^\alpha + \bar{q}} - \frac{(1 - \theta)\mu n}{A_m - L_m - t_m n} = 0,$$

$$c + \frac{w_f e_f}{w_f e_f L_f + w_m e_m L_m} - \frac{\theta \mu}{A_f - L_f - t_f n} = 0,$$

$$d + \frac{w_m e_m}{w_f e_f L_f + w_m e_m L_m} - \frac{(1 - \theta)\mu}{A_m - L_m - t_m n} = 0,$$

$at_f = 0$, $bt_m = 0$, $cL_f = 0$, $dL_m = 0$, $t_f \geq 0$, $t_m \geq 0$, $L_f \geq 0$, $L_m \geq 0$, $a \geq 0$, $b \geq 0$, $c \geq 0$ and $d \geq 0$. Where a, b, c and d are the KT multipliers. As the Hessian matrix of U is negative definite (so U is strictly concave) and the KT coefficients are positive, the sufficient condition for a maximum is satisfied.

There are 12 cases solving this problem. Only the solutions on childcare and labor are shown. Appendix B.1 provides details on KT coefficients and the conditions under which each case arises.

A1. Cases **A1a**, **A1b** and **A1c** characterize households in which parents provide only the minimal amount of childcare to their children, $\bar{t}_i$, so $t_f = 0$ and $t_m = 0$.[13] It concerns couples with low levels of education. Labor supplies for these first three cases are shown in Appendix B.1.

A1a. L'enfant Sauvage. If $a, b > 0$ and $c, d = 0$; $L_f, L_m > 0$.

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[12]Allowing for the bargaining parameter θ to depend on relative earnings does not change the results of the collective setup. The reason not to make it dependent on earnings allows us to focus on the differences between the commitment vs. non-commitment setups and leave aside the spousal bargaining channel.

[13]The cases where none of the parents provides quality childcare are labeled "l'enfant sauvage", meaning "the wild child". This makes illusion to the 1970's French movie of François Truffaut.

A1b. L'Enfant Sauvage and an Unemployed Husband. If $a, b, d > 0$ and $c = 0$; $L_f > 0$ and $L_m = 0$.

A1c. L'Enfant Sauvage and an Unemployed Wife. If $a, b, c > 0$ and $d = 0$; $L_f = 0$ and $L_m > 0$.

A2. The next three cases **A2a**, **A2b** and **A2c** characterize households where both parents contribute to the production of child quality; $t_f > 0$ and $t_m > 0$.

A2a. If $a, b, c, d = 0$, there is a continuum of positive values of $\{t_i, L_i\}$ that maximize the couple's utility. In this case men and women are equivalent in both working and providing quality childcare to children. Hence, any arrangement is efficient.

A2b. A Busy Wife. If $d > 0$ and $a, b, c = 0$; $L_m = 0$,

$$t_f = \frac{(\gamma + (1 - \theta)\mu)A_f e_f^\alpha - (1 + \theta\mu)(A_m e_m^\alpha + \bar{q}n)}{(1 + \gamma + \mu)n e_f^\alpha},$$

$$t_m = \frac{-(1 - \theta)\mu(A_f e_f^\alpha + \bar{q}n) + (1 + \gamma + \theta\mu)A_m e_m^\alpha}{(1 + \gamma + \mu)n e_m^\alpha} \quad \text{and}$$

$$L_f = \frac{A_f e_f^\alpha + A_m e_m^\alpha + \bar{q}n}{(1 + \gamma + \mu)e_f^\alpha}.$$

We can check that $\partial t_f/\partial e_f > 0$, $\partial t_f/\partial e_m < 0$, $\partial t_m/\partial e_f < 0$, $\partial t_m/\partial e_m > 0$, $\partial L_f/\partial e_f < 0$ and $\partial L_f/\partial e_m > 0$. As the time supplied by one partner is a substitute to the time supplied by the other, childcare decreases as the education of the partner increases. Fixing the education of the wife, an increase in the education of the husband increases her amount of labor supplied because she can reduce her amount of childcare. For a given education of the husband, an increase in the education of the wife increases the amount of child quality she provides. This decreases the total number of hours worked.

A2c. A Busy Husband. If $c > 0$ and $a, b, d = 0$; $L_f = 0$,

$$t_f = \frac{(1 + \gamma + (1 - \theta)\mu)A_f e_f^\alpha - \theta\mu(A_m e_m^\alpha + \bar{q}n)}{(1 + \gamma + \mu)n e_f^\alpha},$$

$$t_m = \frac{-(1 + (1 - \theta)\mu)(A_f e_f^\alpha + \bar{q}n) + (\gamma + \theta\mu)A_m e_m^\alpha}{(1 + \gamma + \mu)n e_m^\alpha} \quad \text{and}$$

$$L_m = \frac{A_f e_f^\alpha + A_m e_m^\alpha + \bar{q}n}{(1 + \gamma + \mu)e_m^\alpha}.$$

This case is symmetric to **A2b**: $\partial t_f/\partial e_f > 0$, $\partial t_f/\partial e_m < 0$, $\partial t_m/\partial e_f < 0$, $\partial t_m/\partial e_m > 0$, $\partial L_m/\partial e_f > 0$ and $\partial L_m/\partial e_m < 0$.

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A3. The next three cases **A3a**, **A3b** and **A3c** characterize households where the woman provides the minimal amount of childcare while her partner provides time to produce child quality; $t_f = 0$ and $t_m > 0$.

A3a. A Multi-Task Husband. If $a > 0$ and $b, c, d = 0$;

$$t_m = \frac{\gamma(A_f w_f e_f + A_m w_m e_m) - (1 + \mu)\bar{q}n w_m e_m^{1-\alpha}}{(1 + \gamma + \mu)n w_m e_m},$$

$$L_f = \frac{(1 + \gamma + (1-\theta)\mu)A_f w_f e_f - \theta\mu(A_m e_m^\alpha + \bar{q}n)w_m e_m^{1-\alpha}}{(1 + \gamma + \mu)w_f e_f} \quad \text{and}$$

$$L_m = \frac{-(\gamma + (1-\theta)\mu)A_f w_f e_f + (1+\theta\mu)(A_m e_m^\alpha + \bar{q}n)w_m e_m^{1-\alpha}}{(1 + \gamma + \mu)w_m e_m}.$$

In this case, we can check that $\partial t_m/\partial e_f > 0$, $\partial L_f/\partial e_f > 0$, $\partial L_m/\partial e_f < 0$,

$$\frac{\partial t_m}{\partial e_m} > 0 \iff e_m^{1-\alpha} > \frac{\gamma A_f w_f e_f}{\alpha(1+\mu)\bar{q}n w_m}, \quad \frac{\partial L_f}{\partial e_m} > 0 \iff e_m^\alpha < \frac{(\alpha-1)\bar{q}n}{A_m}$$

$$\text{and} \quad \frac{\partial L_m}{\partial e_m} > 0 \iff e_m^{1-\alpha} < \frac{(\gamma + (1-\theta)\mu)A_f w_f e_f}{\alpha(1+\theta\mu)\bar{q}n w_m}.$$

The sign of the derivatives of t_m, L_f and L_m with respect to e_f is unambiguous. An increase in the education of the wife increases her opportunity cost to be out of the labor market, hence, her labor supply increases. This allows the husband to substitute labor with childcare. The effects of an increase in the education of the husband are more complex and depend on the education levels of both partners. For instance, if both have a similar education level, an increase in e_m increases his opportunity cost in terms of labor income and also in terms of child quality. This implies that both labor and childcare supplied by the husband could increase. The increase in the labor supplied by the husband decreases the amount of labor supplied by the wife.

A3b. Modern Specialization. If $a, d > 0$ and $b, c = 0$; $L_m = 0$,

$$t_m = \frac{A_m \gamma e_m^\alpha - (1-\theta)\mu\bar{q}n}{(\gamma + (1-\theta)\mu)n e_m^\alpha} \quad \text{and} \quad L_f = \frac{A_f}{1 + \theta\mu}.$$

We can check that $\partial t_m/\partial e_m > 0$. Time allocated to childcare increases with the education of the husband because an increase in his education increases the cost of his leisure relative to child quality. L_f does not depend on e_f because of the logarithmic specification of the utility function. Choices on childcare and labor do not depend on the education of the partner because of specialization; as the wife only works and the husband only provides childcare, an increase in e_m has no effect on total consumption.

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A3c. An Unemployed Wife. If $a, c > 0$ and $b, d = 0$; $L_f = 0$,

$$t_m = \frac{A_m \gamma e_m^\alpha - (1 + (1 - \theta)\mu)\bar{q}n}{(1 + \gamma + (1 - \theta)\mu)ne_m^\alpha} \quad \text{and} \quad L_m = \frac{A_m e_m^\alpha + \bar{q}n}{(1 + \gamma + (1 - \theta)\mu)e_m^\alpha}.$$

Here choices do not depend on e_f as the wife does not provide any income and does not contribute to rising the quality of the children. As child quality is a luxury good for parents, $\partial t_m/\partial e_m > 0$ and $\partial L_m/\partial e_m < 0$.

A4. The last three cases **A4a**, **A4b** and **A4c** are symmetric to the last three ones. Now, it is the male partner who does not provide time for the production of child quality, $t_m = 0$, while the wife does, $t_f > 0$.

A4a. A Multi-Task Wife. If $b > 0$ and $a, c, d = 0$;

$$t_f = \frac{\gamma(A_f w_f e_f + A_m w_m e_m) - (1 + \mu)\bar{q}nw_f e_f^{1-\alpha}}{(1 + \gamma + \mu)nw_f e_f},$$

$$L_f = \frac{(1 + (1 - \theta)\mu)(A_f e_f^\alpha + \bar{q}n)w_f e_f^{1-\alpha} - (\gamma + \theta\mu)A_m w_m e_m}{(1 + \gamma + \mu)w_f e_f} \quad \text{and}$$

$$L_m = \frac{(1 + \gamma + \theta\mu)A_m w_m e_m - (1 - \theta)\mu A_f w_f e_f + \bar{q}nw_f e_f^{1-\alpha}}{(1 + \gamma + \mu)w_m e_m}.$$

The relationships of t_f, L_f and L_m with respect to e_m are monotonous: $\partial t_f/\partial e_m > 0$, $\partial L_f/\partial e_m < 0$ and $\partial L_m/\partial e_m > 0$. An increase in the education of the husband increases the amount of childcare provided by the wife because it increases the earnings of the couple so the wife can work less. The sign of the derivatives with respect to e_f depend, as in **A3a**, in the education of both partners:

$$\frac{\partial t_f}{\partial e_f} > 0 \iff e_m < \frac{\alpha(1 + \mu)\bar{q}nw_f e_f^{1-\alpha}}{\gamma A_m w_m}, \quad \frac{\partial L_f}{\partial e_f} > 0 \iff e_m > \frac{\alpha(1 + (1 - \theta)\mu)\bar{q}nw_f e_f^{1-\alpha}}{(\gamma + \theta\mu)A_m w_m}$$

$$\text{and} \quad \frac{\partial L_m}{\partial e_f} > 0 \iff e_f^\alpha < \frac{(\alpha - 1)\bar{q}n}{A_f}.$$

A4b. An Unemployed Husband. If $b, d > 0$ and $a, c = 0$; $L_m = 0$,

$$t_f = \frac{A_f \gamma e_f^\alpha - (1 + \theta\mu)\bar{q}n}{(1 + \gamma + \theta\mu)ne_f^\alpha} \quad \text{and} \quad L_f = \frac{A_f e_f^\alpha + \bar{q}n}{(1 + \gamma + \theta\mu)e_f^\alpha}.$$

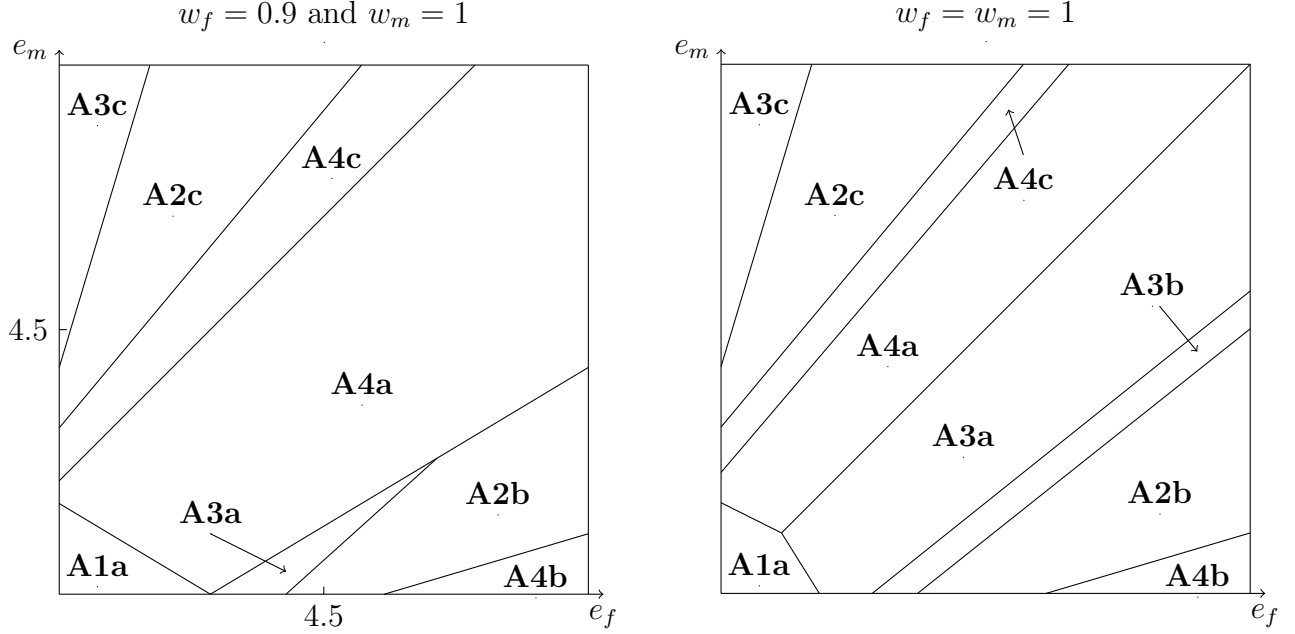Here choices do not depend on e_m and $\partial t_f/e_f > 0$ and $\partial L_f/\partial e_f < 0$.

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Figure 1: Possible cases with respect to e_f and e_m, with (left) and without (right) gender gap.

A4c. Traditional Specialization. If $b, c > 0$ and $a, d = 0$; $L_f = 0$,

$$t_f = \frac{\gamma A_f e_f^\alpha - \theta\mu\bar{q}n}{(\gamma + \theta\mu)ne_f^\alpha} \quad \text{and} \quad L_m = \frac{A_m}{1 + (1-\theta)\mu},$$

with $\partial t_f/\partial e_f > 0$.

Figure 1 provides some intuition on the the behavior of an efficient household, with respect to the education of each partner and the gender wage gap. In the left panel, $w_f = 0.9$ and $w_m = 1$ while in the right panel $w_f = w_m = 1$. The rest of the parameters take the following values: $\mu = 1.5$, $\bar{t}_f = 0.05$, $\bar{t}_m = 0.02$, $\bar{q} = 0.7$, $n = 2$, $\theta = 0.5$, $\alpha = 0.85$ and $\gamma = 0.9$. This means that in the right-hand side, the only difference between a man and a woman is the minimal time required with their children (due to childbearing).

Within couples where both partners have low education, none of them provides more childcare than $\bar{t}_f$ and $\bar{t}_m$ and both are in the labor market (**A1a**). This case is sensitive to the value of the innate ability of children, $\bar{q}$. Keeping the other parameters constant, a larger $\bar{q}$ increases the region **A1a**. This is because lowly educated partners have low income and hence, low consumption. Marginally, an increase in education increases more the couple's utility through increasing consumption than through increasing children's quality because of the positive value of $\bar{q}$.

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Couples in which the e_f and e_m are very different may either be in **A3c** or **A4b**, where only the most educated works and provides quality childcare to the children.[14] An increase in the education of the uneducated partner brings the couple to **A2c** or **A2b** where both parents contribute in the production of child quality. The partner with the highest education works and provides childcare as the opportunity cost in terms of both income and child quality is high. As the unemployed partner has a low level of education, he or she is a low substitute in terms of supplying quality childcare. It is then efficient that the employed partner also participates in providing child quality.

We will now focus on couples where e_f and e_m are large and close enough for not being in the cases that were just described (**A1a**, **A2b**, **A2c**, **A3c** and **A4b**).

When the man has an education level larger than that of his wife, the relevant cases are **A4c** or **A4a**. In either case, he is in the labor market because his labor-income is higher than the (potential) one of his wife. Only the woman (the one with the lowest earnings) provides time in producing child quality in these cases. The reason for this is due to the innate quality of the children, $\bar{q}$ that makes spouses closer substitutes in terms of utility gain from producing child quality compared to the utility gain in terms of providing income. For a similar education than that of her husband, the wife also works as her opportunity cost in terms of income is large (**A4a**).

The gender wage gap affects the situation of women who are relatively more educated than their male partner. When there is no gender wage gap, the relevant cases for a woman with more education than her husband are symmetric to those described in the last paragraph (**A3a** and **A3b**). The gender wage gap makes the woman a lower substitute to her husband in providing income to the household. This implies that both will be in the labor market (because the non participation cost is high when education is large) but the wife continues to be the only one to supply quality childcare. Only when her education is large enough as to have higher earnings than those of her husband, the man will take the role of supplying childcare.

To summarize, when the woman has a higher education than that of her husband and there is no gender wage gap, the household is either in **A3a** or in **A3b**. In such cases, only the husband allocates time to the production of child quality. With a positive gender gap, the couple is very likely to choose to be in case **A4a**. This means that in cooperative couples where women's earnings are lower than that of men due to gender discrimination, childcare is much more likely to be entirely supplied by women.

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[14]As we focus on couples with a positive assortative matching in terms of education (Table 2), **A3c** and **A4b** are unlikely to appear in the simulations of the model (Section 5).
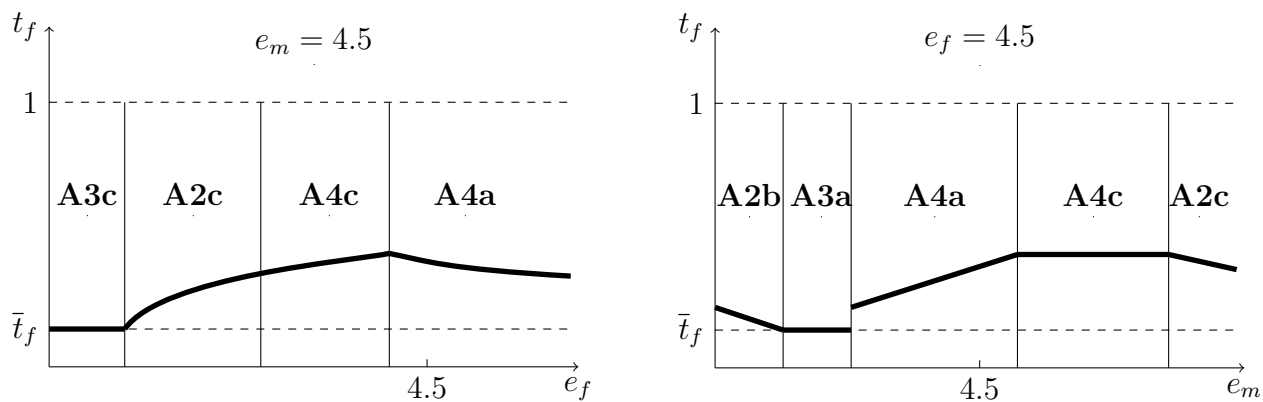
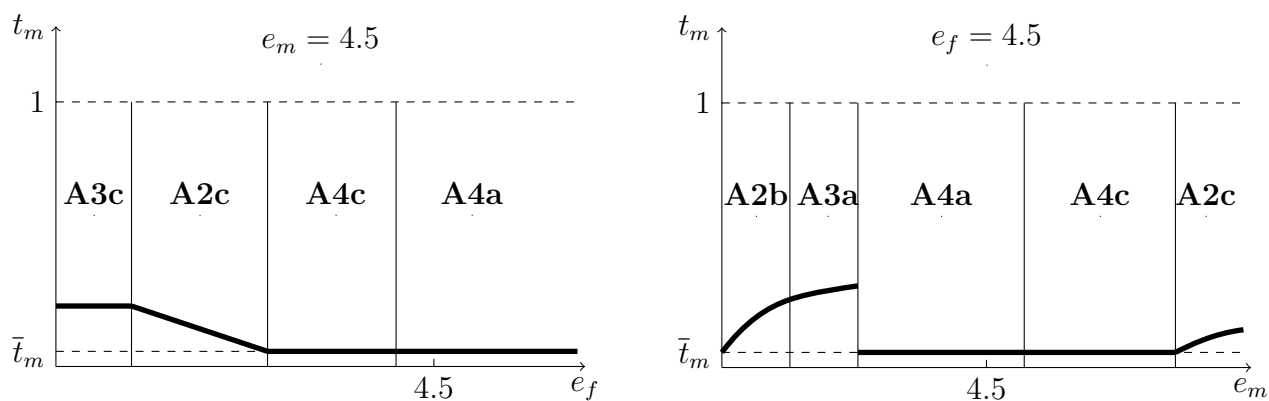Figure 2: Female childcare as a function of e_f (left) and e_m (right).

Figure 3: Male childcare as a function of e_f (left) and e_m (right).

Figures 2 and 3 show childcare supplies for the woman and the man of a couple with respect to their education and that of the partner. The values of the parameters remain the same than above, $w_f = 0.9$ and $w_m = 1$.

The left panel of Figure 2 shows the amount of childcare supplied by a woman with respect to her education. For an uneducated woman, the opportunity cost of not providing childcare is very low and she will not provide more childcare than the minimum amount, $\bar{t}_f$. As her education increases, the cost of not providing child quality increases implying an increase in the time supplied to quality childcare (**A2c** and **A4c**) until reaching case **A4a**. For $e_m = 4.5$, a further increase in e_f increases her amount of labor supplied in the labor market (as her opportunity cost in terms of income is high) and this decreases childcare.[15] The right panel of Figure 3 shows the amount of childcare supplied by a man with respect to his education. A husband with low education, will supply childcare and be in cases **A2b** or **A3a** where

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[15]Note that this is true even in the case of increasing returns of parental education to producing child quality due to $\bar{q}$.

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childcare increases with his education. In both cases, it is efficient that the female works because her education is large enough with respect to that of her partner to compensate for the gender wage gap. Once this is no longer the case, the husband will stop supplying childcare up to the point where his wife is no longer a close substitute for providing child quality (**A2c**).

The relationship between childcare and the education of the partner also differs across genders (right panel of Figure 2 and left panel of Figure 3). Starting again with childcare provided by the woman, when her husband is lowly educated, both partners supply childcare. As they are substitutes, an increase in the education of the man decreases her amount of childcare (**A2b**). For a further increase in e_m, it becomes efficient that the husband works. This decreases the amount of labor supplied by the wife and allows her to increase her supply of childcare (**A4a** and **A4c**). When the husband is very educated, childcare supplied by the wife becomes a lower substitute to that supplied by the husband, so female childcare decreases with the education of her spouse (**A2c**). Turning to childcare supplied by the man, an increase in the education of his wife decreases his amount of childcare (left panel of Figure 3). As for the same education his earnings are higher, he will only provide childcare for very low education levels of his wife. Childcare of men is then decreasing and then flat with respect to e_f.

The next subsection studies the choices on childcare for a semi-cooperative couple.

3.3   Semi-Cooperative Setup

In the semi-cooperative framework, choices are made in two steps. First, couples cooperatively choose their labor supply, L_i. In a second step, each partner individually chooses the amount of his or her time to allocate into the production of child quality. The intuition behind this setup is that there is a commitment on labor supplies as one usually signs an employment contract at the beginning of adult life but that partners do not commit themselves on supplying a certain amount of childcare duties (i.e. there is no clause in the marriage contract on how much time each parent should provide to the children and there is no monitoring on this either). Partners are then more likely to deviate from any agreement regarding childcare. Hence, we resort to Cournot-Nash non-cooperative equilibrium to model the second step. The problem is solved by backward induction, starting with the decisions on childrearing.

3.3.1 Second Step: Non-Cooperative Game over Childcare

Each individual within a couple chooses the amount of childcare that maximizes

$$\ln c + \mu \ln l_i + \gamma \ln (qn)$$

subject to $t_i \geq 0$, (2) and (3), L_f and L_m being given by the outcome of the first step. Individual choices on childcare implies that parents do not internalize the positive externality of childcare on the utility of the couple. This will further lead to choices on childcare that are lower than in the collective model. The KT conditions for this maximization problem are,

$$a + \frac{e_f^\alpha}{t_f e_f^\alpha + t_m e_m^\alpha + \overline{q}} - \frac{\mu n}{A_f - L_f - t_f n} = 0,$$

$$b + \frac{e_m^\alpha}{t_f e_f^\alpha + t_m e_m^\alpha + \overline{q}} - \frac{\mu n}{A_m - L_m - t_m n} = 0,$$

$at_f = 0$, $bt_m = 0$, $t_f \geq 0$, $t_m \geq 0$, $a \geq 0$ and $b \geq 0$, where a and b are the KT multipliers. There are four possible solutions to this problem, $\{t_{f1}, t_{m1}\}$, $\{t_{f2}, t_{m2}\}$, $\{t_{f3}, t_{m3}\}$ or $\{t_{f4}, t_{m4}\}$, depending on the amount of labor supplied by each spouse. Definition 1 provides the thresholds on labor supplies characterizing the possible solutions.

Definition 1 (Labor Thresholds) $L_1 \equiv \dfrac{\gamma A_f e_f^\alpha - \mu \overline{q} n}{\gamma e_f^\alpha}$, $L_2 \equiv \dfrac{\gamma A_m e_m^\alpha - \mu \overline{q} n}{\gamma e_m^\alpha}$,

$L_3(L_m) \equiv A_f - \dfrac{\mu(A_m - L_m)e_m^\alpha + \mu \overline{q} n}{(\gamma + \mu)e_f^\alpha}$, $L_4(L_m) \equiv A_f - \dfrac{(\gamma + \mu)(A_m - L_m)e_m^\alpha - \mu \overline{q} n}{\mu e_f^\alpha}$ and

$L_4(L_f) \equiv A_m - \dfrac{\mu(A_f - L_f)e_f^\alpha + \mu \overline{q} n}{(\gamma + \mu)e_m^\alpha}$.

B1. If $a, b > 0$; $t_f = t_{f1} \equiv 0$, $t_m = t_{m1} \equiv 0$,

$$a = \frac{\mu n}{A_f - L_f} - \frac{\gamma e_f^\alpha}{\overline{q}} \quad \text{and} \quad b = \frac{\mu n}{A_m - L_m} - \frac{\gamma e_m^\alpha}{\overline{q}}.$$

As both a and b should be positive to satisfy the KT conditions, the conditions on labor supplies to be in this case are $L_f > L_1$ and $L_m > L_2$.

B2. If $a, b = 0$; $t_f > 0$ and $t_m > 0$. The best-response functions in this case are,

$$t_m = r_1 \equiv \frac{(A_m - L_m)\gamma e_m^\alpha - \mu \bar{q} n}{(\gamma + \mu)n e_m^\alpha} - \frac{\mu e_f^\alpha}{(\gamma + \mu)e_m^\alpha}t_f$$

$$t_m = r_2 \equiv \frac{(A_f - L_f)\gamma e_f^\alpha - \mu \bar{q} n}{\mu n e_m^\alpha} - \frac{(\gamma + \mu)e_f^\alpha}{\mu e_m^\alpha}t_f$$

so childcare is a strategic substitute variable as $\partial r_1/\partial t_f < 0$ and $\partial r_2/\partial t_m < 0$. The Nash equilibrium is,

$$t_f = t_{f2} \equiv \frac{(\gamma + \mu)(A_f - L_f)e_f^\alpha - \mu(A_m - L_m)e_m^\alpha - \mu n \bar{q}}{(\gamma + 2\mu)n e_f^\alpha}$$

$$t_m = t_{m2} \equiv \frac{(\gamma + \mu)(A_m - L_m)e_m^\alpha - \mu(A_f - L_f)e_f^\alpha - \mu n \bar{q}}{(\gamma + 2\mu)n e_m^\alpha}.$$

The conditions to be in this case are such that $t_f > 0$ and $t_m > 0$ which hold if and only if $L_f < L_3(L_m)$ and $L_m < L_4(L_f)$.[16]

B3. If $a > 0$ and $b = 0$; $t_f = t_{f3} \equiv 0$,

$$t_m = t_{m3} \equiv \frac{(A_m - L_m)\gamma e_m^\alpha - \mu \bar{q} n}{(\gamma + \mu)n e_m^\alpha} \quad \text{and} \quad a = -\frac{(\gamma + \mu)n e_f^\alpha}{(A_m - L_m)e_m^\alpha + \bar{q} n} + \frac{\mu n}{A_f - L_f}.$$

The conditions to be in this situation are such that $t_m > 0$ and $a > 0$ which hold if and only if $L_m < L_2$ and $L_f > L_3(L_m)$. For this case to exist, the education of the man has to be large enough so that $L_2 > 0$.

B4. If $a = 0$ and $b > 0$; $t_m = t_{m4} \equiv 0$,

$$t_f = t_{f4} \equiv \frac{(A_f - L_f)\gamma e_f^\alpha - \mu \bar{q} n}{(\gamma + \mu)n e_f^\alpha} \quad \text{and} \quad b = -\frac{(\gamma + \mu)n e_m^\alpha}{(A_f - L_f)e_f^\alpha + \bar{q} n} + \frac{\mu n}{A_m - L_m}.$$

Both $t_f > 0$ and $b > 0$ must hold to be in this situation, implying that $L_f < L_1$ and $L_m > L_4(L_f)$. For this case to exist, the education of the woman has to be large enough so that $L_1 > 0$.

If $L_1 > 0$ and $L_2 > 0$, one possible representation of these four equilibria is shown in Figure 4, where the thick lines delimit the zones of each possible case. In this particular configuration, if both L_f and L_m are large, none of the parents will choose to provide childcare other than $\bar{t}_f$ and $\bar{t}_m$ (**B1**). If the choice on the amount of labor supplies are low, both will provide quality childcare (**B1**). When one of the spouses provides relatively more labor than the

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[16] As the absolute value of the slope of r_2 is higher than 1, this equilibrium is stable.

other, only this later will supply time to raise the quality of the children (**B3** and **B4**).
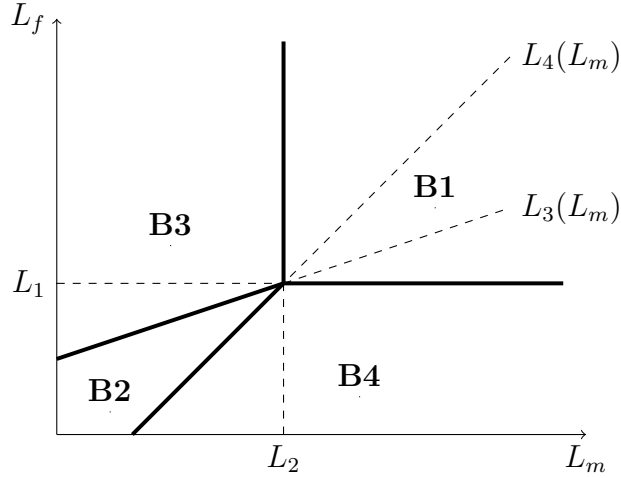

Figure 4: Possible equilibria in the second stage.

3.3.2 First Step: Cooperative Decisions over Labor Choices

In the first stage of the game, couples collectively choose L_f and L_m to maximize (6) subject to the time constraint, (2), the budget constraint, (4), and to either $\{t_{f1}, t_{m1}\}$, $\{t_{f2}, t_{m2}\}$, $\{t_{f3}, t_{m3}\}$ or $\{t_{f4}, t_{m4}\}$, depending on labor supplies. For each of the four cases, the choices on labor could be: both household members work or only one does. As in Section 3.2, there are 12 possible cases. Details on the conditions for each of these outcomes to arise are provided in Appendix B.2. The solutions on labor supplies for each case are equal to the corresponding ones of the cooperative framework.

B1. Let us assume that labor supplies are such that none of the partners provides childcare, $\{t_{f1}, t_{m1}\}$. The KT conditions are,

$$c + \frac{w_f e_f}{w_f e_f L_f + w_m e_m L_m} - \frac{\theta \mu}{A_f - L_f} = 0,$$

$$d + \frac{w_m e_m}{w_f e_f L_f + w_m e_m L_m} - \frac{(1 - \theta)\mu}{A_m - L_m} = 0,$$

$cL_f = 0$, $dL_m = 0$, $L_f \geq 0$, $L_m \geq 0$, $c \geq 0$ and $d \geq 0$ where c and d are the KT multipliers. There are three possible solutions to this problem, **B1a**, **B1b** and **B1c**.

B1a. L'Enfant Sauvage. If $c, d = 0$; $L_f, L_m > 0$.

B1b. L'Enfant Sauvage and an Unemployed Husband. If $c = 0$ and $d > 0$; $L_m = 0$ and $L_f > 0$.

B1c. L'Enfant Sauvage and an Unemployed Wife. If $c > 0$ and $d = 0$; $L_f = 0$ and $L_m > 0$.

B2. Assuming that labor supplies are such that both partners provide childcare, $\{t_{f2}, t_{m2}\}$, the KT conditions are,

$$c + \frac{w_f e_f}{w_f e_f L_f + w_m e_m L_m} - \frac{(\gamma + \mu)e_f^\alpha}{(A_f - L_f)e_f^\alpha + (A_m - L_m)e_m^\alpha + \bar{q}n} = 0,$$

$$d + \frac{w_m e_m}{w_f e_f L_f + w_m e_m L_m} - \frac{(\gamma + \mu)e_m^\alpha}{(A_f - L_f)e_f^\alpha + (A_m - L_m)e_m^\alpha + \bar{q}n} = 0,$$

$cL_f = 0$, $dL_m = 0$, $L_f \geq 0$, $L_m \geq 0$, $c \geq 0$ and $d \geq 0$. The three possible solutions to this problem are **B2a**, **B2b** and **B2c**. In order for one of these cases to exist, the following conditions need to be satisfied:

$$e_f > \left(\frac{\mu \bar{q}n}{\gamma A_f}\right)^{1/\alpha} \quad \text{and} \quad e_m > \left(\frac{\mu \bar{q}n}{\gamma A_m}\right)^{1/\alpha}.$$

B2a. If $c, d = 0$, then there is a continuum of positive values $\{t_i, L_i\}$ that are possible. This can happen when partners are identical in both supplying childcare and providing income. As in **A2a**, this implies that any positive labor arrangement maximizes the objective of the couple.

B2b. A Busy Wife. If $c = 0$ and $d > 0$; $L_f > 0$, $L_m = 0$,

$$t_f = \frac{(\gamma + \mu)^2 A_f e_f^\alpha - (\gamma + (2 + \gamma)\mu + \mu^2)(A_m e_m^\alpha + \bar{q}n)}{(1 + \gamma + \mu)(\gamma + 2\mu)ne_f^\alpha} \quad \text{and}$$

$$t_m = \frac{-\mu(\gamma + \mu)(A_f e_f^\alpha + \bar{q}n) + (\gamma + 2\mu + (\gamma + \mu)^2)A_m e_m^\alpha}{(1 + \gamma + \mu)(\gamma + 2\mu)ne_m^\alpha}.$$

B2c. A Busy Husband. If $c > 0$ and $d = 0$; $L_f = 0$, $L_m > 0$,

$$t_f = \frac{(\gamma + 2\mu + (\gamma + \mu)^2) A_f e_f^\alpha - \mu(\gamma + \mu)(A_m e_m^\alpha + \bar{q}n)}{(1 + \gamma + \mu)(\gamma + 2\mu)ne_f^\alpha} \quad \text{and}$$

$$t_m = \frac{-(\gamma + (2 + \gamma)\mu + \mu^2)(A_f e_f^\alpha + \bar{q}n) + (\gamma + \mu)^2 A_m e_m^\alpha}{(1 + \gamma + \mu)(\gamma + 2\mu)ne_m^\alpha}.$$

As in **A2b** and **A2c** we can check that $\partial t_f / \partial e_f > 0$, $\partial t_f / \partial e_m < 0$, $\partial t_m / \partial e_f < 0$ and

$\partial t_m / \partial e_m > 0$ in **B2b** and **B2c**.

B3. Let us assume now that labor supplies are such that only the husband provides childcare, $\{t_{f3}, t_{m3}\}$, then the KT conditions for the maximization problem of couples are,

$$c + \frac{w_f e_f}{w_f e_f L_f + w_m e_m L_m} - \frac{\theta \mu}{A_f - L_f} = 0$$

$$d + \frac{w_m e_m}{w_f e_f L_f + w_m e_m L_m} - \frac{(\gamma + (1 - \theta)\mu)e_m^\alpha}{(A_m - L_m)e_m^\alpha + \bar{q}n} = 0$$

and $cL_f = 0$, $dL_m = 0$, $L_f \geq 0$, $L_m \geq 0$, $c \geq 0$, $d \geq 0$. The next three possible solutions to the household problem, **B3a**, **B3b** and **B3c** can arise if

$$e_m > \left(\frac{\mu \bar{q} n}{\gamma A_m} \right)^{1/\alpha}.$$

B3a. A Multi-Task Husband. If $c, d = 0$; $L_f, L_m > 0$ and

$$t_m = \frac{\gamma(\gamma + (1 - \theta)\mu)\left(A_f w_f e_f + A_m w_m e_m\right) - (\gamma + \mu + (1 + \theta)\gamma\mu + \mu^2))w_m e_m^{1-\alpha} \bar{q} n}{(\gamma + \mu)(1 + \gamma + \mu)n w_m e_m}.$$

We can check that $\partial t_m / \partial e_f > 0$ (as in **A3a**) and that,

$$\frac{\partial t_m}{\partial e_m} > 0 \iff e_f < \frac{\alpha(\gamma + \mu + \gamma(1 + \theta)\mu + \mu^2)\bar{q} n w_m e_m^{1-\alpha}}{\gamma(\gamma + (1 - \theta)\mu)A_f w_f}.$$

This threshold on e_f such that $\partial t_m / \partial e_m > 0$ is higher than in the cooperative setup.

B3b. Modern Specialization. If $c = 0$ and $d > 0$; $L_f > 0$, $L_m = 0$ and

$$t_m = \frac{\gamma A_m e_m^\alpha - \mu \bar{q} n}{(\gamma + \mu)n e_m^\alpha}.$$

As in **A3b**, $\partial t_m / \partial e_m > 0$. In this case, t_m is what we would have in the cooperative setup if male's bargaining power was equal to 1 ($\theta = 0$).

B3c. An Unemployed Wife. If $c > 0$ and $d = 0$; $L_f = 0$, $L_m > 0$ and

$$t_m = \frac{\gamma(\gamma + (1 - \theta)\mu)A_m e_m^\alpha - (\gamma + (1 + \gamma + (1 - \theta)\mu)\mu)\bar{q} n}{(\gamma + \mu)(1 + \gamma + (1 - \theta)\mu)n e_m^\alpha},$$

where $\partial t_m / \partial e_m > 0$.

B4. Finally, let us assume that labor supplies are such that only the wife provides childcare,

$\{t_{f4}, t_{m4}\}$, then the KT conditions for the maximization problem of couples are,

$$c + \frac{w_f e_f}{w_f e_f L_f + w_m e_m L_m} - \frac{(\gamma + \theta\mu)e_f^\alpha}{(A_f - L_f)e_f^\alpha + \bar{q}n} = 0$$

$$d + \frac{w_m e_m}{w_f e_f L_f + w_m e_m L_m} - \frac{(1 - \theta)\mu}{A_m - L_m} = 0$$

and $cL_f = 0$, $dL_m = 0$, $L_f \geq 0$, $L_m \geq 0$, $c \geq 0$, $d \geq 0$. The last four possible solutions to this problem, **B4a**, **B4b** and **B4c** can arise if

$$e_f > \left(\frac{\mu\bar{q}n}{\gamma A_f}\right)^{1/\alpha}.$$

B4a. A Multi-task wife. If $c, d = 0$; $L_f, L_m > 0$ and

$$t_f = \frac{\gamma(1 + \theta\mu)\left(A_f w_f e_f + A_m w_m e_m\right) - (\gamma + \mu + (2 - \theta)\gamma\mu + \mu^2)w_f e_f^{1-\alpha}\bar{q}n}{(\gamma + \mu)(1 + \gamma + \mu)w_f e_f n}.$$

We can check that $\partial t_f/\partial e_m > 0$ as in **A4a** and that

$$\frac{\partial t_f}{\partial e_f} > 0 \iff e_m < \frac{\alpha(\gamma + \mu + \gamma(2 - \theta)\mu + \mu^2)\bar{q}n w_f e_f^{1-\alpha}}{\gamma(\gamma + \theta\mu)A_m w_m}.$$

The threshold on e_m such that $\partial t_f/\partial e_f < 0$ is larger than the cooperative case **A4a**, as in the symmetric case **B3a**.

B4b. An Unemployed Husband. If $c = 0$ and $d > 0$; $L_f > 0$, $L_m = 0$ and

$$t_f = \frac{(\gamma + \theta\mu)A_f e_f^\alpha - (\gamma + (1 + \gamma + \theta\mu)\mu)\bar{q}n}{(\gamma + \mu)(1 + \gamma + \theta\mu)n e_f^\alpha}.$$

where $\partial t_f/\partial e_f > 0$.

B4c. Traditional Specialization. If $c > 0$ and $d = 0$; $L_f = 0$, $L_m > 0$ and

$$t_f = \frac{\gamma A_f e_f^\alpha - \mu\bar{q}n}{(\gamma + \mu)e_f^\alpha n}$$

where $\partial t_f/\partial e_f > 0$ as in the cooperative counterpart **A4c**.

Figure 5 shows when some of the cases that were listed above could arise with respect to e_f and e_m and the existence of a gender wage gap. The values of the parameters are the same as in Figure 1 for the cooperative setup. The zones corresponding to the cases in **B1**, **B3**
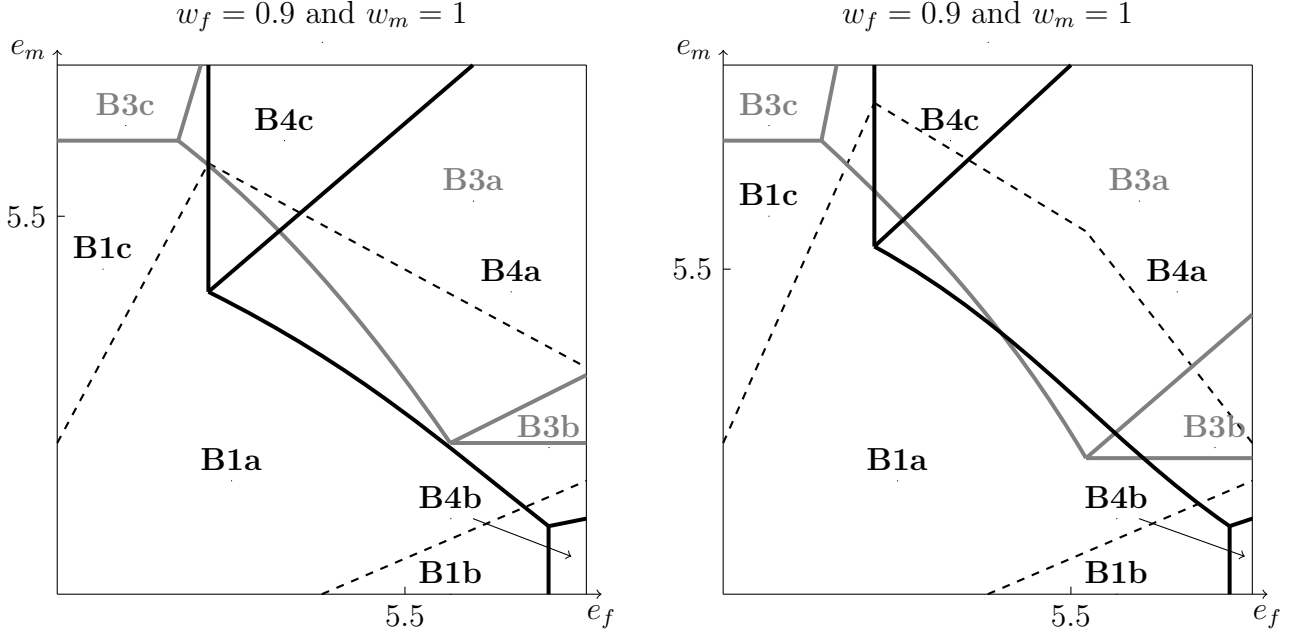
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Figure 5: Possible cases with respect to e_f and e_m, with (left) and without (right) gender gap.

and **B4** are respectively represented with a dashed black line, a gray line and a black line (cases in **B2** do not appear for the given values of the parameters).

The first important difference between Figure 1 and Figure 5, is that indeterminacy of the equilibrium arises in the semi-cooperative setup. The multiplicity of equilibria does not come from the second stage of the semi-cooperative setup: within each zone **B1**, **B2**, **B3** and **B4** (given (L_f, L_m)), there exists a unique solution for (t_f, t_m). This was already shown in Figure 4. Indeterminacy comes from the first stage of the decision process. In stage one, if a couple believes that a given arrangement on childcare will happen, then the couple will choose labor supplies accordingly and in the second stage, it will be a Nash equilibrium that this arrangement occurs.[17] Lets take for instance, in the left panel of Figure 5, the zone where the possible equilibria are **B1a**, **B3a** and **B4a**. Suppose that education levels are such that $e_f = 5$ and $e_m = 4.5$, which belong to this region. If a couple predicts the individual decisions about childcare to be such that $t_f, t_m = 0$, for the given parameters above, labor choices are $L_f = 0.34$ and $L_m = 0.40$. For these labor choices, the conditions for being in region **B1**, $L_f > L_1$ and $L_m > L_2$, are respected implying that the final decisions on childcare such that $t_f, t_m = 0$ is a Nash equilibrium. If instead we assume that a couple predicts that the husband will take care of the children, then the household decision on labor supplies is

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[17]This intuition is similar to that of de la Croix and Doepke (2009) where there is multiplicity in the type of education systems.

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$L_f = 0.40$ and $L_m = 0.26$. Given these choices for labor, individuals choices on childcare are those expected, $t_f = 0$ and $t_m = 0.01$ so that this also constitutes a Nash equilibrium. Finally, if a couple believes that the woman will provide quality childcare to the children, then the couple will choose to supply $L_f = 0.18$ and $L_m = 0.47$ units of labor. Given this choice, the conditions to be in region **B4** are satisfied and the amount of childcare provided by each partner is $t_f = 0.02$ and $t_m = 0$. So **B4a** is also a Nash equilibrium. Note that only one of these equilibria is efficient: for the given education levels, a cooperative couple would be in **A4a** where only the woman supplies quality childcare and both partners work.

Also comparing Figures 1 and 5, we can notice that education levels must be higher in the semi-cooperative setup than in the cooperative one in order for parents to start providing quality childcare to their children (the cases in **B1** take a larger area than those in **A1**). This comes from the individual choice on childcare that does not internalize the positive externality of childrearing on the couple's utility. As there is no multiplicity within **B1**, multiplicity only appears for higher education levels.

The cases that are most concerned with multiplicity are **B3a**, **B3b**, **B4a** and **B4b**. Lets take for instance a couple within the zone **B3b** and **B4a** in the right panel of Figure 5. The possible outcomes for the woman are to specialize in the labor force or to do both working and childcare activities. The final situation will depend on beliefs, as explained above. Beliefs are likely to be influenced by social norms. Imagine for example that the cultural environment tends to favor either men or women, then the prediction on childcare will depend on which situation maximizes men or women's respective utility. For $e_f = 7$, $e_m = 3.5$ and no gender wage gap, the possible equilibria are **B3b**, where the man specializes in childrearing and the woman in labor activities, or **B4a**, where both work and only the woman provides quality childcare. A selection criteria that maximizes the utility of the husband would pick **B3b** while one that maximizes the utility of the woman would choose **B4a**. The husband prefers **B3b**, where he only provides childcare, to **B4a**, where he only works, because the individual choice on childcare does not internalize the externality of childcare on the utility of the couple. This makes him enjoy more leisure than what he would have otherwise in **B4a**. The wife prefers **B4a** to **B3b** because she gains more leisure from not internalizing the externality on child quality and more consumption from the extra income provided by the husband (that reduces her supply of labor).[18]

Different cases can arise in the semi-cooperative setup compared to the cooperative setup. In particular, the cases where spouses specialize replace the cases where both partners provide

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[18]I abstract from social norms here. Including in the setup that the husbands might feel more responsible for bringing income to the household would, of course, change the results.
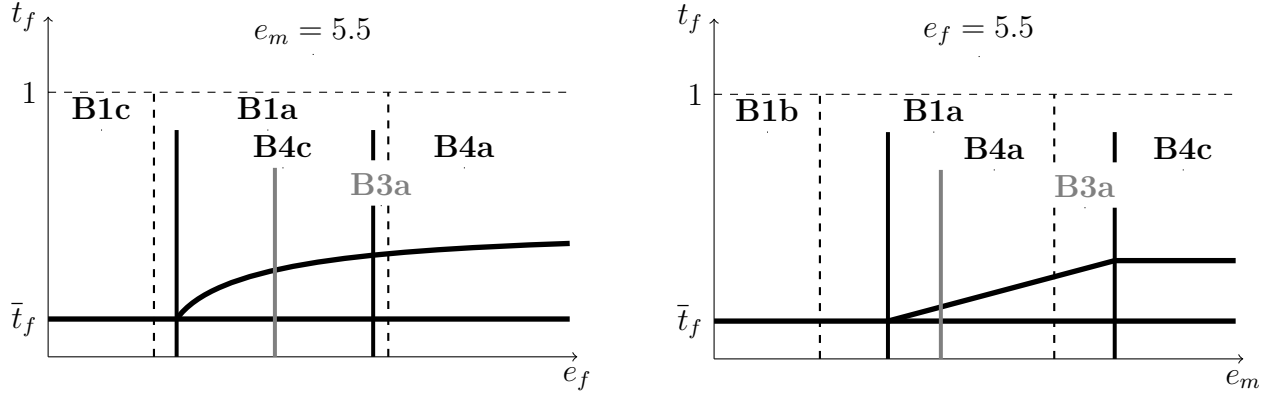
Figure 6: Female childcare as a function of e_f (left) and e_m (right).

childcare. So a husband that would both work and provide childcare in case **A2c**, is more likely to be in case **B4c** in the semi-cooperative setup where he only works. This happens for a highly educated man coupled with a woman with average education, that is a low substitute in providing childcare. The reason for this is, once again, related to the comment above that individual choices on childcare do not internalize the positive externality on the utility of the couple. This pushes down the amount of childcare provided.

Comparing the two panels of Figure 5, we see that the gender wage gap does not affect as much the possible outcomes as in the cooperative setup. In the presence of a gender wage gap, the zones where the woman is the only one providing quality childcare (**B4**) are bigger, but the same cases appear in both panels, which was not the case in the cooperative setup. The reason for this is that the gender gap may affect the number of possible equilibria but not their selection. Only if the gender wage gap affected beliefs (by expecting more to have women taking care of children alone), then a positive gender gap would change the final outcome.
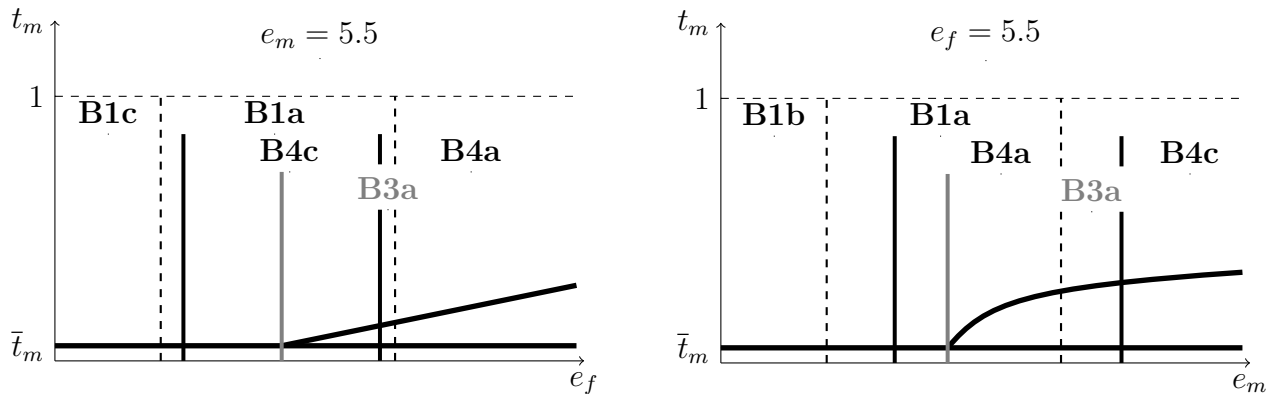

Figure 7: Male childcare as a function of e_f (left) and e_m (right).

Figures 6 and 7 show childcare supplies for men and women relative to their education and the education of their partner, in the presence of a gender wage gap and the same values of the parameters than before. We see that this setup, for the given parameters, allows to reproduce that childcare increases with one's education and also with the education of one's partner. This is due to the multiplicity of equilibria that allows the cases where we can have a positive relationship between childcare an both partners' education (**B3a** and **B4a**) to jointly appear. We also see that in the semi-cooperative setup, the cases where childcare decreases with the education of the partners (**B2b** and **B2c**) do not appear. As explained before, the reason is that the positive externality on child quality pushes parents to invest less time in their children so the cases where both parents supply child quality are less likely to happen.

In the next section, the deep parameters of both the cooperative and the semi-cooperative models are estimated. For the semi-cooperative model, four criteria for selecting the equilibrium are proposed. The first is a random selection criterion: given, $x \in \{1, 4\}$ the number of possible equilibria, the probability for one of these equilibrium to result is $1/x$. The second criterion is to chose the equilibrium that maximizes the utility of the male partner (i.e. machist criterion) and the third choses the equilibrium that maximizes the utility of the female partner (feminist criterion). These two criteria would reflect a very strong social gender biased society.[19] The last selection criteria estimates the degree of gender biasness in the society. I assume that whenever there is more than one equilibrium for a household, the equilibrium that will finally be realized will be the one maximizing the utility of the husband for $\rho\%$ of the households and the one maximizing the utility of the wife for $(1 - \rho)\%$ of the households. It then provides a measure of the degree of machism or feminism in the society.

4   Estimation

The two models exposed in Sections 3.2 and 3.3 are now fit to match the data on childcare supplied by men and women, shown in Section 2.

Parameters θ and n are a priori fixed. The bargaining parameter, θ, is set to 0.5, so to focus on the commitment vs. non-commitment issue of household's decisions, leaving aside the bargaining channel. Including θ as one of the parameters to be identified does not increase the ability of either model in reproducing the facts. Fertility, n, is exogenous and fixed to

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[19]Lundberg and Pollak (1993) also suggest that gender roles assign responsibility to each spouse for an activity rather than another. In their case, they assume that each partner supplies one public good (the one falling into his or her "separate sphere") and partners do not jointly provide public goods.

2. Using the observed fertility rates for each type of couple, $\{e_f, e_m\}$ does not change the results. The amount of time that an individual spends in childcare, labor and leisure in the data is between 475 an 645 minutes for women and between 601 and 719 for men, depending on the education of both partners. I arbitrarily fix the number of minutes that individuals have per day to 600 minutes.

Education levels are introduced as potential earnings and are computed using a Mincer equation:

$$e_i = \exp 0.1e \tag{7}$$

where 0.1 is the rate of return of one extra year of education and e is the number of years of education given in Table 1.[20] The gender wage gap, w_f/w_m, is set to 0.9. This changes the interpretation of parameter α: an increase in one year of parent education will increase the efficiency of their time in providing child quality by 0.1α.

The remaining seven parameters are estimated using the Simulated Method of Moments. The objective, f, to minimize is the following,

$$f(p) = \left(\frac{d - s(p)}{d}\right)^2 \tag{8}$$

where p is the vector of the parameters, d is the vector of the 74 empirical moments (amount of childcare for men and women) and s is the vector of the moments simulated by the model. In order to compute the simulated moments, I draw 10,000 households for the seven different e_f and e_m (this makes a total of 49,000 households where 37,000 are used to match the data). Each household has a different innate quality for their children, drawn from a log-normal distribution from which the mean, $\bar{q}_m$, and standard error, $\bar{q}_{se}$, are estimated. The minimization of the objective f was computed in two steps. I first estimate p using a genetic algorithm called PIKAIA that is used to find the region where the global extreme is located. The results of this estimation are then used as initial values for another, faster and more local algorithm, UOBYQA (Powell (2002)). These two procedures were run in FORTRAN 90.

Table 3 provides the estimates of the parameters for the five different settings: (1) co-operative, (2) semi-cooperative with random selection criterion, (3) semi-cooperative with machist selection criterion, (4) semi-cooperative with feminist selection criterion and (5) semi-cooperative where the degree of gender biasness is estimated. The last row of Table 3 shows the values of f for the estimated parameters in each setup.

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[20]The value 0.1 as an estimate of the economic return of one extra year of schooling is the usual one found in the literature (see for example Table 4.1.1 in Angrist and Pischke (2009)).

p	Name of the Parameter	(1)	(2)	(3)	(4)	(5)
$\bar{q}_{me}$	Mean of the lognormal distribution for $\bar{q}$	1.593	0.257	1.688	1.182	0.241
$\bar{q}_{se}$	S.E. of the lognormal distribution for $\bar{q}$	2.808	0.240	1.089	2.662	0.254
μ	Preference for leisure	0.832	1.189	0.371	1.599	1.112
γ	Preference for child quality	3.349	1.559	1.082	3.397	1.480
α	Returns to parent education on childcare	1.089	1.019	1.287	0.473	1.014
$\bar{t}_f$	Fixed time providing childcare (female)	0.000	0.051	0.079	0.031	0.050
$\bar{t}_m$	Fixed time providing childcare (male)	0.027	0.021	0.010	0.025	0.021
ρ	Degree of gender biasness towards men					0.476
f	Value of the objective function	4.718	1.026	3.438	2.258	1.121

Table 3: Estimated Parameters for the Cooperative (1), Semi-Cooperative with Random Criterion (2), Semi-Cooperative with Machist Criterion (3) and Semi-Cooperative with Feminist Criterion (4) Models.

We can see that the estimated values of the parameters are very different from one model to the other, except for setups (2) and (5). This is however not surprising as the parameter measuring the degree of gender biasness, ρ, is close to 0.5. We can however retrieve that the preference for child quality, γ, is for all the setups larger than the preference for leisure, μ, and for consumption (fixed to 1). The estimates of minimal childcare in the cooperative model are such that $\bar{t}_f < \bar{t}_m$. This is strange and is because the gender wage gap causes women to be more dedicated to childcare than their husbands. So the cooperative model does not need $\bar{t}_f$ to match data on female childcare. The values of the parameters for the semi-cooperative model with random selection criterion (2) provide the best match with the data compared to the other models ($f = 1.026$). We will use this setup for the interpretation of the parameters. The lower $\bar{q}_{se}$ for the semi-cooperative setup with a random selection criteria compared to the other setups is explained by the fact that this criterion alone generates heterogeneity. The returns to parent education on childcare are close to the returns on potential earnings (fixed to 0.1). A value of α of 1.019 implies that an increase of one year of the parent education increases the efficiency of the parent time in providing child quality by 10.19%. The estimate on $\bar{t}_f$ that we obtain is similar to that estimated by Echevarria and Merlo (1999). We add to their findings that there is also a positive fix cost in terms of time for fathers equal to 2.1% of his adult life, for one child. In terms of minutes per day, it corresponds to 30.6 minutes for the mother and to 12.6 minutes for the father.

Next section compares the simulations of each model to the data of Table 2.

5 Simulations

5.1 Comparison with the Data

Figures 9 to 13 show, for each theoretical framework and the estimated parameters of Table 3, the simulations for male and female childcare with respect to their education and that of their partner. All the left panels show in the vertical axis t_f, in the horizontal axis e_f and each curve is traced for a different category of the male partner's education, e_m. All the right panels show in the vertical axis t_m, in the horizontal axis e_m and each curve is traced for a different category of e_f. Figure 8 is a graphical representation of the facts given in Table 2 to facilitate the comparison between data and theory.
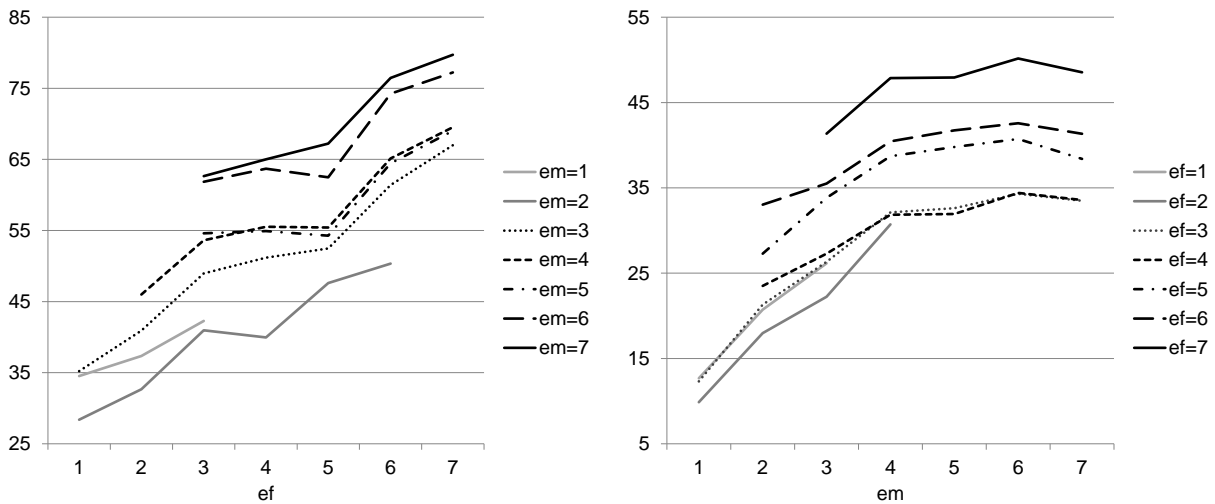

Figure 8: Data on Female (Left) and Male (Right) Childcare with respect to e_f and e_m.

In Figure 9, we see that the cooperative model is able to reproduce that the amount of childcare supplied by an individual increases with his or her education (Fact 1). This model is also able to reproduce that female childcare increases with respect to the education of the husbands. It is, however, unable to reproduce that male childcare increases with the education of the wives. This was already predicted from the left panel in Figure 3. The reason is that the women have a comparative advantage in childrearing activities compared to men, due to the gender wage gap. Couples in which the woman is educated will either specialize and be in **A4c** or be in **A4a** where both work and only the woman provides childcare.

Setups (2) and (5) allow to replicate all the facts introduced in Section 2 (Figures 10 and 11). This result is driven by the multiplicity of equilibria, as has been shown in Figures 6 and 7.
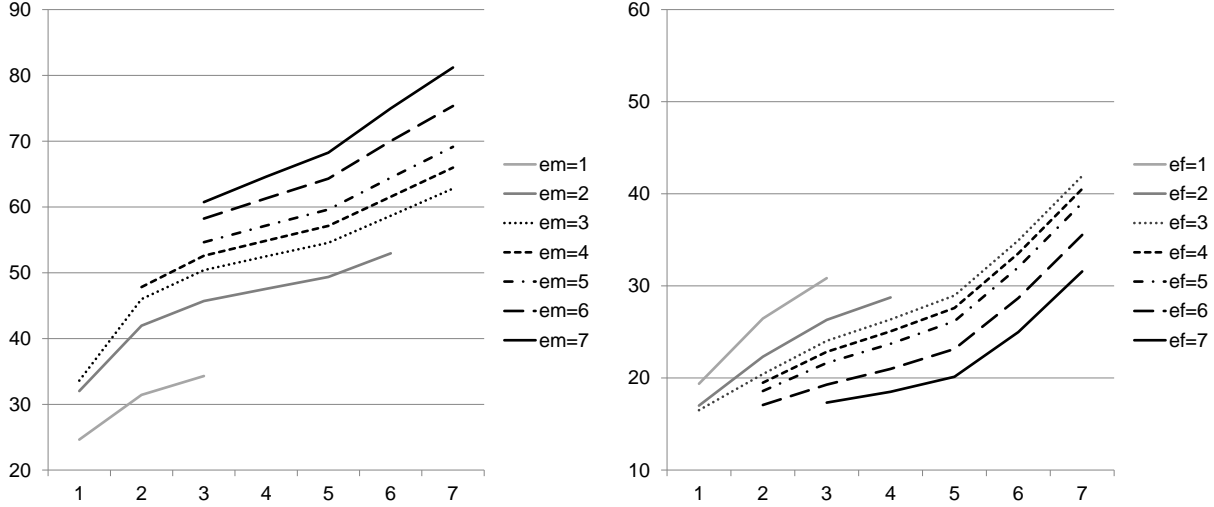
Figure 9: Simulations for Female (Left) and Male (Right) Childcare with respect to e_f and e_m in the Cooperative Model.
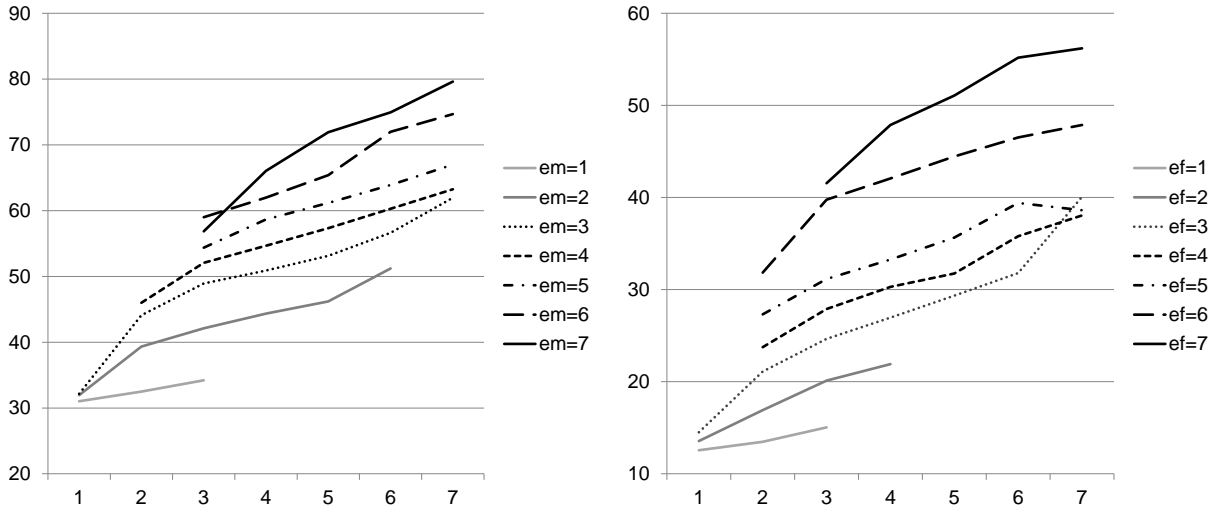

Figure 10: Simulations of the Semi-Cooperative Model for Female (Left) and Male (Right) Childcare with respect to e_f and e_m Using the Random Selection Criterion.

In particular for the case **B4a** (**B3a**), t_f (t_m) increases with e_m (e_f) because a higher e_m (e_f) decreases L_f (L_m) by relaxing the budget constraint. In the cooperative setup **A3a**, did not appear due to the wage gap that made it efficient for the couple with an educated woman to be in **A4c** or **A4a**. The possibility for a couple of given e_f and e_m to be in **B4a** or **B3a** is what at the aggregate level allows to reproduce the facts. As the estimate for the gender bias of the society is close to 0.5, both setups look similar (drawing a large number of household implies that each equilibrium has an equal chance to appear when using the random selection criteria).
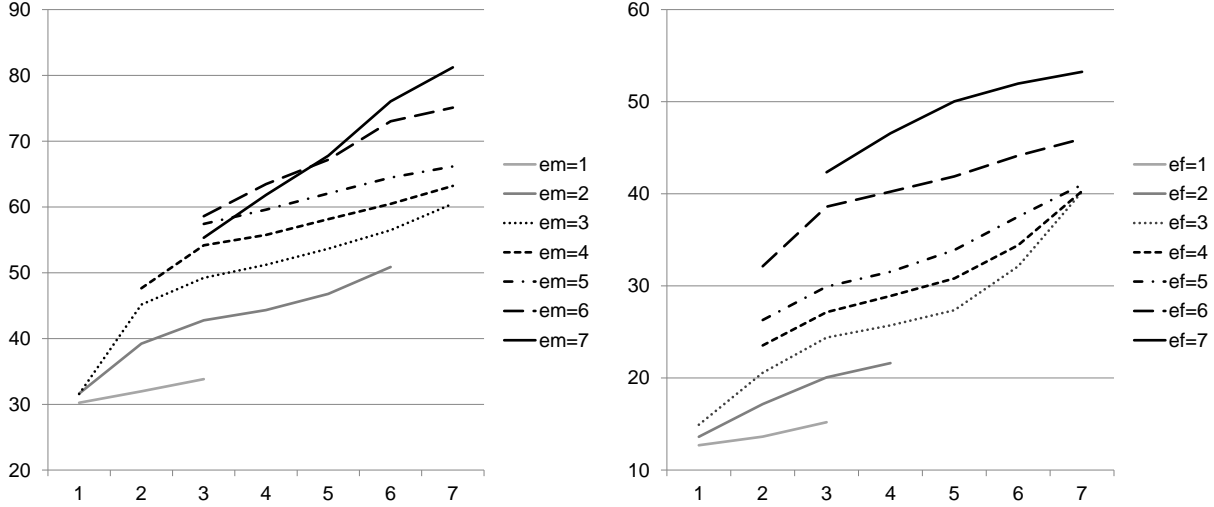
Figure 11: Simulations of the Semi-Cooperative Model for Female (Left) and Male (Right) Childcare with respect to e_f and e_m Using the Criterion that estimates the gender bias of society.
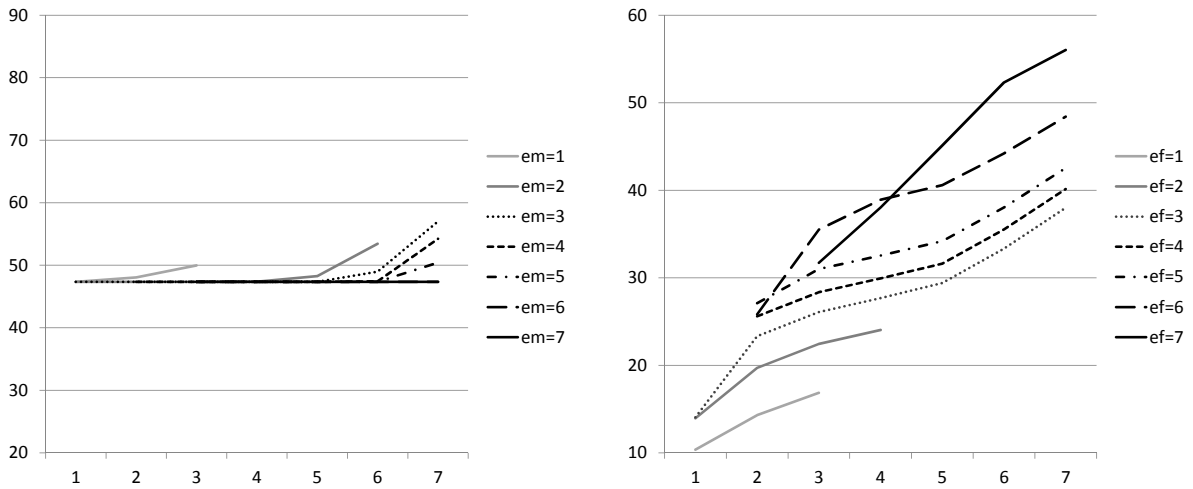

Figure 12: Simulations of the Semi-Cooperative Model for Female (Left) and Male (Right) Childcare with respect to e_f and e_m Using the Machist Selection Criterion.

Figures 12 and 13 show the simulations for the semi-cooperative setup with machist and feminist selection criteria respectively. Applying the machist selection criterion, the model is able to reproduce that t_m increases with e_f and e_m but is not able to reproduce the facts for t_f. The reason for this is that the individual choice on childcare makes parents enjoy more leisure than what they would in the cooperative setting, with the same amount of labor. This implies that the final situations that are more likely to appear are those where the husband provides all the quality childcare (**B3a** or **B3b**). This explains why the model is able to replicate the choices for men. As in **B3a** or **B3b** women do not provide quality
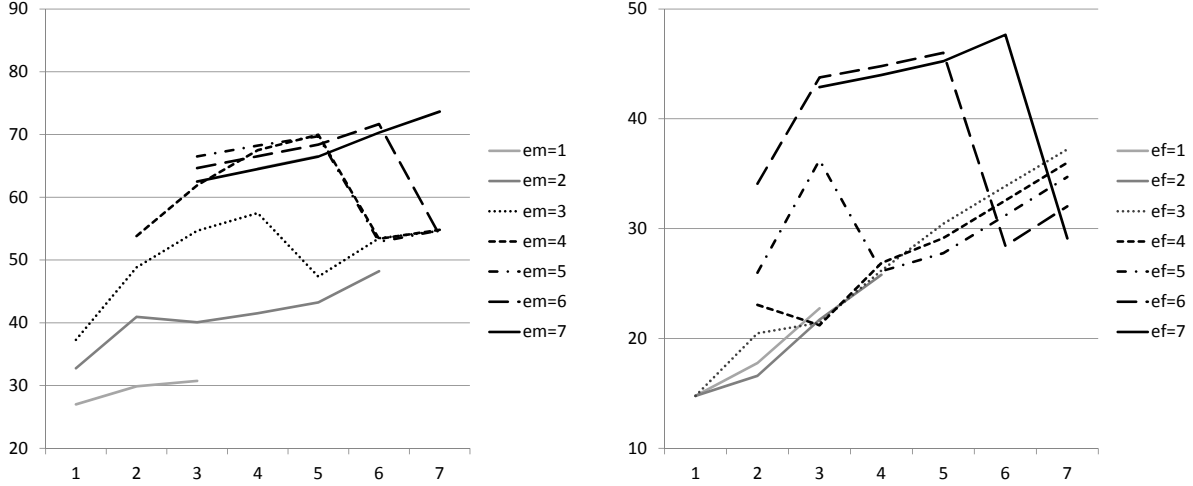
Figure 13: Simulations of the Semi-Cooperative Model for Female (Left) and Male (Right) Childcare with respect to e_f and e_m Using the Feminist Selection Criterion.

childcare, the relationship between female childcare and either e_f or e_m is flat.

Figure 13 is not symmetric to Figure 12 as we would have expected. The high level of the estimated value of γ (Table 3) makes the cases where both partners supply childcare (**B2b** and **B2c**) more likely to appear. A woman will always prefer the situations in **B4** to those in **B2** and the situations in **B2** to those in **B3**. This is due to the individual choice on childcare and hence, leisure. This setup is able to reproduce that female childcare increases with her education. The reason is that couples are more likely to be in cases **B4a** and **B4c** as the education of the wife increases. Female childcare also increases with respect to the education of the husband. This is because a couple with an uneducated husband is more likely be in **B4a** whereas one where the husband is educated will be in **B4c**. Male childcare increases with respect to his education for couples where the wife is lowly educated. For these couples, as men and women are not close substitutes in producing childcare, **B2b** can arise for educated men. This setup does not do very well at reproducing that male childcare increases with the education of his wife because when the wife's education is large, the couple will most likely be in case **B4c** where only the woman supplies childcare.

Figure 14 compares the prediction of each model with Fact 3 (that the amount of childcare time supplied by women relative to men decreases as the education of both members of the couple increases). Neither the cooperative model nor the semi-cooperative with feminist selection criterion allow to reproduce this fact. Lets call the education of the couple $e_c = e_f = e_m$. In both the cooperative setup and semi-cooperative setup with feminist selection criterion male childcare increases with e_c (as the large estimate for γ makes couples more likely to be in **A2c** or **B2c** as education increases). Female childcare also increases with e_c
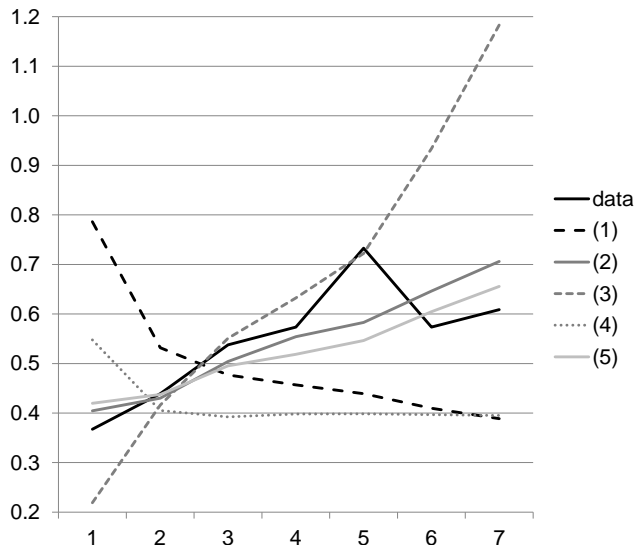
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Figure 14: Gap between childcare provided by men and women in couples where both parties have the same education level, data and simulations.

but more sharply (as more couples are in **B4c** where the woman specializes in childcare). This pushes t_m/t_f to decrease as e_c increases. As for the semi-cooperative setup with feminist selection criterion couples will mostly be in **B4c**, t_m/t_f looks very flat. The semi-cooperative with machist selection criterion is able to reproduce Fact 3 because it is able to reproduce the facts on childcare for men and the relationship between t_f and education levels is flat. Semi-cooperative models (2) and (5) are both able to match this last fact. This is due to a composition effect: as e_c increases there is a more egalitarian fraction of couples in **B4a** compared to **B3a** meaning that at the macro level, the proportion of couples where the husband provides quality childcare to their children increases.

To conclude, we can say that the setup that better fits the U.S. data on childcare, under the assumption that spouses are substitutes in the time that increase the human capital of their children, is the semi-cooperative setup either with the random selection criteria or with a the criteria that estimates how society is biased towards men because they allow for multiplicity of equilibria. This heterogeneity is not explained by the model and should be interpreted as another dimension affecting households' decisions regarding the family organization such as the social environment.

5.2 Efficiency

A first question that is addressed here is what would be the efficient amount of time to allocate to childcare when using the parameters estimated with the semi-cooperative setup
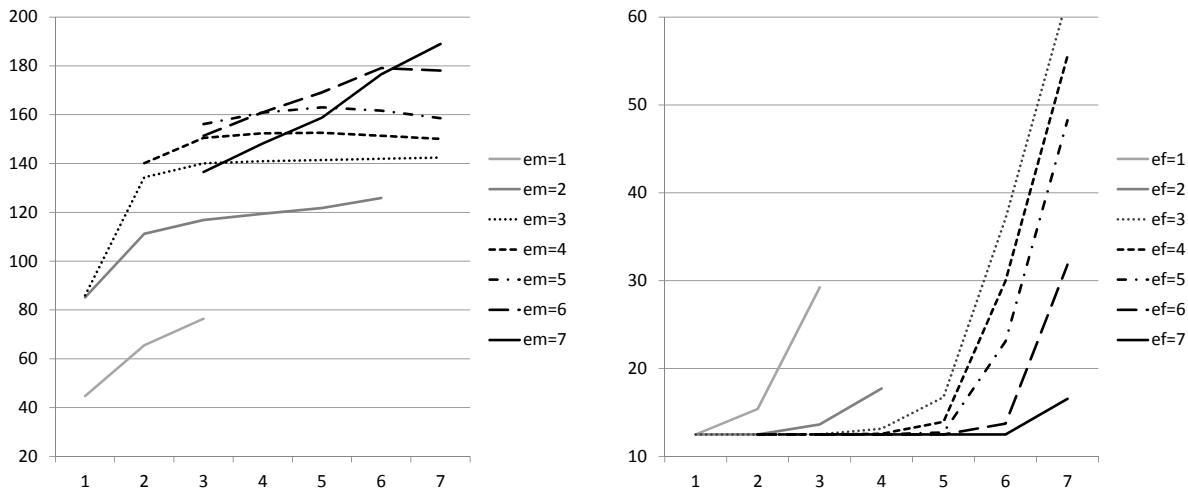
32

and the random selection criterion.

Figure 15: Simulations of the Cooperative Model for Female (Left) and Male (Right) Child-care with respect to e_f and e_m Using the Estimated Parameters for (2).

Figure 15 shows childcare supplies for men and women given by the cooperative model with the parameters of the semi-cooperative model (2), provided in Table 3. Compared to Figure 10, efficient couples have women supplying more childcare than what they would in semi-cooperative couples, as they are in either **A4a** or **A4c**. In either one of these cases her partner does not supply quality childcare. Men only supply childcare when their wives are a poor substitute in producing child quality (**A2c**). This explains the right panel in Figure 15 where childcare supplied by men increases with their education, in particular when they are coupled with uneducated women.

The conclusion is that the amount of childcare supplied by semi-cooperative spouses is inefficient. In the presence of a gender wage gap, women should spend more time with children. Men should spend less time with their children, unless if they are married with an uneducated woman, in which case they should provide more quality childcare. The semi-cooperative model estimates that women spend in average 54.64 minutes per day in childcare activities. Men spend in average 32.41 minutes. This implies that children from a semi-cooperative couple receive 87.06 minutes of attention from their parents per day. If, for the same parameters, individuals cooperated, women would spend 138.88 minutes and men 18.94 minutes of childcare per day. This sums to 157.82 minutes. So the inefficiency in terms of minutes supplied to children from the non-cooperative setup is around 80% of all the childcare supplied.

5.3 Comparative Statics: changes in the gender wage gap

As said in Section 3, the cooperative model is more sensitive to changes in the wage gap than the semi-cooperative model. The question that is addressed here is what is the effect of closing the gender wage gap on the amount of childcare supplied by parents. As the only difference between men and women is the gender wage gap (and fixed time costs $\bar{t}_f$ and $\bar{t}_f$ that do not affect much the appearance of cases), only the effect of changes in the gender wage gap on female childcare is shown.
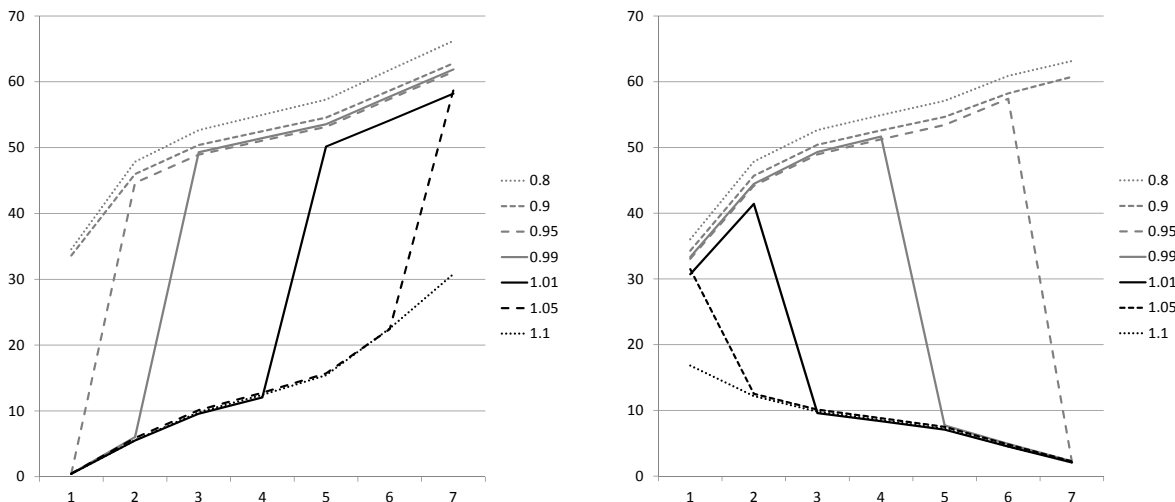

Figure 16: Impact of the Wage Gap on Female Childcare with respect to her Education (left) and The Education of her Male Partner (right) in the Cooperative Setup.

The left panel of Figure 16 shows the impact of changes in the gender wage gap on childcare supplied by women in cooperative couples with respect to their education when coupled with men in category 3 of education. We can see that for any value of the wage gap, childcare increases with her education. We also see that a larger gender gap will lead to more childcare by women. More interestingly, we see that when the gender wage gap is very small, the amount of childcare depends very much on her education. For a positive gender wage gap and a woman with average education, the relevant regimes (in order of appearance as e_f increases) are **A2c**, **A4c** and **A4a**, as shown in Figure 1. When $w_m < w_f$, the relevant cases are **A3a**, **A3b** and **A2b** (symmetric to the previous). In either situation, childcare can increase with her education, but in the case of a negative gender wage gap, the jump in childcare from low educated women to highly educated women is due to the passage from cases where the husband supplies all the quality childcare to the case where both partners supply some. With a positive gender wage gap, it is always efficient having

the woman supplying some of the quality childcare, while for a negative gender wage gap, it is only efficient if the husband is a low substitute in producing child quality.

The right panel of Figure 16 shows how the gender wage gap affects childcare supplied by women with respect to the education of men in the cooperative setup fixing female education to that of category 3. We see that the gender wage gap changes the monotonicity of the relationship between t_f and e_m. When there is a positive gender wage gap, the relationship between childcare and the education of the husband is positive because the relevant cases are (in order of appearance as e_m increases) **A2b** (or **A3a**), **A4a** and **A4c**. When w_f/w_m is close to one, the possible situations for the couple are **A2b**, **A3b** and **A3a**, so childcare of women is first increasing (**A2b**) and then decreasing because in **A3b** and **A3a** she does not supply quality childcare because her husband is more efficient in producing child quality while she has an advantage in producing labor income.
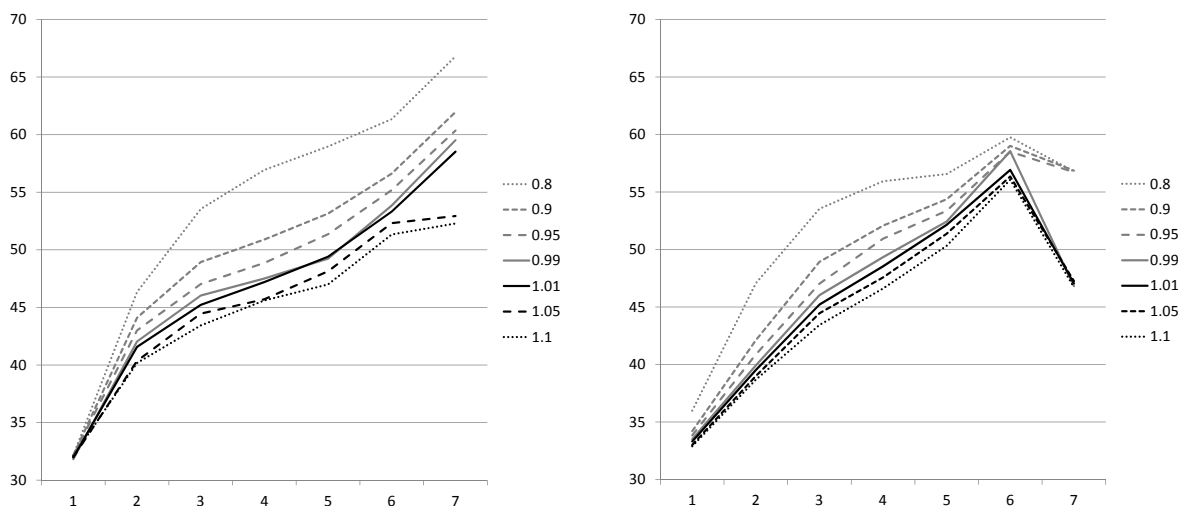

Figure 17: Impact of the Wage Gap on Female Childcare with respect to her Education (left) and the Education of her Male Partner (right) in the Semi-Cooperative setup with Random Selection of the Equilibrium.

Figure 17 replicates the exercise that was shown in Figure 16 for the semi-cooperative setup with the random selection of the equilibrium. In this case, a lower wage gap decreases the amount of childcare supplied by women because it increases the opportunity cost in terms of labor income. The last decrease in childcare with respect to the education of the husband in the right panel of Figure 17 for a low and negative gender wage gap is due to a larger probability to be in case **B3a**. As the husband has a larger education than his wife and she earns more than him, it is likely that the couple will be in the case where he both works and supplies childcare, while she only works. The comparison between Figures 17 and 16 shows

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that childcare supplied by parents reacts less to changes in the wage gap when decisions on childcare are individually taken within couples.

6   Conclusion

This paper proposes a semi-cooperative model of family decision making to explain the observed relationship between childcare and education in the United States. In a first step, couples collectively choose labor supplies. The lack of a monitoring on the amount of childcare supplied by each spouse induces them to make non-cooperative choices regarding this variable in the second step. This implies that the amount of childcare supplied is inefficiently low as individuals do not internalize the positive externality of childcare on the utility of the partner. If there was a credible commitment on the amount of childcare allocated by each partner that could push couples to choose childcare efficiently, children would gain in average 70 minutes more of childcare per day.

The theoretical semi-cooperative framework of this paper generates indeterminacy of the equilibrium. This introduces another dimension of heterogeneity. I interpret this heterogeneity as a social norm driving couples to one type of equilibrium rather than another. To shed some light on this, one could think that this social norm encompasses a certain degree of machism in the society. The estimated proportion of couples that are in an equilibrium that maximizes men's welfare is estimated to be 0.476. This source of heterogeneity is important for explaining why spouses choose one type of family rather than another, for given education levels of partners. This does not mean however that education does not matter for explaining childcare decisions. Education does determine when, given couples' beliefs, a certain time-allocation arrangement within the family will be chosen and also how childcare relates to education within each of these arrangements.

Compared to efficient choices of the collective model, semi-cooperative outcomes are less sensitive to the presence of a gender wage gap. This implies that changes in social norms might then be more effective than changes in the gender wage gap in pushing men to take the responsibility of childrearing.

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A   Total Childcare

Without dividing by the number of children in the households, the facts given in Table 2 would be those exposed in Table 4. We see that facts (1), (2) and (3) continue to hold.

Education, Women	Education, Men						
	1	2	3	4	5	6	7
1	34.407	62.169	76.978				
2	27.066	54.191	72.574	81.904			
3	41.723	55.300	70.487	65.443	69.145	101.540	64.982
4		79.895	58.630	75.603	75.963	83.119	73.946
5		93.885	71.609	97.860	107.795	106.193	113.072
6		67.467	66.687	86.942	86.616	91.435	94.085
7			101.591	104.291	115.980	102.557	105.333

Education, Women	Education, Men						
	1	2	3	4	5	6	7
1	102.995	64.508	109.966				
2	131.507	80.747	142.540	217.137			
3	82.572	109.418	111.698	122.050	143.070	166.126	123.013
4		135.844	114.243	123.702	108.153	170.017	181.968
5		119.009	102.398	131.554	121.807	132.259	196.364
6		134.666	126.200	125.480	127.761	169.267	181.424
7			141.146	145.234	126.710	150.139	157.417

Table 4: Top: Minutes per day spent in childcare by men as a function of his education and of his wife or female partner's. Down: Minutes per day spent in childcare by women as a function of her education and of her husband or male partner's.

B Conditions for Each Case to Arise

B.1 Cooperative Model

A1a. L'enfant Sauvage. Labor supplies are

$$L_f = \frac{(1 + (1 - \theta)\mu)A_f w_f e_f - \theta\mu A_m w_m e_m}{(1 + \mu)w_f e_f} \quad \text{and}$$

$$L_m = \frac{(1 + \theta\mu)A_m w_m e_m - (1 - \theta)\mu A_f w_f e_f}{(1 + \mu)w_m e_m}.$$

The positive KT coefficients are equal to

$$a = \frac{(1 + \mu)n w_f e_f}{A_f w_f e_f + A_m w_m e_m} - \frac{\gamma e_f^\alpha}{\bar{q}} \quad \text{and} \quad b = \frac{(1 + \mu)n w_m e_m}{A_f w_f e_f + A_m w_m e_m} - \frac{\gamma e_m^\alpha}{\bar{q}}.$$

This case arises if $(1 + \mu)\bar{q}n \times \min\{w_f e_f^{1-\alpha}, w_m e_m^{1-\alpha}\} - \gamma(A_f w_f e_f + A_m w_m e_m) > 0$,

$$e_f > \frac{\theta\mu}{1 + (1 - \theta)\mu} \frac{A_m w_m e_m}{A_f w_f} \quad \text{and} \quad e_m > \frac{(1 - \theta)\mu}{1 + \theta\mu} \frac{A_f w_f e_f}{A_m w_m}.$$

A1b. L'Enfant Sauvage and an Unemployed Husband. The labor supply of the wife is equal to
$$L_f = \frac{A_f}{1 + \theta\mu}.$$

The positive KT coefficients are

$$a = \frac{(1 + \theta\mu)n}{A_f} - \frac{\gamma e_f^\alpha}{\bar{q}}, \quad b = \frac{(1 - \theta)\mu n}{A_m} - \frac{\gamma e_m^\alpha}{\bar{q}} \quad \text{and} \quad d = \frac{(1 - \theta)\mu}{A_m} - \frac{(1 + \theta\mu)w_m e_m}{A_f w_f e_f}.$$

This case arises if

$$e_f < \left(\frac{(1 + \theta\mu)\bar{q}n}{\gamma A_f}\right)^{1/\alpha}, \quad e_m < \left(\frac{(1 - \theta)\mu\bar{q}n}{\gamma A_m}\right)^{1/\alpha} \quad \text{and} \quad e_m < \frac{(1 - \theta)\mu}{1 + \theta\mu} \frac{A_f w_f e_f}{A_m w_m}.$$

A1c. L'Enfant Sauvage and an Unemployed Wife. The labor supply of the husband is equal to
$$L_m = \frac{A_m}{1 + (1 - \theta)\mu}.$$

The KT coefficients are,

$$a = \frac{\theta \mu n}{A_f} - \frac{\gamma e_f^\alpha}{\bar{q}}, \quad b = \frac{1 + (1-\theta)\mu n}{A_m} - \frac{\gamma e_m^\alpha}{\bar{q}} \quad \text{and} \quad c = \frac{\theta \mu}{A_f} - \frac{(1 + (1-\theta)\mu)w_f e_f}{A_m w_m e_m}.$$

This case arises if

$$e_m < \left(\frac{(1 + (1-\theta)\mu)\bar{q}n}{\gamma A_m} \right)^{1/\alpha}, \quad e_f < \left(\frac{\theta \mu \bar{q} n}{A_f \gamma} \right)^{1/\alpha} \quad \text{and} \quad e_f < \frac{\theta \mu}{1 + (1-\theta)\mu} \frac{A_m w_m e_m}{A_f w_f}.$$

A2a. Solving the KT conditions for $a, b, c, d = 0$ and $w_f e_f^{1-\alpha} \neq w_m e_m^{1-\alpha}$ leads to the following results:

$$t_f = \frac{A_f w_f e_f + (A_m e_m^\alpha + \bar{q}n)\, w_m e_m^{1-\alpha}}{(w_f e_f^{1-\alpha} - w_m e_m^{1-\alpha})ne_f^\alpha}, \quad t_m = -\frac{\left(A_f e_f^\alpha + \bar{q}n \right) w_f e_f^{1-\alpha} + A_m w_m e_m}{(w_f e_f^{1-\alpha} - w_m e_m^{1-\alpha})ne_m^\alpha},$$

$$L_f = -\frac{\left(A_f e_f^\alpha + A_m e_m^\alpha + \bar{q}n \right) w_m e_m^{1-\alpha}}{(w_f e_f^{1-\alpha} - w_m e_m^{1-\alpha})e_f^\alpha} \quad \text{and} \quad L_m = \frac{\left(A_f e_f^\alpha + A_m e_m^\alpha + \bar{q}n \right) w_f e_f^{1-\alpha}}{(w_f e_f^{1-\alpha} - w_m e_m^{1-\alpha})e_m^\alpha}.$$

This is impossible as t_f, t_m, L_f and L_m cannot be positive altogether. From the KT conditions, it follows that the condition for this case to arise is $w_m e_m^{1-\alpha} = w_f e_f^{1-\alpha}$. This would lead to a continuum of possible combinations of childcare and labor supplies such that $t_f n + L_f \in (0, 1 - A_f)$ and $t_m n + L_m \in (0, 1 - A_m)$.

A2b. A Busy Wife. The KT coefficient is

$$d = \frac{(1 + \gamma + \mu)(w_f e_f^{1-\alpha} - w_m e_m^{1-\alpha})e_m^\alpha}{(A_f e_f^\alpha + A_m e_m^\alpha + \bar{q}n)w_f e_f^{1-\alpha}}.$$

This case arises when $w_m e_m^{1-\alpha} < w_f e_f^{1-\alpha}$,

$$e_m^\alpha < \frac{(\gamma + (1-\theta)\mu)A_f e_f^\alpha - (1 + \theta\mu)\bar{q}n}{(1 + \theta\mu)A_m} \quad \text{and} \quad e_m^\alpha > \frac{(1-\theta)\mu A_f e_f^\alpha - (1 + \gamma + \theta\mu)\bar{q}n}{(1 + \gamma + \theta\mu)A_m}.$$

A2c. A Busy Husband. The only positive KT coefficient is

$$c = \frac{(1 + \gamma + \mu)(w_m e_m^{1-\alpha} - w_f e_f^{1-\alpha})e_f^\alpha}{(A_f e_f^\alpha + A_m e_m^\alpha + \bar{q}n)w_m e_m^{1-\alpha}}.$$

This case arises when $w_m e_m^{1-\alpha} > w_f e_f^{1-\alpha}$,

$$e_f^\alpha > \frac{\theta\mu A_m e_m^\alpha - \bar{q}n}{(1+\gamma+(1-\theta)\mu)A_f} \quad \text{and} \quad e_f^\alpha < \frac{-(\gamma+\theta\mu)A_m e_m^\alpha + (1+(1-\theta)\mu)\bar{q}n}{(1+(1-\theta)\mu)A_f}.$$

A3a. A Multi-Task Husband. The KT coefficient is

$$a = \frac{e_f^\alpha(w_f e_f^{1-\alpha} - w_m e_m^{1-\alpha})n(1+\gamma+\mu)}{A_f w_f e_f + A_m w_m e_m + w_m e_m^{1-\alpha}\bar{q}n}.$$

This case will arise for the education levels such that $w_m e_m^{1-\alpha} < w_f e_f^{1-\alpha}$,

$$e_f > \frac{(1+\mu)\bar{q}n w_m e_m^{1-\alpha} - \gamma A_m w_m e_m}{\gamma A_f w_f}, \quad e_f > \frac{\theta\mu}{1+\gamma+(1-\theta)\mu}\frac{(A_m e_m^\alpha + \bar{q}n)w_m e_m^{1-\alpha}}{A_f w_f}$$

$$\text{and} \quad e_f < \frac{1+\theta\mu}{\gamma+(1-\theta)\mu}\frac{(A_m e_m^\alpha + \bar{q}n)w_m e_m^{1-\alpha}}{A_f w_f}.$$

A3b. Modern Specialization. The KT coefficients are

$$a = \frac{(1+\theta\mu)n}{A_f} - \frac{(\gamma+(1-\theta)\mu)n e_f^\alpha}{A_m e_m^\alpha + \bar{q}n} \quad \text{and} \quad d = \frac{(\gamma+(1-\theta)\mu)e_m^\alpha}{A_m e_m^\alpha + \bar{q}n} - \frac{(1+\theta\mu)w_m e_m}{A_f w_f e_f}.$$

This case will arise for the education levels such that

$$e_f > \frac{(1+\theta\mu)(A_m e_m^\alpha + \bar{q}n)w_m e_m^{1-\alpha}}{(\gamma+(1-\theta)\mu)A_f w_f}, \quad e_m > \left(\frac{(1-\theta)\mu\bar{q}n}{\gamma A_m}\right)^{1/\alpha}$$

$$\text{and} \quad e_m^\alpha > \frac{(1+(1-\theta)\mu)A_f e_f^\alpha - (1+\theta\mu)\bar{q}n}{(1+\theta\mu)A_m}.$$

A3c. An Unemployed Wife. The KT coefficients are

$$a = \frac{\theta\mu n}{A_f} - \frac{(1+\gamma+(1-\theta)\mu)n e_f^\alpha}{A_m e_m^\alpha + \bar{q}n} \quad \text{and} \quad c = \frac{\theta\mu}{A_f} - \frac{(1+\gamma+(1-\theta)\mu)w_f e_f}{(A_m e_m^\alpha + \bar{q}n)w_m e_m^{1-\alpha}}.$$

This case arises for education levels such that

$$e_m > \left(\frac{(1+(1-\theta)\mu)\bar{q}n}{\gamma A_m}\right)^\alpha$$

and $\theta\mu(A_m e_m^\alpha + \bar{q}n)w_m e_m - (1+\gamma+(1-\theta)\mu)A_f \times \max\{w_f e_f e_m^\alpha, w_m e_m e_f^\alpha\} > 0$.

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A4a. A Multi-Task Wife. The KT coefficient is,

$$b = \frac{(1+\gamma+\mu)n(w_m e_m^{1-\alpha} - w_f e_f^{1-\alpha})e_m^\alpha}{A_f w_f e_f + A_m w_m e_m + \bar{q}n w_f e_f^{1-\alpha}}.$$

This case arises for education levels such that $w_m e_m^{1-\alpha} > w_f e_f^{1-\alpha}$,

$$e_m > \frac{(1+\mu)\bar{q}n w_f e_f^{1-\alpha} - \gamma A_f w_f e_f}{\gamma A_m w_m}, \quad e_m < \frac{(1+(1-\theta)\mu)(A_f e_f^\alpha + \bar{q}n)w_f e_f^{1-\alpha}}{(\gamma+\theta\mu)A_m w_m}$$

$$\text{and} \quad e_m > \frac{((1-\theta)\mu A e_f^\alpha - \bar{q}n)w_f e_f^{1-\alpha}}{(1+\gamma+\theta\mu)A_m w_m}.$$

A4b. An Unemployed Husband. The KT coefficients are,

$$b = \frac{(1-\theta)\mu n}{A_m} - \frac{(1+\gamma+\theta\mu)n e_m^\alpha}{\bar{q}n + A_f e_f^\alpha} \quad \text{and} \quad d = \frac{(1-\theta)\mu}{A_m} - \frac{(1+\gamma+\theta\mu)w_m e_m}{(\bar{q}n + A_f e_f^\alpha)w_f e_f^{1-\alpha}}.$$

This case arises for education levels such that $(A_f \gamma e_f^\alpha - (1+\theta\mu)\bar{q}n)w_f e_f^{1-\alpha} > 0$ and $(1-\theta)\mu(A_f e_f^\alpha + \bar{q}n)w_f e_f - (2+\theta\mu)A_m \max\{w_f e_f e_m^\alpha, w_m e_m e_f^\alpha\} > 0$.

A4c. Traditional Specialization. The KT coefficients are,

$$b = \frac{(1+(1-\theta)\mu)n}{A_m} - \frac{(\gamma+\theta\mu)n e_m^\alpha}{A_f e_f^\alpha + \bar{q}n} \quad \text{and} \quad c = \frac{(\gamma+\theta\mu)e_f^\alpha}{A_f e_f^\alpha + \bar{q}n} - \frac{(1+(1-\theta)\mu)w_f e_f}{A_m w_m e_m}.$$

This case arises for education levels such that

$$e_f > \left(\frac{\theta\mu\bar{q}n}{\gamma A_f}\right)^{1/\alpha}, \quad e_f > \left(\frac{(\gamma+\theta\mu)A_m e_m^\alpha - (1+(1-\theta)\mu)\bar{q}n}{(1+(1-\theta)\mu)A_f}\right)^{1/\alpha}$$

$$\text{and} \quad e_m > \frac{(1+(1-\theta)\mu)(A_f e_f^\alpha + \bar{q}n)w_f e_f^{1-\alpha}}{(\gamma+\theta\mu)A_m w_m}.$$

Note that $b, c > 0 \Rightarrow w_m e_m^{1-\alpha} > w_f e_f^{1-\alpha}$.

B.2   Semi-Cooperative Model

B1a. L'Enfant Sauvage. A couple can be in this case if labor supplies are positive and the conditions needed to be in **B1** are satisfied. This holds if,

$$\begin{cases} e_f & > \dfrac{\theta\mu}{1+(1-\theta)\mu}\dfrac{A_m w_m e_m}{A_f w_f} \\[2mm] e_m & > \dfrac{(1-\theta)\mu}{1+\theta\mu}\dfrac{A_f w_f e_f}{A_m w_m} \\[2mm] e_m & < \dfrac{(-\theta A_f e_f^\alpha + (1+\mu)\bar{q}n)w_f e_f^{1-\alpha}}{\theta A_m w_m} \\[2mm] e_f & < \dfrac{(-(1-\theta)A_m e_m^\alpha + (1+\mu)\bar{q}n)w_m e_m^{1-\alpha}}{(1-\theta)A_f w_f}. \end{cases}$$

B1b. Enfant Sauvage and an Unemployed Husband. The positive KT multiplier is

$$d = \frac{(1-\theta)\mu}{A_m} - \frac{(1+\theta\mu)w_m e_m}{A_f w_f e_f}$$

This case can arise if

$$e_f < \left(\frac{(1+\theta\mu)\bar{q}n}{\theta A_f}\right)^{1/\alpha}, \quad e_m < \left(\frac{\mu\bar{q}n}{A_m}\right)^{1/\alpha} \quad \text{and} \quad e_m < \frac{(1-\theta)\mu}{1+\theta\mu}\frac{A_f w_f e_f}{A_m w_m}.$$

B1c. L'Enfant Sauvage and an Unemployed Wife. The positive KT multiplier is

$$c = \frac{\theta\mu}{A_f} - \frac{(1+(1-\theta)\mu)w_f e_f}{A_m w_m e_m}.$$

This case arises if

$$e_f < \left(\frac{\mu\bar{q}n}{A_f}\right)^{1/\alpha}, \quad e_m < \left(\frac{(1+(1-\theta)\mu)\bar{q}n}{(1-\theta)A_m}\right)^{1/\alpha} \quad \text{and} \quad e_f < \frac{\theta\mu}{1+(1-\theta)\mu}\frac{A_m w_m e_m}{A_f w_f}.$$

B2a. Lets first assume that $w_m e_m^{1-\alpha} \neq w_f e_f^{1-\alpha}$. Labor supplies would be equal to

$$L_f = \frac{-w_m e_m^{1-\alpha}\left(A_f e_f^\alpha + A_m e_m^\alpha + \bar{q}n\right)}{e_f^\alpha(w_f e_f^{1-\alpha} - w_m e_m^{1-\alpha})} \quad \text{and} \quad L_m = \frac{w_f e_f^{1-\alpha}\left(A_f e_f^\alpha + A_m e_m^\alpha + \bar{q}n\right)}{e_m^\alpha(w_f e_f^{1-\alpha} - w_m e_m^{1-\alpha})}$$

which do not satisfy both $L_m, L_f > 0$. As for **A2a**, both parents work and provide some childcare (other than the minimal required levels) will never arise, unless $w_m e_m^{1-\alpha} = w_f e_f^{1-\alpha}$.

B2b. A Busy Wife.

$$d = \frac{(1 + \gamma + \mu)(w_f e_f^{1-\alpha} - w_m e_m^{1-\alpha})e_m^{\alpha}}{(A_f e_f^{\alpha} + A_m e_m^{\alpha} + \overline{q}n)w_f e_f^{1-\alpha}}$$

This case can arise if $w_m e_m^{1-\alpha} < w_f e_f^{1-\alpha}$ and

$$e_m^{\alpha} < \frac{(\gamma + \mu)^2 A_f e_f^{\alpha} - (\gamma + 2\mu + (\gamma + \mu)^2)\,\overline{q}n}{(\gamma + 2\mu + (\gamma + \mu)^2)\,A_m} \quad \text{and} \quad e_m^{\alpha} > \frac{\mu(\gamma + \mu)(A_m e_f^{\alpha} + \overline{q}n)}{(\gamma + 2\mu + (\gamma + \mu)^2)\,A_m}.$$

B2c. A Busy Husband.

$$c = \frac{(1 + \gamma + \mu)(w_m e_m^{1-\alpha} - w_f e_f^{1-\alpha})e_f^{\alpha}}{(A_f e_f^{\alpha} + A_m e_m^{\alpha} + \overline{q}n)w_m e_m^{1-\alpha}}$$

This case arises if the conditions to be in **B2** are satisfied, $c > 0$ and $L_m > 0$ which hold when $w_m e_m^{1-\alpha} > w_f e_f^{1-\alpha}$ and

$$e_f^{\alpha} > \frac{\mu(\gamma + \mu)(A_m e_m^{\alpha} + \overline{q}n)}{(\gamma + 2\mu + (\gamma + \mu)^2)\,A_f} \quad \text{and} \quad e_f^{\alpha} < \frac{(\gamma + \mu)^2 A_m e_m^{\alpha} - (\gamma + (2 + \gamma)\mu + \mu^2)\overline{q}n}{(\gamma + (2 + \gamma)\mu + \mu^2)A_f}.$$

B3a. A Multi-Task Husband. In order for this case to happen, labor supplies must be positive and satisfy the conditions needed to be in **B3**, which hold if,

$$\begin{cases} e_f & > \dfrac{\theta\mu}{1 + \gamma + (1 - \theta)\mu}\dfrac{(A_m e_m^{\alpha} + \overline{q}n)w_m e_m^{1-\alpha}}{A_f w_f} \\[2mm] e_f & < \dfrac{1 + \theta\mu}{\gamma + (1 - \theta)\mu}\dfrac{(A_m e_m^{\alpha} + \overline{q}n)w_m e_m^{1-\alpha}}{A_f w_f} \\[2mm] e_m & < \dfrac{\mu(A_f w_f e_f + A_m w_m e_m)((\gamma + (1 - \theta)\mu)w_f e_f^{1-\alpha} - \theta(\gamma + \mu)w_m e_m^{1-\alpha})e_m^{\alpha}}{\overline{q}n w_m} \\[2mm] e_f & > \dfrac{(\gamma + \mu + (1 + \theta)\gamma\mu + \mu^2)\overline{q}n w_m e_m^{1-\alpha}}{\gamma(\gamma + (1 - \theta)\mu)A_f w_f} - \dfrac{A_m w_m e_m}{A_f w_f}. \end{cases}$$

B3b. Modern Specialization.

$$d = \frac{(\gamma + (1 - \theta)\mu)e_m^{\alpha}}{A_m e_m^{\alpha} + \overline{q}n} - \frac{(1 + \theta\mu)w_m e_m}{A_f w_f e_f}$$

In order for this case to happen, we need

$$e_f > \frac{(1 + \theta\mu)(A_m e_m^{\alpha} + \overline{q}n)w_m e_m^{1-\alpha}}{(\gamma + (1 - \theta)\mu)A_f w_f} \quad \text{and} \quad e_m^{\alpha} > \frac{\theta(\gamma + \mu)A_f e_f^{\alpha}}{(1 + \theta\mu)A_m} - \frac{\overline{q}n}{A_m}.$$

B3c. An Unemployed Wife.

$$c = \frac{\theta\mu}{A_f} - \frac{(1 + \gamma + (1 - \theta)\mu)w_f e_f}{A_m e_m^\alpha + \bar{q}n}$$

The conditions under which this case may happen are such that:

$$
\begin{cases}
e_m^\alpha & > \dfrac{(\gamma + (1 + \gamma + (1 - \theta)\mu)\mu)\bar{q}n}{(\gamma + (1 - \theta)\mu)A_m} \\[2mm]
e_f & < \theta\mu\dfrac{(A_m e_m^\alpha + \bar{q}n)w_m e_m^{1-\alpha}}{(1 + \gamma + (1 - \theta)\mu)A_f w_f} \\[2mm]
e_f^\alpha & < \dfrac{\mu(\gamma + (1 - \theta)\mu)}{(\gamma + \mu)(1 + \gamma + (1 - \theta)\mu)}\dfrac{A_m e_m^\alpha + \bar{q}n}{A_f}.
\end{cases}
$$

B4a. A Multi-Task Wife. In order for this case to happen, labor supplies must be positive and satisfy the conditions needed to be in **B4**:

$$
\begin{cases}
e_m & > \left(\dfrac{(1 - \theta)(\gamma + \mu)}{(\gamma + \theta\mu)}\dfrac{w_f}{w_m}\right)^{\frac{1}{1-\alpha}} e_f \\[2mm]
e_m & < \dfrac{(1 + (1 - \theta)\mu)(A_f e_f^\alpha + \bar{q}n)w_f e_f^{1-\alpha}}{(\gamma + \theta\mu)B w_m} \\[2mm]
e_m & > \dfrac{(1 - \theta)\mu}{1 + \gamma + \theta\mu}\dfrac{(A_f e_f^\alpha + \bar{q}n)w_f e_f^{1-\alpha}}{A_m w_m} \\[2mm]
e_m & > \dfrac{(-\gamma(\gamma + \theta\mu)A_f e_f^\alpha + (\gamma + \mu + (2 - \theta)\gamma\mu + \mu^2)\bar{q}n)w_f e_f^{1-\alpha}}{\gamma(\gamma + \theta\mu)A_m w_m}.
\end{cases}
$$

B4b. An Unemployed Husband.

$$d = \frac{(1 - \theta)\mu}{A_m} - \frac{(1 + \gamma + \theta\mu)w_m e_m}{(A_f e_f^\alpha + \bar{q}n)w_f e_f^{1-\alpha}}$$

In order for this case to happen, L_f must be positive and satisfy the conditions needed to be in **B4** and $d > 0$. This holds if

$$
\begin{cases}
e_f^\alpha & > \dfrac{(\gamma + (1 + \gamma + \theta\mu)\mu)\bar{q}n}{(\gamma(\gamma + \theta\mu)A_f} \\[2mm]
e_m & < \dfrac{(1 - \theta)\mu}{(1 + \gamma + \theta\mu)}\dfrac{(A_f e_f^\alpha + \bar{q}n)w_f e_f^{1-\alpha}}{A_m w_m} \\[2mm]
e_m^\alpha & < \dfrac{\mu(\gamma + \theta\mu)(A_f w_f e_f e_f^\alpha + \bar{q}n w_f e_f)}{(\gamma + \mu)(1 + \gamma + \theta\mu)A_m w_f e_f}.
\end{cases}
$$

B4c. Traditional Specialization.

$$c = \frac{(\gamma + \theta\mu)e_f^\alpha}{A_f e_f^\alpha + \overline{q}n} - \frac{(1 + (1 - \theta)\mu)w_f e_f}{A_m w_m e_m}$$

In order for this case to happen, we need to have $L_m, c > 0$ and the conditions needed to be in **B4** must be checked. This holds if

$$e_m > \frac{(1 + (1 - \theta)\mu)(A_f e_f^\alpha + \overline{q}n)w_f e_f^{1-\alpha}}{(\gamma + \theta\mu)A_m w_m} \quad \text{and} \quad e_f^\alpha > \frac{(1 - \theta)(\gamma + \mu)A_m e_m^\alpha - (1 + (1 - \theta)\mu)\overline{q}n}{(1 + (1 - \theta)\mu)A_f}.$$

C Complementarity vs. Substitutability of Parents

C.1 Discussion and Previous Findings

More research should investigate the underlying form of the production function that relates parental time and education to the cognitive and non-cognitive ability of children. Common sense would lead to think of parents as substitutes instead of as complements in producing child quality as we have seen a rise in the amount of single mothers in the last decades (Regalia et al. (2011)) or the rise in divorce (Greenwood et al. (2012)), leading one parent to be more in charge of the education of the children. Del Boca and Ribero (2001) also provide a non-cooperative theoretical framework in which divorced fathers exchange monetary transfers for visitation time to custodial mothers. Historically, fathers were also much less present in the education of their children than what they are today (Bianchi et al. (2004)). Moreover, family policies in Europe are usually gender neutral, suggesting that parents can replace each other.

Pailhé and Solaz (2008) look at how experiencing unemployment affects the distribution of parental tasks within French couples. They show that some childcare activities are substitutable when one parents is unemployed while others are not. The most substitutable activities are transportation ("taxi parents" activities) and care (which is the heaviest time consuming task and includes eating, washing, medical care...). The less are the social and leisure activities, suggesting that parents do not easily give up to these activities as they enjoy them most.[21] Which of these activities increases most the human capital of the children is not clear as all of them seem to be important. However, as the "care" activity is the one

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[21] Joint parental time is very little compared to the sum overall time that parents spend in childrearing activities (see Table 2 in Pailhé and Solaz (2008)).

Nb.	t_{fs}
1	48.436
2	61.690
3	55.035
4	61.187
5	67.742
6	71.967
7	104.691

Table 5: Minutes per day spent in childcare by single women, per child.

in which parents devote most of their parental time and it is substitutable, this seems to push for an overall substitutability between parents.

We can also compare the amount of childcare supplied by single mothers (living alone with their children), denoted t_{fs}, relative to that supplied by married mothers. Table 5 shows the estimated minutes per day per child of childcare of a white single women, aged 35-39 and whose youngest child is between 3 and 5 years old. These estimates are obtained from the same linear regression than in Section 2. Comparing these estimate to those in the bottom panel of Table 2, we see that in most cases, single women supply more childcare than married women. This is particularly true for uneducated women. For women who have completed high school, only those who are married with a more educated men than them may supply more childcare than single, for similar education levels. This comparison also goes in favor of substitutability between parents.

C.2   Results of the Collective Model when Parents are Complements in Producing Child Quality

In this Appendix, I assume that parental time of both the mother and the father are complementary inputs in producing child quality:

$$q = \min\{t_f e_f^\alpha, k t_m e_m^\alpha\} + \overline{q} \tag{9}$$

where k is a technological parameter to denote how efficient is the father relative to the mother. The couple maximizes (6) with respect to t_f, t_m, L_f and L_m, subject to (2), (4),
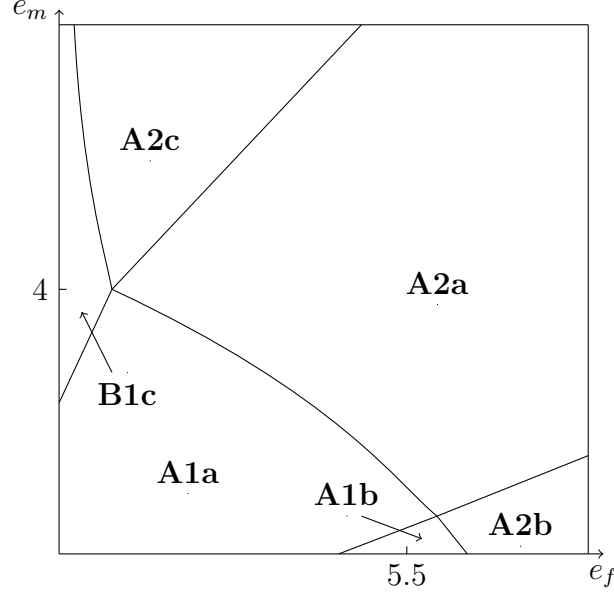
Figure 18: Possible cases with respect to e_f and e_m when parents are complements in producing child quality.

(9), $t_i \geq 0$ and $L_i \geq 0$. Replacing by

$$t_m = \frac{t_f e_f^\alpha}{k e_m^\alpha},$$

we can now solve the maximization problem just for t_f, L_f and L_m. The Kuhn-Tucker (KT) conditions are,

$$a + e_f^\alpha \left(\frac{\gamma}{t_f e_f^\alpha + \overline{q}} + \frac{(1-\theta)\mu n}{t_f e_f^\alpha n - k(A_m - L_m)e_m^\alpha} \right) - \frac{\theta \mu n}{A_f - L_f - t_f n} = 0,$$

$$c + \frac{w_f e_f}{w_f e_f L_f + w_m e_m L_m} - \frac{\theta \mu}{A_f - L_f - t_f n} = 0,$$

$$d + \frac{w_m e_m}{w_f e_f L_f + w_m e_m L_m} - \frac{k(1-\theta)\mu e_m^\alpha}{k(A_m - L_m)e_m^\alpha - t_f e_f^\alpha n} = 0,$$

$at_f = 0$, $cL_f = 0$, $dL_m = 0$, $t_f \geq 0$, $L_f \geq 0$, $L_m \geq 0$, $a \geq 0$, $c \geq 0$ and $d \geq 0$. Where a, c and d are the KT multipliers. When parents are complements in producing child quality, cases in **A3.** and **A4.** do not appear and there are 6 cases solving the problem. I will only interpret the numerical results as the analytical solutions are not tractable.

Figure 18 shows when cases **A1a**, **A1b**, **A1c**, **A2a**, **A2b** and **A2c** appear, with respect to the education of parents. The value of the parameters that were chosen are the same than in Section 3.2, $w_f = 0.9$, $w_m = 1$ and k was set to 3. For this parametrization, we have

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the following relationships between childcare and education. For households in **A2a**, **A2b** and **A2c**; $\partial t_f/\partial e_f > 0$, $\partial t_f/\partial e_m > 0$, $\partial t_m/\partial e_f > 0$, $\partial t_m/\partial e_m > 0$. This implies that if parental time of the mother and the father are complemetary inputs in the production of child quality, the collective framework can be compatible with the facts shown in Section 2.