Environmental Policy: An Evolutionary Perspective

Lorenzo Cerda Planas*

WORKING PAPER

Last revision: May 11th, 2012

Abstract

In the present work, I address the polluting problem from a 'political' point of view. Is it possible to reach a sufficient amount of people who are actually willing to sustain a good environment quality? Or will we always be in a state dominated by the less environmentally aware people? To address this question I will use the concepts of 'moral gain' and 'population dynamics'. With the help of a simple evolutionary model, I will find that the result mainly depends on the initial proportion of green/brown people in the society. I then expand the model to a more 'realistic' one and I show that the society can 'switch' sides, arising the possibility of having tipping points in the dynamics.

JEL Classification: Q50, C73, C61, D64, H41, C62

Keywords: Environmental motivation, replicator dynamics, evolutionary equilibrium, population dynamics

*C.E.S., Paris School of Economics, C.E.S. - Université de Paris 1 Panthéon-Sorbonne; 106-112, Boulevard de l'Hôpital, 75647 Paris Cedex 13, France. Email: lorenzo.cerda@malix.univ-paris1.fr

1 Introduction

This paper is motivated by the fact that, sometimes, long term objectives with undeniable social benefits are not always reached. Worse, the course of political action may not necessarily pursue these objectives at all. The question that therefore arises is: are we, as a society, able to reach said objectives, bearing in mind that we live in a politically framed society?

Narrowing this idea to a more specific topic, I choose to address the environmental problem. It seems quite clear that enjoying a good and sustainable environment would be beneficial to all, and governments should therefore act accordingly to achieve this outcome. But preserving the environment also entails costs which some members of society may not be willing to pay. This unwillingness could derive from various causes: Different people can evaluate the quality of the environment in different ways. Others may possess divergent preferences between environmental quality and, for example, their own consumption. To some extent, poorer people will always prefer to have food and housing above all, regardless of the state of the environment. Therefore, we are faced with heterogeneity from the position of individual actors regarding environmental issues and actions.

This might explain, at least partially, why some countries act in one way and others in another. Questions such as, 'Why did Europe sign the Kyoto Protocol when the USA did not?' could actually be related to cultural features. How can there prevail different environmental situations in countries which are similar from both an economic and a development perspective? It also might be the explanation of why, in the same country or society, some people are more environmentally aware than others. It might be the case that some other country specific characteristic could impact this heterogeneity just as they affect their income distributions or inequality.

Therefore, the topic that I address here consists of whether such a long term objective as good environment quality can be 'socially' or 'politically' achievable? Do we have (or will have) enough 'green' people in our society in order to actually reach a 'green' equilibrium? How does the interplay between the utility of goods consumption and the disutility of pollution evolve? Can these 'green' preferences win over the classical homo-economicus preferences in future generations? Will our children be more or less sensitive to the environmental status?

It seems that, so far, the environmental issue has been addressed only in such a perspective, that is, the pollution and the environment. Which limits can be imposed on polluting? Are there any incentives that can be put in place so as to abate it? Multiple papers on the matter usually approach the issue from this perspective: best abatement policies, information on the products we buy, etc. However, could 'political' forces actually deter good environment quality? In this vein, social preferences can be considered a living phenomenon in that there is an evolution of the 'general' preferences of a society, which usually translates into political and economic decisions.

In order to model the subsequent idea, I base this paper on two main ideas. The first one considers the 'evolution' of types of people in a society. This evolution comes from the fact that people are in constant comparison with their peers and the rest of the society. This fact has been studied in varied literature, as in Alvarez et al. (2004) [2], Bisin and Verdier (2000) [7], Saez-Marti and Zenou (2005) [30], Schlag (1995) [32]. The main concept I borrow is that the proportion of different type of people evolves and this evolution is due to the comparison of some utility function or 'fitness'.

This idea brings us to what is called 'population dynamics' which is based on the 'replicator dynamics', the later developed by Taylor and Jonker (1978) [39]. The former consists of the study of short and long-term changes in the size and age composition of populations, and the biological and environmental processes influencing those changes. The latter is the mathematical area in which this evolution is modeled. Both concepts are quite closely related. An introduction to the mathematical background can be found in Bomze (1983) [8], Hofbauer and Sigmund (1998) [19], Hofbauer and Sigmund (2003) [20] and Sigmund (1984) [36]. A comprehensive link to the economical framework has been developed in Friedman (1991) [13], Friedman (1998) [14] and Silverberg (1997) [38]. A good 'summary' can be found in Faber and Frenken (2008) [11] and van der Bergh (2007) [5].

Returning to the environmental side of the present work, I now consider the concept of 'moral gain', inspired by Brekke et al. (2003) [9] and Nyborg et al. (2003) [28]. In short, the concept is that (some) individuals get utility from being 'good citizens', in the same line as the 'warm-glow' of Andreoni (1990) [3]. Being good to the environment could mean, for example, that one recycles. In this context, individuals feel better ('moral gain') because they contribute to maintaining a good environment.

Therefore, using the previous concepts, I build a model in which two different types of people, which I term green and brown people, will have different types of utility functions with respect to the environment (one with moral gain, the other without). Moreover, these proportions will evolve accordingly. According to the proportion of the population, a green government could be elected, implementing some green policies. It is of interest to analyze the possibility of reaching this point in order to actually have a green government and therefore, a better environmental quality.

To present the model, I begin with a simple framework and thereafter I relax some constraints in order to find more general conclusions. Therefore, I will start in section 2 with a two type of people society (the 'green' ones and the 'brown' ones) and assuming the pollution as a flux. In section 3 I will use a more sophisticated representation of the moral gain. In section 4, I will use the standard definition of pollution as a stock, which will gives us some new insights. Finally, section 5 concludes.

2 The simple model

2.1 The Framework

As stated in the introduction, the idea here is to study the possible 'behaviour' of a society according to a 'green' and 'brown' people distribution and preferences. Therefore, in the present model, I will have a society with a constant population (normalized to 1) which will be compound of a share of 'green' people q_t and its consequent share of 'brown' people $(1 - q_t)$.

This society can consume two types of goods, which I will call the 'green good' (c_t) and 'brown good' (d_t) - abusing a little with the nomenclature. The difference between these goods is that the green one does not pollute, whereas the brown one does. With this, we will have that the level of pollution P_t will depend on the amount of brown good consumed. The relationship will simply be: $P_t = \gamma d_t$.[1] The 'drawback' of the green product is that it is more expensive than its brown counterpart. In this model, I will assign a normalized price of 1 to the brown good and a price of $(1 + \rho)$ to the green one. Therefore the value of ρ represents the extra amount (with respect to the whole original price) to be paid to buy a green product.

Finally, before going into the details of the model, I will assume that each agent has a fixed wage of w, which they will use to consume these goods. The share of each good will be discussed in the following sections.

2.2 The People

2.2.1 The Agent

In this model, the difference between the green and brown agents is due to the first one having a 'moral gain' of being a responsible and environmentally friendly person. Therefore, this agent will be willing to 'contribute' to the environment by buying green products, even thought these are more expensive. I call this predisposition to contribute the *green willingness* = δ, which will simply be the percentage (in goods) that the green person is willing to give up in order to contribute to the environment. It can be also be thought as the percentage of net income given to the green cause (both are equivalent). I assume that δ is fixed. It is actually the measure of *how green* these people are.

With the last definition I can now write the two utility functions, one for the green person, the other for the brown one, as follows:

$$W_{green}(n_{1t}, P_{t-1}) = f(n_{1t}) - h(P_{t-1}) + m(r, P_t) \tag{2.1}$$

$$W_{brown}(n_{2t}, P_{t-1}) = f(n_{2t}) - h(P_{t-1}) \tag{2.2}$$

In the next table I present a summary of the parameters and functions:

[1] A more 'realistic' evolution equation, as $P_t = (1 - \eta)P_{t-1} + \gamma d_t$, will be used in section 4.

Variable	Meaning
η	Environment "self" restoring.
γ	Impact of brown (total) consumption on the environment.
ρ	"Mark up" for green products (with respect to brown ones).
P_{t-1}	Pollution level in period $(t-1)$, which is observed at the beginning of period t.
n_{1t}	Total amount of consumed goods (brown and green products) by a **green** agent. Subsequently it will be just called n_1.
n_{2t}	Total amount of consumed goods (brown and green products) by a **brown** agent. Subsequently it will be just called n_2.
$f(n)$	It is the economic part of the utility function. As usual, it will be an increasing function, with diminishing marginal returns.
$h(P_t)$	It is the disutility of pollution. It is increasing function with $\frac{\partial h}{\partial P_t} > 0$ and $\frac{\partial^2 h}{\partial P_t^2} > 0$.
$m(r, P_t)$	Moral utility (only for green people) which is increasing function with r and P_t.

2.2.2 The brown people

Since at the beginning of period t the pollution level is given, the brown people will only maximize the consumption part of their utility function, subject to the budget constraint, which in this case is simply the wage w. Therefore their consumption n_2 will be governed by the simple identity $n_2 = w$.

2.2.3 The green people

We have that a green person will consume less quantity in order to be green (defined by δ, the green willingness). Since the price of the green products is higher than the price of its green counterpart, we will have that he/she will consume a mix of green and brown products. I will call 'r' the share of green products he/she buys and the rest $(1-r)$ the brown share. Since the total amount of product bought by a green person is n_1, then he/she will buy $r \cdot n_1$ green products (at a price of $(1+\rho)$) and $(1-r)n_1$ brown products (at price equal to 1). Therefore their budget constraint is:

$$(1+\rho)r \cdot n_1 + (1-r)n_1 = w$$

and solving for n_1 we get:

$$n_1 = \frac{w}{(1+r\rho)} \tag{2.3}$$

Following the '*green willingness*' idea, as defined before, this is the proportion of products (in quantity) that the green person is willing to 'give up' in order to be green, in comparison of just behaving as a brown person. Consequently, the equations are:

$$
\begin{aligned}
\frac{(n_2 - n_1)}{n_2} &= \delta \\
1 - n_1/n_2 &= \delta \\
n_1/n_2 &= 1 - \delta \\
\frac{1}{(1 + r\rho)} &= (1 - \delta) \\
r &= \frac{\delta}{(1 - \delta)\rho}
\end{aligned}
$$

And, therefore, the fraction r is determined by the green willingness (δ) and the extra price of the green goods (ρ). Obviously, this fraction is bounded by 1, and therefore the final expression for r is:

$$r = \min\left(1, \frac{\delta}{(1 - \delta)\rho}\right) \tag{2.4}$$

2.2.4 The moral gain

As stated in the introduction, besides from the consumption utility ($f(\cdot)$), I will use another source of utility in my model, the 'moral gain'. Varied literature shows different examples on this matter, with Nyborg and Rege (2003) [27] being a comprehensive 'summary' of different types of moral motivation.[2] The general idea is that it is not possible to explain the human behavior only with the *homo economicus* paradigm (Nyborg (2000) [26]). Actually this is not a novel idea, as it was stated in the 'warm-glow giving' of Andreoni (1990) [3], where the solely altruist action of donating gives an utility to the subject. Following a similar idea, but now connected to the environmental issue, Frey (1999) [12] talks about the 'environmental morale'.

In my case, I will use the moral gain or moral satisfaction in a similar fashion addressed by Nyborg et al.(2003) [28], Holländer (1990) [21] and Rege (2004) [29]. In the first case, Nyborg talks of a *'self image improvement S_i'*, whereas in the second two cases, the moral gain comes from *'social approval'*. Although the ideas presented in this papers are not exactly the same, they address the same issue and more interesting, they translate into the same formulation: The moral gain comes from a comparison of the agents contribution with respect to the average one.[3] I will call this type of gain, the *relative moral utility*. Further in this paper I will add to the moral gain what I will call the 'absolute moral utility'.

Therefore, to start with a simple model, I will suppose that green people gain their moral satisfaction from contributing in preserving a good environment and/or improving the environmental quality. Therefore, their gain will come from their contribution, which in our case would be rep-

[2]Although this paper focused on how these different type of moral motivations can crowd out private contribution, it has a good presentation of different types of moral motivation.

[3]The differences of them will be discussed in section 3.

resented by r. But as stated before, this gain is due to a relative contribution, which I translate as a moral gain that exists if one does 'better' than the average citizen, $\bar{r}$. The total moral utility (which for now it is only the relative moral utility) will also be an increasing function with respect to the pollution level P_t.

The intuition here is that people care when there is something to care about. If the environmental quality is good, then helping not to pollute does not have the same strength (from a moral rewarding point of view) as when the environment is heavy polluted and our help has a much higher impact (at least psychologically). This premise is consistent with the fact that thirty or forty years ago people were not concerned about not polluting too much, even though the emissions of CO_2, for example, were already quite high. The thing was that back then, the total pollution level (for example CO_2 concentration levels) were not too high in order to actually be a big problem to the society.

Therefore, their moral satisfaction will be an increasing function of $(r - \bar{r})$. Acknowledging that $\bar{r} = r \cdot q + 0 \cdot (1 - q) = r \cdot q$ and that $m(\cdot)$ is an increasing function in both arguments, we have:

$$\begin{aligned} moral\ gain &= m((r - \bar{r}), P_t) \\ &= m((r - rq), P_t) \\ &= m(r(1 - q), P_t) \end{aligned}$$

In my model I will use, for simplicity in the following calculations, the subsequent multiplicative function:

$$m(r(1 - q), P_t) = \alpha r(1 - q)P_t$$

Where α is just the weight of the moral satisfaction of the individual, in his/her utility function. Recalling what was seen at the beginning of this section, we have that $P_t = \gamma \cdot d_t$, being d_t the (total) brown good consumption. The brown consumption is made of the consumption of brown people (which is all brown) and the proportion of brown consumption of green people. Therefore we get the following relationship: $d_t = n_2(1 - r)q + n_1(1 - q)$. Recalling that $n_1 = w$, $n_2 = \frac{w}{(1+r\rho)}$ and equation 2.4, we have:[4]

$$d_t = 1 + q_t(r\delta - \delta - r)$$

Replacing this in the previous equation and rearranging, we get:

$$m(r, q) = \alpha\gamma r(1 - q)(1 - q(\delta + r - r\delta)) \tag{2.5}$$

Where γ is the impact of brown consumption on the environment (pollution).

Note about the 'real' mean person perception:

I have assumed so far that the green person can actually perceive, with perfect precision, the

[4] For simplicity I assume, from now on, that $w = 1$.

real mean value of r: $\bar{r}$. Let us now suppose that this perception is biased. Let us call Ω (with $0 < \Omega < \infty$) the *distorting factor of perception*.[5] In this case, the previous equation will become:

$$m(r,q) = \alpha\gamma r(1-\Omega q)(1-q(\delta+r-r\delta))$$

We will analyze this bias effect in section 2.3, where we will see the impact of Ω in the equilibrium point. One interesting thing to point out is that the Ω parameter is (mathematically) equivalent to the 'α' weighting parameter in Holländer's work [21], or the 'viscosity' one in Rege's work [29].

It could also be thought that not only the perception of the mean contribution is biased, but also the perception of the pollution level. In this second case though, it will suffice use (or vary) the parameter γ associated with the impact of d_t on the pollution level.

2.2.5 The cost of being green

Now I will address how the green willingness translates into the cost that green people bear, due to a lower consumption, when they 'behave' green. This 'cost of being green' will be the difference of the consumption part of the utility functions:

$$\Delta U_e = f(n_1) - f(n_2)$$

As usual, we have that $f(\cdot)$ has to be an increasing function with diminishing marginal returns. I will continue the present development using $f(n) = \beta \ln(n)$. The possible impacts of using another type of consumption utility functions are analyzed in Appendix A. This difference gives us what I will call the 'cost of being green', which in this particular case, it is independent of the wage level w:

$$\Delta U_e = \beta \ln(1-\delta) \tag{2.6}$$

Nevertheless, if another consumption utility function is used, then the value of ΔU_e will increase or decrease with the wage level. But since this equivalent to change the value of the parameter β, I will omit this analysis and refer to it when verifying what happens when β changes.

2.3 The evolution of society

Now I will focus on how society evolves. In this work, 'evolves' is to be interpreted as how the proportion of green people (and conversely brown people) changes from time to time. I will use the concept known as 'replicator dynamics' from Taylor and Jonker (1978) [39]. The replicator dynamics says that the growth rate of a strategy is proportional to its 'gain' or 'fitness'. In my setting this translates into how *being green* gets better off. Therefore this welfare difference between a green person and a brown one can be stated as the moral payoff minus its 'cost of being green'.

[5] The biased perception could be due to the results of different effects, as for example: publicizing the effort made by some 'famous' people ($\Omega > 1$) or, on the contrary, by reports of how people do not care of environment ($\Omega < 1$).

Using the replicator dynamics motion equation, we have:[6]

$$\Delta q_{t+1} = q_{t+1} - q_t = q_t(1 - q_t)(m(r, q) + \Delta U_e) \tag{2.7}$$

where $m(r, q)$ is the moral gain and ΔU_e (which is always negative) is the constant cost of being green.[7] As it can be seen from the motion equation, there are at least two equilibrium points: $q^* = 0$ and $q^* = 1$, plus the one(s) coming from the third parentheses. Since $(m(r, q) + \Delta U_e)$ is positive when $q_t \approx 0$ and it is negative when $q_t \approx 1$, then these two points are unstable.[8]

Replacing the functions $m(r, q)$ and ΔU_e with equations 2.5 and 2.6 respectively, we get:

$$\Delta q_{t+1} = q_t(1 - q_t)[\alpha\gamma r(1 - q)(1 - q(\delta + r - r\delta)) + \beta \ln(1 - \delta)] \tag{2.8}$$

Finally, using the previous functions and some 'sound' parameters,[9] we can have two possible outcomes: A green equilibrium ($q^* > 1/2$) or a brown one ($q^* < 1/2$). These equilibria can be observed in the following figures 1 and 2.

Curve	Meaning
$m(r, q)$	**Green**: The moral gain or satisfaction at a level q_t of green population proportion.
ΔU_e	**Red**: The cost, in utility units, of being green. It does not depend on q_t.
$m(r, q) + \Delta U_e$	**Blue**: The sum of the two previous curves. It is the total gain (or loss) of being green.
Δq_{t+1}	***Dotted line***: The result of the motion equation. A positive value means that q_t will be grow in the next period, where a negative value means a decline of q.
q_{t+1}	**Black:** The value of q_{t+1} as a function of q_t

The fact that ΔU_e is independent of q_t is quite obvious, since ΔU_e has to do with the level of green willingness δ and the relative prices between brown and green goods, represented by ρ. Given these two parameters, the value of ΔU_e is defined, which does not have relationship with the proportion of green people, q_t.

Now looking at $m(r, q)$, we can see that it is a decreasing function of q_t. This also makes sense, since the moral satisfaction comes from the personal level of 'green help' r compared to the average level of the same concept, $\bar{r}$. Since the mean value $\bar{r}$ will increase with q_t, then the resulting moral satisfaction will decline with q_t, and therefore the decreasing function $m(r, q)$.

[6] When 'replicator dynamics' is applied to population evolutions, it is also referred as 'population dynamics'. Since in my work they have the same implementation, I will use both terms indistinctly.

[7] The cost of being green is constant with respect to q_t.

[8] To be verified in the following pages, assuming some 'sound' parameters.

[9] The parameters were chosen as follows: δ at a 10% level, which might seem a little high, although changing the value, towards 5% for example, does not change the results. The green mark up ρ between 20% and 30%, which seems reasonable for an average green mark up. α and γ, which get combined together, result in being the measure of the moral gain versus β which is the weight of the consumption gain. It seems reasonable for β to be higher than $\alpha\gamma$, since the U_e has diminishing marginal returns, whereas the former does not.

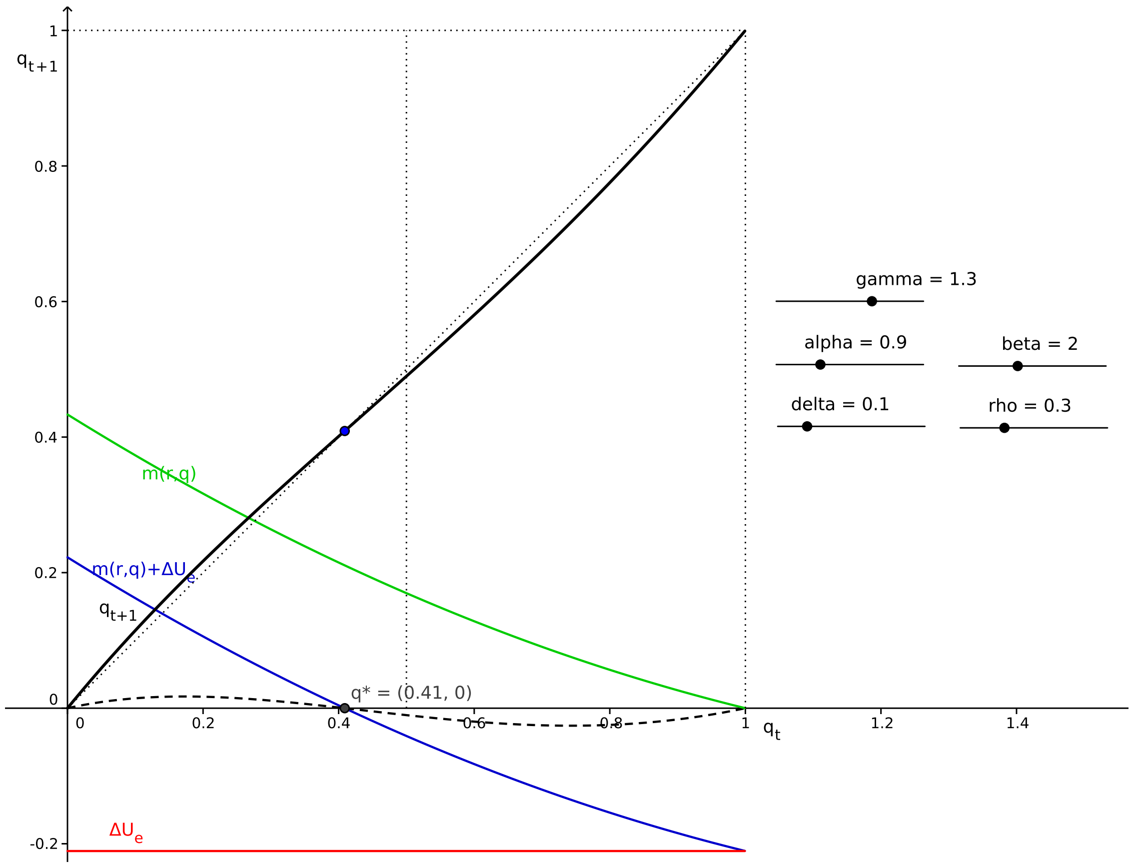


Figure 1: Case with *brown* equilibrium

Recalling the concept of the *'distorting factor of perception'* Ω from section 2.2.4, it is interesting to see that the equilibrium point will change with Ω. If $\Omega > 1$, the perceived $\bar{r}$ will be higher and therefore the equilibrium point will move to the left, becoming a more brown equilibrium, compared to the base case. As expected, the converse is also true, having that a value of $\Omega < 1$ will move the equilibrium point to the right, to a greener equilibrium. The idea here, as it is discussed in Nyborg et al.(2003) [28] and Brekke et al.(2003) [9], is that when $\bar{r}$ seems to be higher, people might think that the rest of the society *has already taken over the responsibility* of environment conservation and therefore their moral gain will decline.

As it was mentioned at the beginning of the present section, we can now verify that when $q_t = 0$ or $q_t = 1$ the equilibria are actually unstable (observing Δq_{t+1}, the dashed line at $q_t = 0$ and $q_t = 1$.) The intuition is straight forward: When nobody is green, the moral gain will be very high. Conversely, when everybody is green, the moral gain will be null.

2.4 The equilibrium point q^*

Now I will calculate the equilibrium point q^* and then see how a green tax/subsidy can impact on it. Recalling the motion equation of q_t (equation **??**) and setting the last term to zero, we have

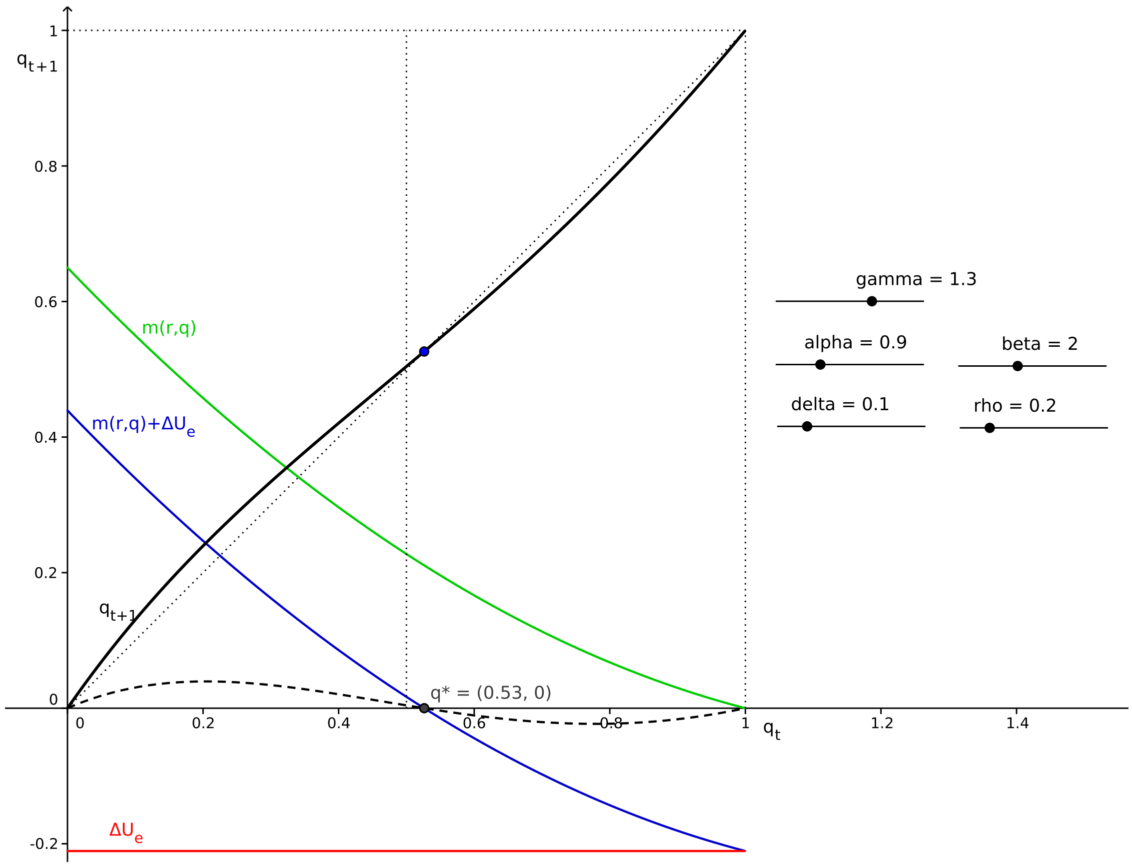


Figure 2: Case with *green* equilibrium

that the equilibrium point q^* must satisfy:

$$\alpha\gamma r(1-q)(1-q(\delta+r-r\delta))+\beta\ln(1-\delta)=0 \tag{2.9}$$

Performing the required algebra and using the fact that for small values of δ, $\ln(1-\delta)\approx-\delta$, I get only one solution for q^*:

$$q^*\approx\frac{1}{2}+\frac{\rho-\sqrt{(\delta+\delta\rho-\rho)^2+\frac{4\beta\delta\rho^2(1+\rho)(1-\delta)}{\alpha\gamma}}}{2\delta(1+\rho)} \tag{2.10}$$

For the mathematical development of the solution (and its uniqueness) and some sensibility analysis, please refer to Appendix B. Observing previous equation 2.10 and relying on simulations,[10] we can see that the effect of α (and in the same manner γ) will be a shift up and down of the curve (with some deformations). In the same fashion, but with opposite direction, will be the effect of β on the curve $q^*(\delta)$.

For the matter of the introduction of a political system (green or brown government), it is

[10]Simulations use the actual utility function, without the approximation of $\ln(1-\delta)\approx-\delta$.

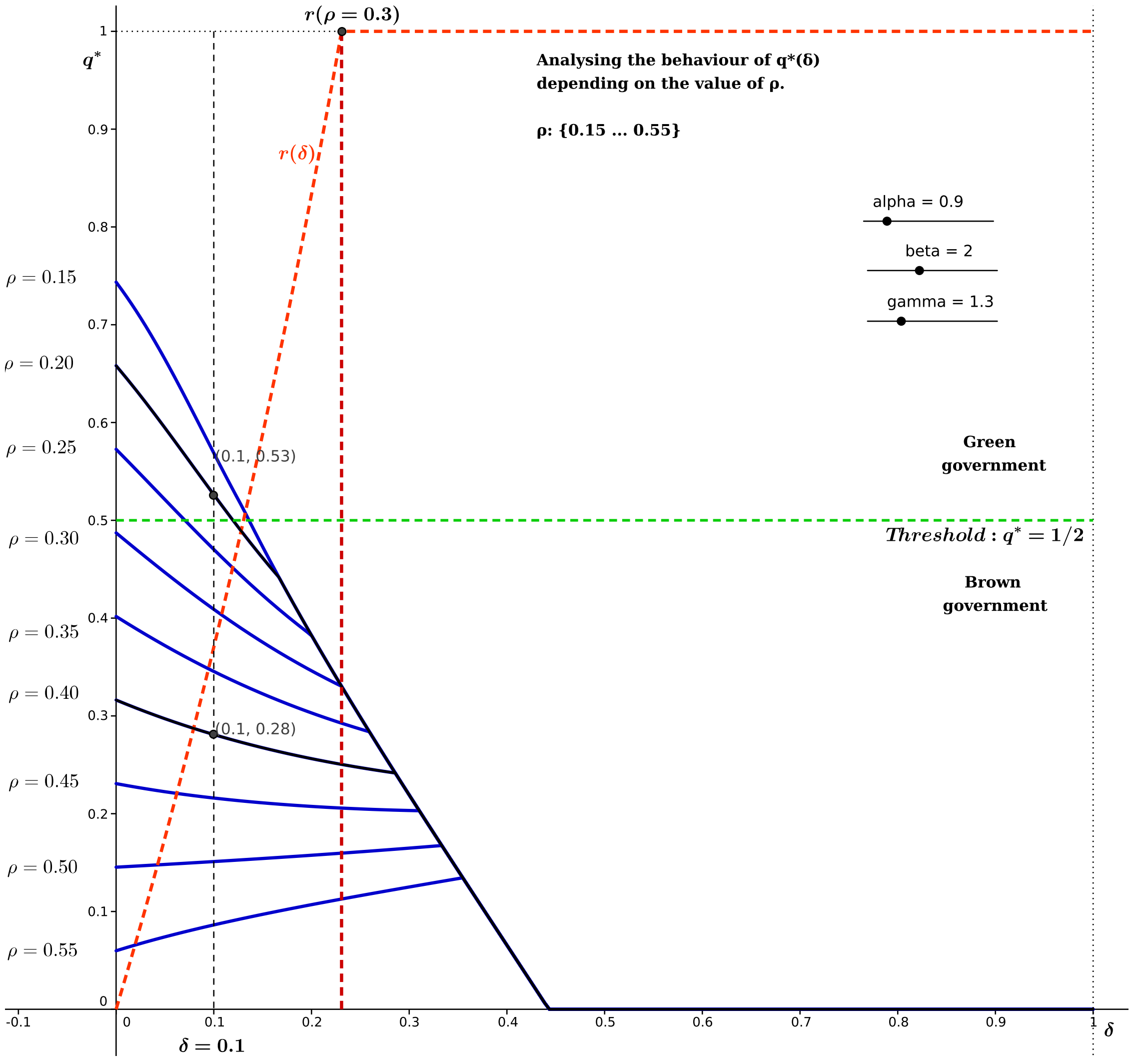


Figure 3: Analysis q*

useful to detain a moment to analyze how δ and ρ impact on the equilibrium point q^*. The one important thing to notice here is the fact that q^* will shift when the values of δ and ρ change. As expected, q^* will increase as the value of ρ decreases (Figure 3). On the other hand, the effect of δ on the value of q^* depends actually on the value of ρ too. In short, since the moral gain depends on the proportion of green people q_t, the effect of the green willingness δ on the final equilibrium point q^* depends on the proportion of green people, which in turn depends also on ρ. Again, for a detailed explanation refer to Appendix B.

2.5 The green government and final dynamics

In the following section I will introduce the difference between having a brown or a green government. As announced in the introduction, the type of government elected will respond to a simple majority rule, meaning that if $q_t < 1/2$ we will have a brown government, where if

$q_t \geq 1/2$ we will get a green one.

In the first case, the government will just leave the market work, i.e. it will not change the prices of the brown or green goods, nor implement any kind of incentive to pollute less. It is just not in the best direct (economic) interests of its constituents, the brown people.

But in the second instance, the government will actually make some changes. In this model, when in the presence of a green majority, the government will introduce a lump sum tax τ on everybody's income. Therefore the available income will change from w to $(1-\tau)w$. With this tax, the government will subsidy the green goods, changing it price from $(1+\rho)$ to $(1+\rho-\epsilon)$, where ϵ is the value of the subsidy.[11]

It is easy to see that the case with a green government is the same as the base case, with the only difference that now we have a lower mark-up for the brown products. Since the goal of this paper is to study the dynamics that we can get, the analysis of the tax rate τ and its impacts on welfare are developed in Appendix C.

Therefore, combining the case with and without the tax, we get the dynamics shown in figure 4. We can see that the curve that has a jump in $q_t = 1/2$. I have used the same values as before, as in figures 1 and 2. The two equilibria points q_1^* and q_2^*, that is, the brown and green equilibria respectively, can be observed in this figure.

As it can be deducted from the graph, both points are stable. Moreover, we have a 'pollution trap' effect, meaning that the final result depends on the initial value of q_t. In other words, if we start from a value of $q_0 < 1/2$, we will end up in the brown equilibrium, whereas if we start in a point $q_0 \geq 1/2$, we will end in the second equilibrium point q_2^*, the green one.

At this point, some comments can be made with respect to the possible impact of the income level w on the previous result. So far, I have assumed that the consumption utility function $f(n)$ has a logarithmic shape, giving us that the resulting ΔU_e does not depend on the level of w. However, if that were not the case, then the wage level would have an impact on the equilibria.[12] In this case, a change in w will shift up and down the red curve ΔU_e, which conversely will do the same with the blue curve $m(r,q) + \Delta U_e$.

The fact of having two stable equilibrium points has important significance, since it could explain why there are, at a given point, countries with high income, like the United States, which pollute more than others with a lower income, who turn out to be much more environmentally

[11] Some other options could be added to the model, as for example a method to change the value of γ, the impact on the environment of the brown consumption. Mechanisms such as these are not considered, since they get out of the scope of the present model, although it could be thought as extensions of the present work.

[12] For example, a function $f(n) = \sqrt{n}$ will give us an increasing value of ΔU_e with respect to w, whereas a function $f(n) = \frac{(x-1)}{x}$ will give us the opposite effect.

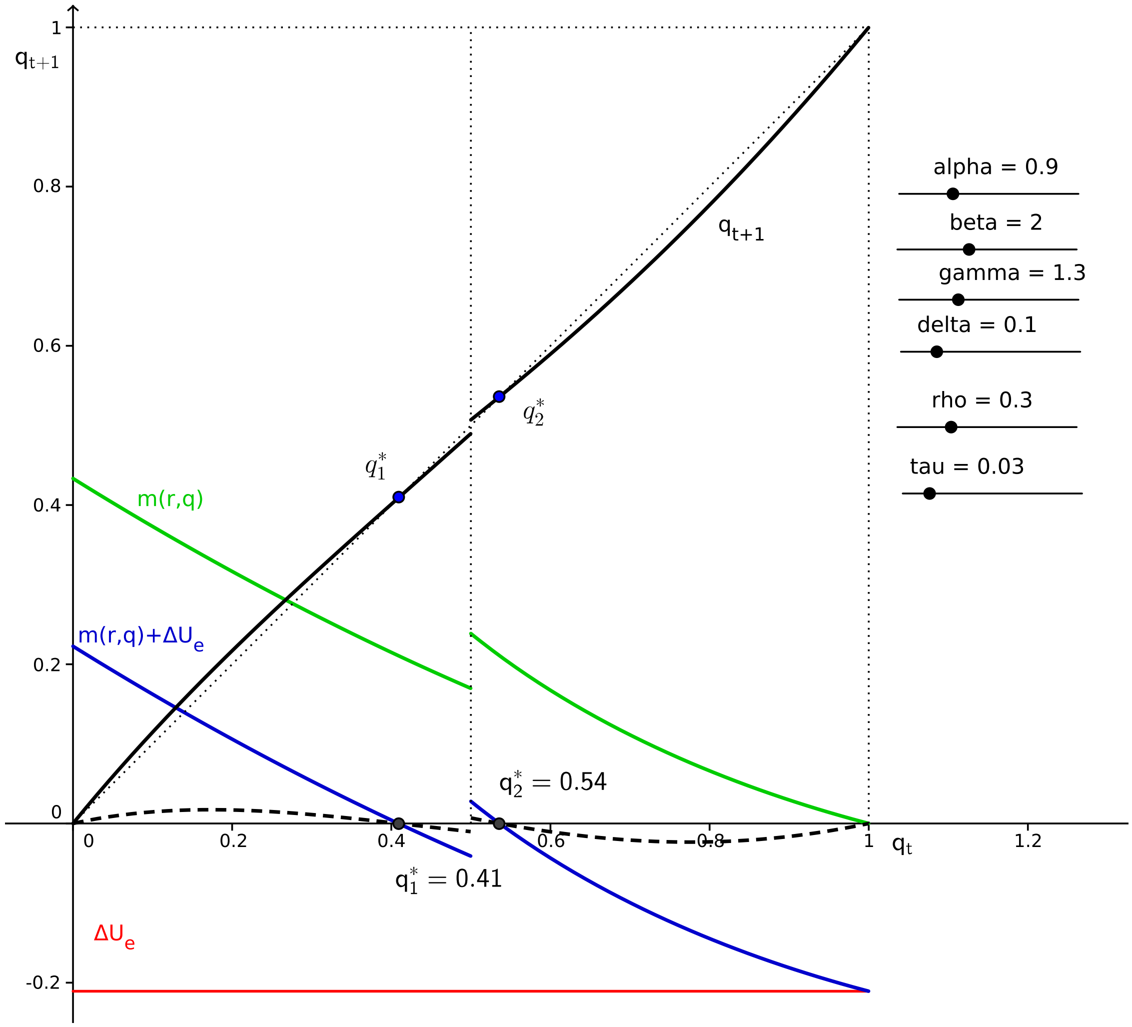


Figure 4: Final Dymanics

friendly.[13] Obviously, this is not the only effect that can actually shift the curves and therefore the equilibrium points. The personal green satisfaction intensity could have a cultural background. Therefore changing it (α in my model) will have a similar impact, giving us again different equilibria results for two, initially, similar countries.

We can now recall Ω, the perception of the 'mean' green contribution ($\bar{r}$). Remembering that this parameter represents how $\bar{r}$ is perceived, we can see that a change in this parameter, due to the action of a NGO for example, would impact on $m(\cdot)$, the moral gain. For example, it might be the case that a NGO would like to push this transmission by exposing how little the society contributes with the environment, therefore 'distorting' the perception of $\bar{r}$. This impact will translate into a displacement of the curve and therefore we could actually shift from a brown dynamics to

[13]Information about this fact can be gathered from the International Energy Agency [41] and the World Bank's CO2 emissions [42], to mention a few.

a green one. In other words, this could allow us to escape from this 'pollution trap'.

3 A more sophisticated moral gain

3.1 The absolute moral gain

So far I have used a moral utility which arose due to the difference of contribution between the agent and the mean person of the society. I will call this type of moral utility the *'relative moral utility'*. But there can be other 'sources' of moral gain as for example as the 'Warm-Glow of Giving' of Andreoni (1990) [3]. The idea here is that the individual gains utility just from the act of giving (in my case, contributing with the environment). A nice example of this can me summarized in this ad taken from the same reference:

> Feel good about yourself - Give blood!
>
> *Advertisement, The American Red Cross*

The underlying reason, if I can say so, is a Kantian morale: *Which general rule of action would maximize social welfare, as I perceive it, given that everyone acted according to the same general rule as I?* I will work on this precise idea, translated in my model as 'how much should I contribute in other to have a good environment, if everyone did the same?' Therefore, starting from this idea, I will try to find how an *'absolute utility function'* (in contrast with the previous one) should be. To do so, I will suppose that the agent thinks like this, but actually he is not a 100% optimistic. Therefore he will guess that only 75% of the population will behave like him (the rest will just not contribute). Doing so, I get that the pollution decreases as I increase the contribution r. Since the agent has a disutility function $h(P_t)$ with $\frac{\partial^2 h}{\partial P_t^2} > 0$, he will get an increasing disutility function with respect to r. For simplicity I use $h(P_t) = P_t^2$, although different function will provide similar results.

Finally, since this is actually a gain, we would expect that when $r = 1$ we should get a positive value of absolute moral utility. Also, as I commented before, when $r = 0$ the green agent will feel kind of guilty or ashamed, as in Elster (1998)[10]. Therefore I should have a negative value of the absolute moral utility and hence, a crossing point we its value is zero. This is what is called *emotional indifference* (also in [10]). Consequently, I have added a constant value in order to obtain the later. With this, we get a emotional indifference point, which I have set to 0.09.[14] Finally, the result can be observe in Figure 5, where two pollution functions were used, being the solid line $h(P_t) = P_t^2$ and, as an example, $h(P_t) = P_t^3$ the dashed line.

The absolute moral utility ends up being the following expression:

$$m_a(r) = K_a - \phi\gamma^2(1 - q_a r(1+\rho)/(1+r\rho))^2 \tag{3.1}$$

[14] Changing the value of the emotional indifference does not change the results, if it remains in a *sound* range.

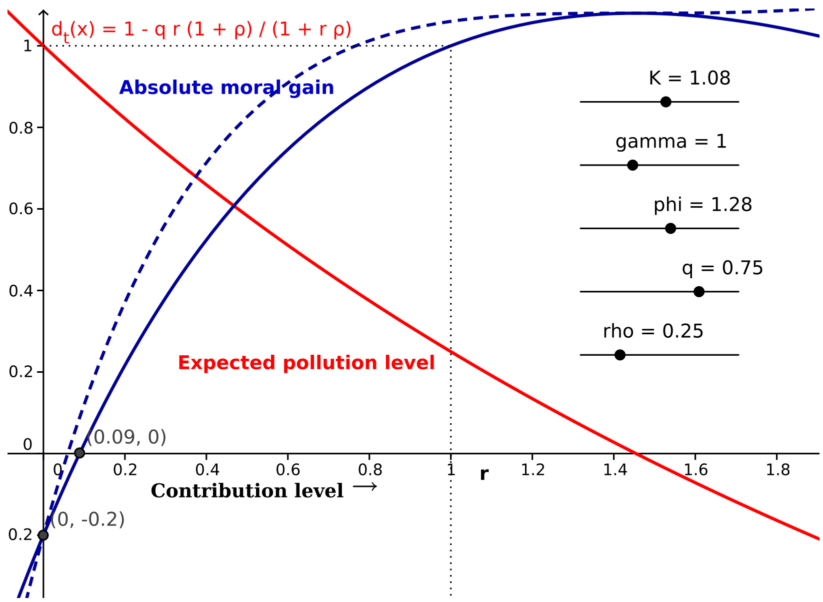


Figure 5: Finding an absolute moral utility

Where K_a is the constant I referred in the previous paragraph, γ and ρ are the usual constants and q_a is the proportion of people the agents assumes will behave like him.[15] The parameter ϕ is just the multiplier of the pollution and the agent's disutility.

3.2 Endogenous green willingness

At last I can write now the new moral utility function. This will be the sum of the absolute moral utility (previous section) and the original (relative) moral utility (section 2.2.4). For simpleness of the formulation, I have assumed that these moral gains are additive.

Concerning the green willingness, I will now consider that the agent will choose his level (δ), such that his total utility is maximized. As we have seen, the new moral utility depends on the value of q_t and previously on r and δ. Since now the agent is maximizing his utility by changing δ and therefore r, we get a moral function which depends on the proportion of green people, q_t. In Figure 6 we have different utility functions for different values of q_t. The maximum of each curve are highlighted.

As we can observe, for small values of q_t the optimum reached when the value of r becomes equal to 1. This is an intuitive result: When a little proportion of the society is green, then being green gives the agent a big amount of **relative** moral utility (a high $(r - \bar{r})$). Any higher value of δ will only imply a higher cost (the agent will consume less quantity) with a partial amount of moral utility increment (only from the absolute component). As q_t increases, the value of $(r - \bar{r})$

[15] I assume he has perfect knowledge of the state of the nature. In case this is not the case, the results do not change much.

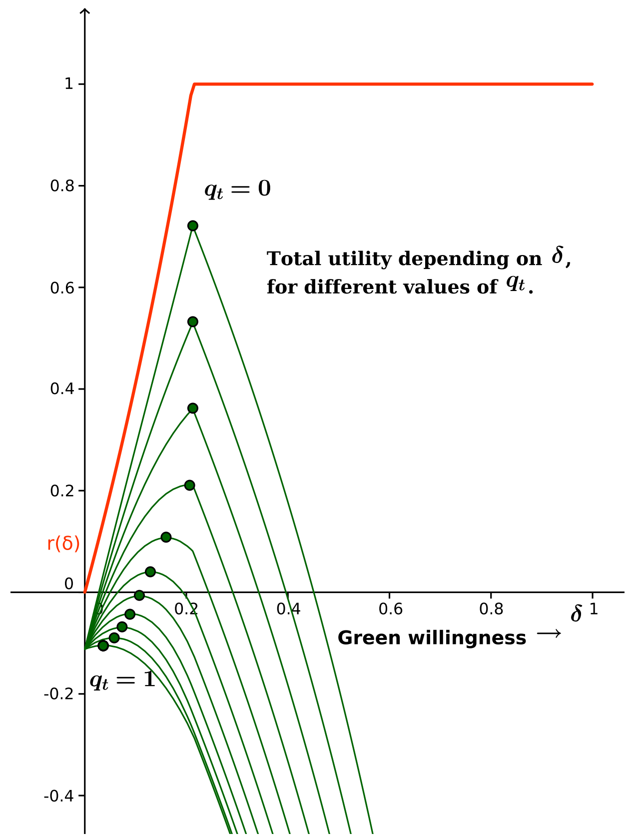


Figure 6: Total Utility function with endogenous delta

will decrease, hence being *too green* will not pay as as before, and the balance between the cost of being green and the gain of an extra contribution is reached in an intermediate level of δ. Finally, when q_t is big, the main source of moral gain comes from the *absolute* side of the equation. Since for small values of δ the agent will feel ashamed (negative value of the **absolute** moral gain), therefore the optimum is reached at a $\delta \gg 0$. Summarizing, we have a new value of δ, which is not endogenously chosen, but endogenously determined, depending how green the society is.

3.3 Returning to the dynamics

Incorporating the previous discussion into the original dynamics will allow us to verify if the previous results hold or not. Happily they do, as we can observe in Figure 8. In order to build this dynamics I relied on simulations. In this case, since the function $\delta(q_t)$ is the result of a maximization process at each value of q_t (big green dots in the graphic), I have used an polynomial approximation of it.[16] The minimum and maximum values of $\delta(q_t)$ (named δ_{min} and δ_{max} respectively) are 0.06 and 0.19. It is easy to see that we get the same dynamics as in section 2.3, with the main difference that in the green side the evolution will develop faster than in the original case.

[16] I have used a polynomial of degree 5 and 21 values of $\delta(q_t)$: $(0, 0.05 \ldots 0.95, 1)$. In any case, this approximation will definitely not change the results.

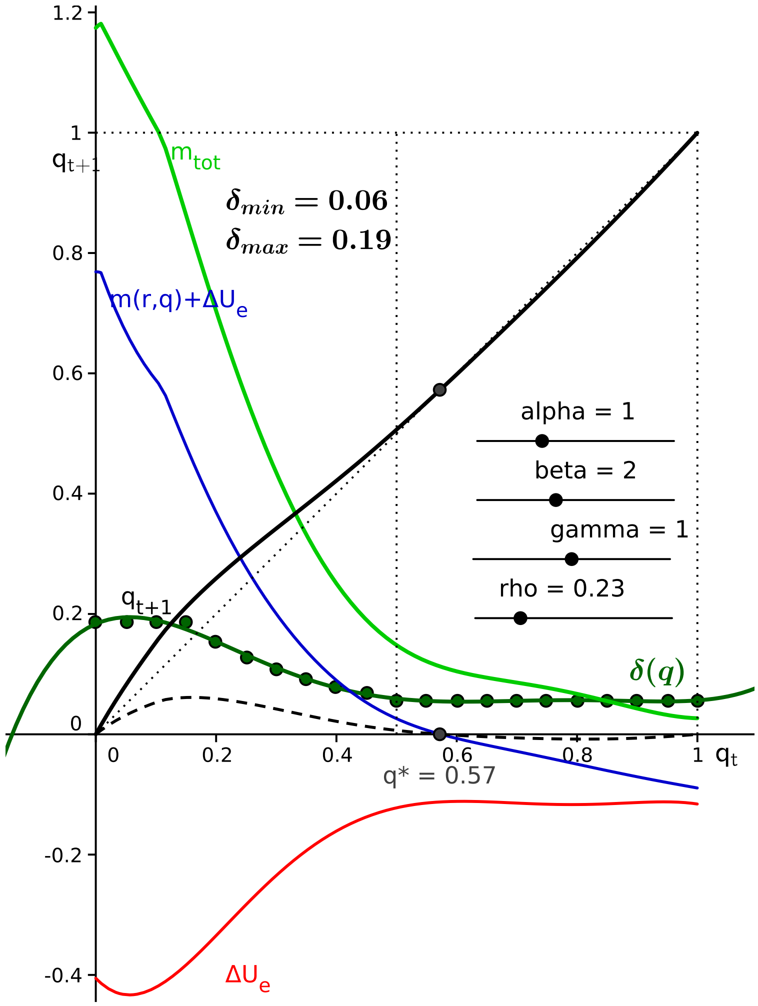


Figure 7: Dynamics with endogenous delta

Finally, I incorporate the political framework back to the dynamics. This translates into the graph observed in figure 8. As we can see, the original idea, i.e. two equilibrium points, one green and one brown, remains.

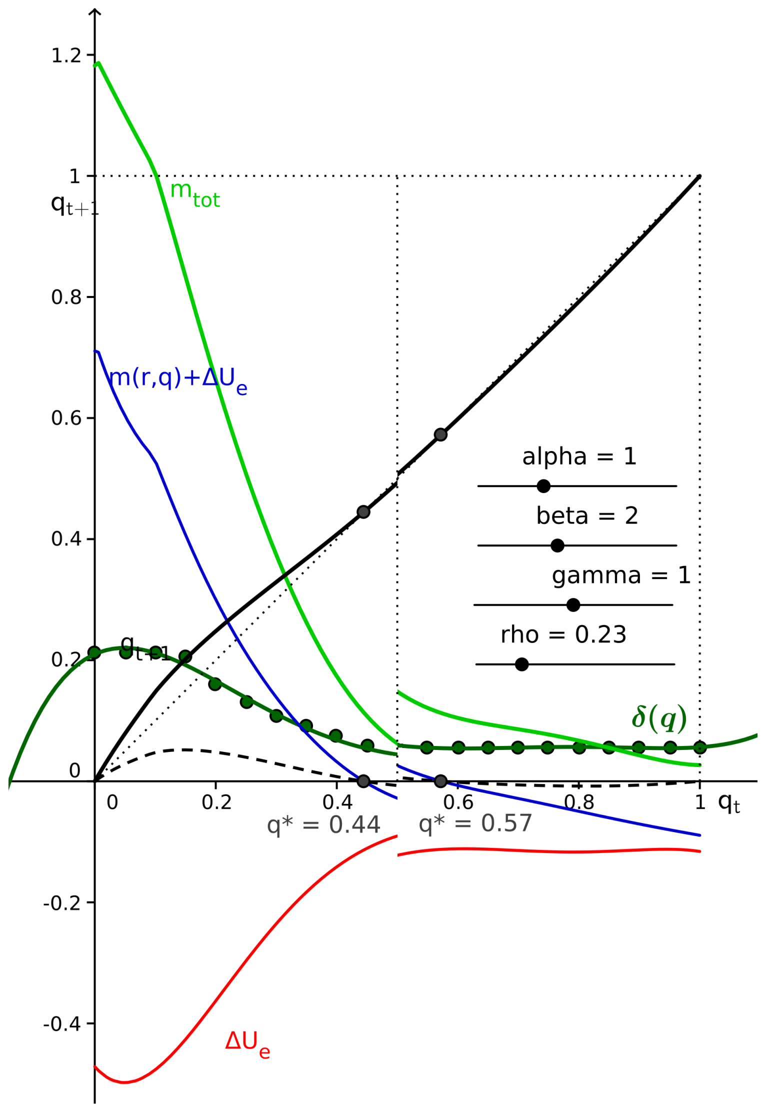


Figure 8: Dynamics with endogenous delta

In the next section I will analyze the possible repercussions of using the pollution as a stock, instead of as a flux, as I have done so far.

4 Pollution as a stock

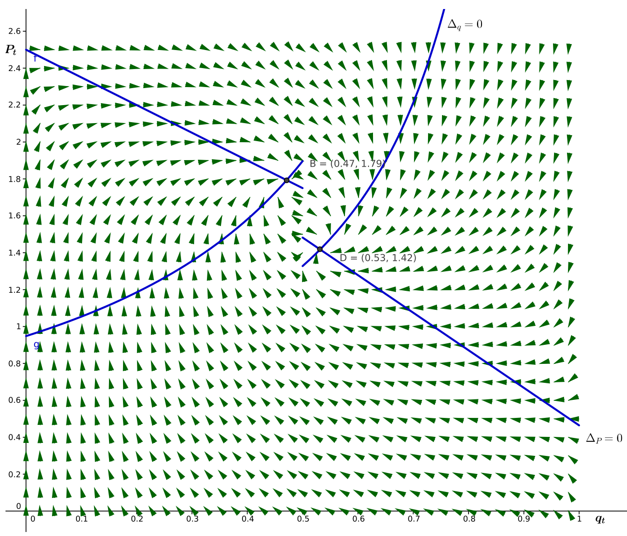
In this section I will study the possible differences that can arise with the introduction of the pollution as a stock. Up to this point, the paper has treated the pollution as a flux, meaning that the resulting pollution at time t is proportional to the total brown good consumption d_t. Undoubtedly this is not a very realistic case, which has be done in this way in order to keep the model simple.

In this section I will switch from the previous equation of pollution $P_t = \gamma d_t$, to the standard equation $P_t = (1-\eta)P_{t-1} + \gamma d_t$.[17] As it can be observed immediately, we will now have a two state variable system. Therefore the study of the evolution of the system becomes more complex and the goal of this section is to show that the previous results hold, with some possible deviations.

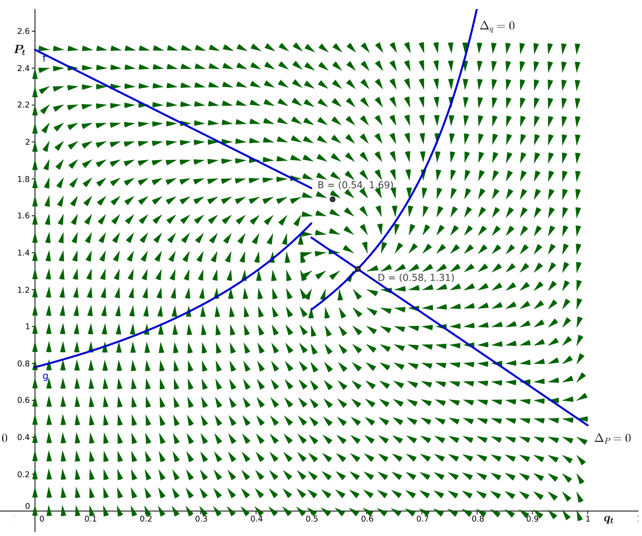
Before entering into the equations, it is useful to observe that the change of the pollution equation will introduce an 'inertia' to the pollution variable. In other words, the pollution will still be responding to the total brown good consumption d_t, but it will 'retain' some part of its previous value $(1-\eta)$, the 'inertia'. Therefore we can expect similar results of those found in the previous sections, with some possible changes due to this inertia.

Using again the aid of simulation, I will show three cases that we could get with this new implementation, displayed in Figure 9. In the first case (Figure 9a) I get the same result that I found in section 2.5. The main difference here is that even though if we start from a brown (green) situation ($q_t \lesseqgtr 1/2$) we could actually end up in the green (brown) equilibria, i.e. in the other 'side' of the portrait. This is the case when we start with higher (lower) levels of pollution and its 'inertia' makes the system to actually cross the threshold ($q_t \lesseqgtr 1/2$) and end up in the other side of the system. We can observe these phenomena in figure 10 and 11. We can also observe the cases where the system ends up in the same side where it started, as in the case with pollution as a flux. These last cases can be observed in Figure 12. The other two possible cases can be observed in figures 9b and 9c. In these instances, the equilibrium point of one side ends up in the other side of the portrait. Therefore, even if the system starts in the brown side (as in figure 9b), it will end up in the green side. The symmetric case is also possible, as it can be seen in figure 9c. These cases can be found depending on the pollution evolution equation parameters γ and η.
The interesting result here is that we get now a system that can actually switch from the brown side into the green side (and vice-versa), if the initial pollution conditions are 'far away' from the ones of its equilibria. This is a new outcome that we did not get in the first part of the paper and that seems to be a more realistic one. Regardless of this last result, it is interesting to see that the original outcome holds, whereas the only difference is that we get some cases where we can cross the brown/green threshold, although maintaining the same equilibria found before. Actually this new possibility of crossing the green/brown threshold is more appealing. It seemed strange that if we started from a very polluted society, we would not end up with a green government and vice-versa.

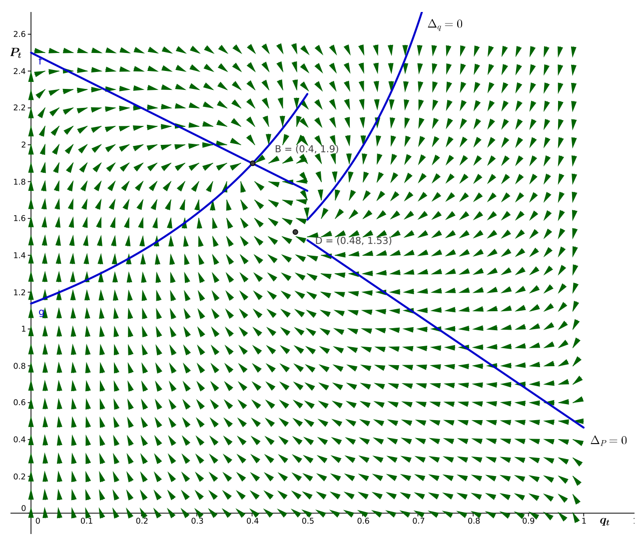
[17] It is important to notice that in this new equation the value of γ does not have the same impact as it did before.



(a) The 2-equilibria result



(b) The green equilibrium



(c) The brown equilibrium

Figure 9: Possible cases with pollution as a stock.

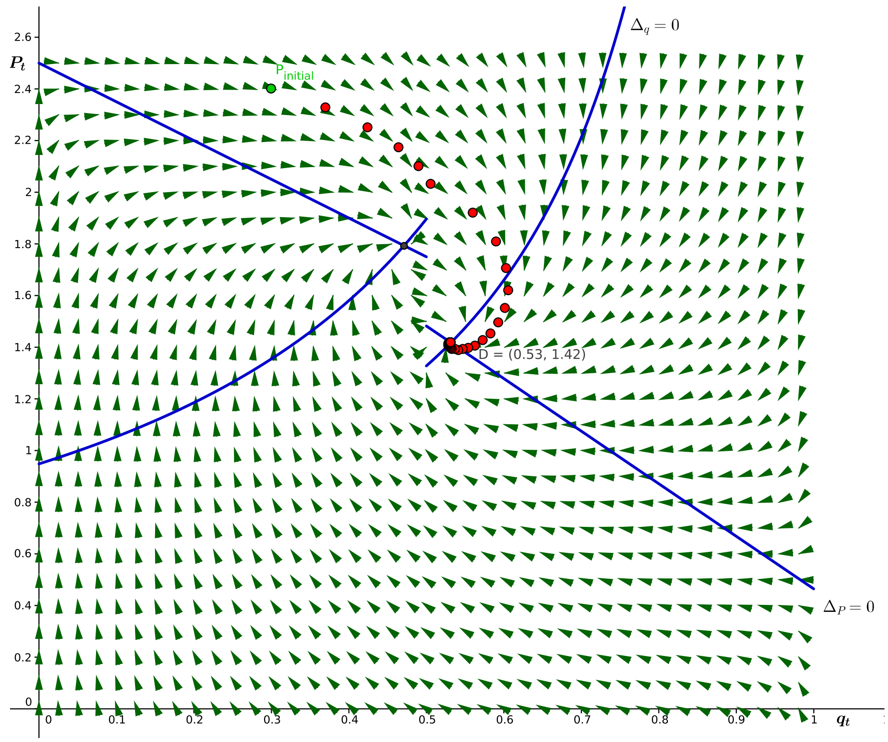


Figure 10: Starting brown and with a high level of pollution.

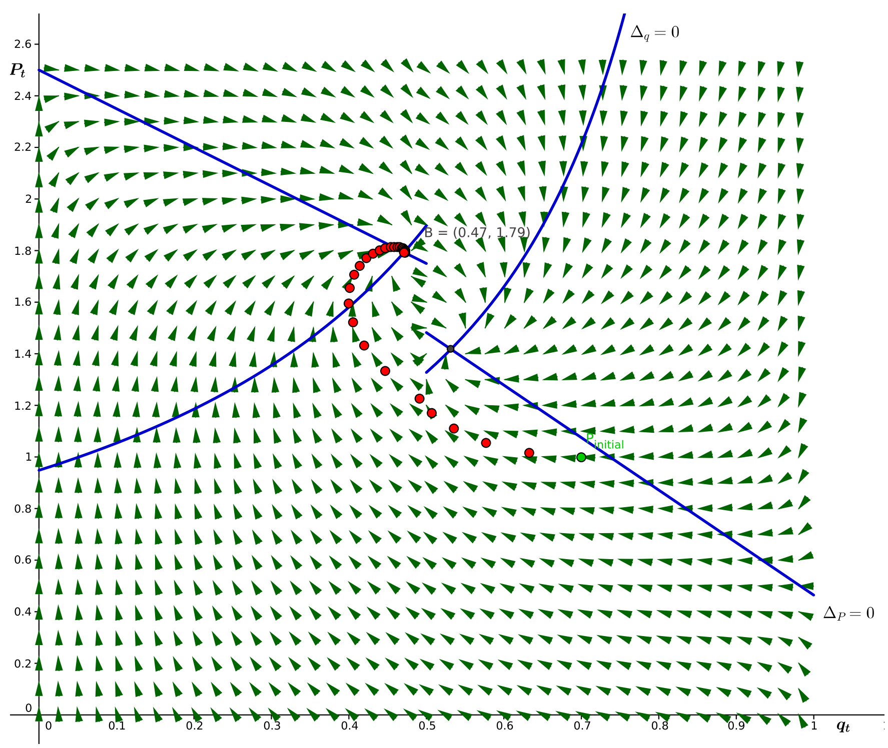
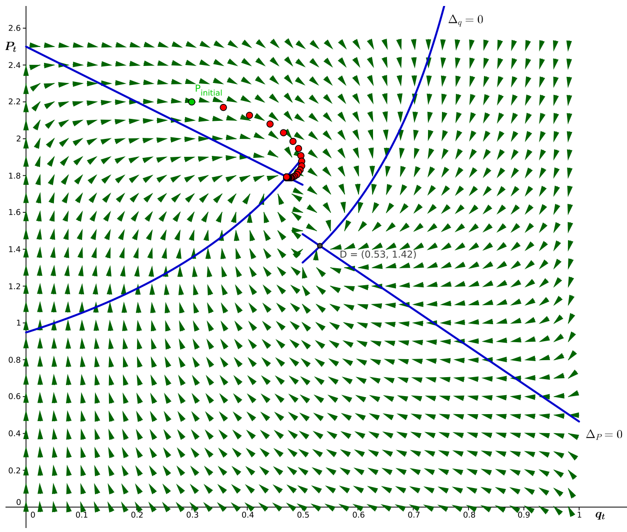
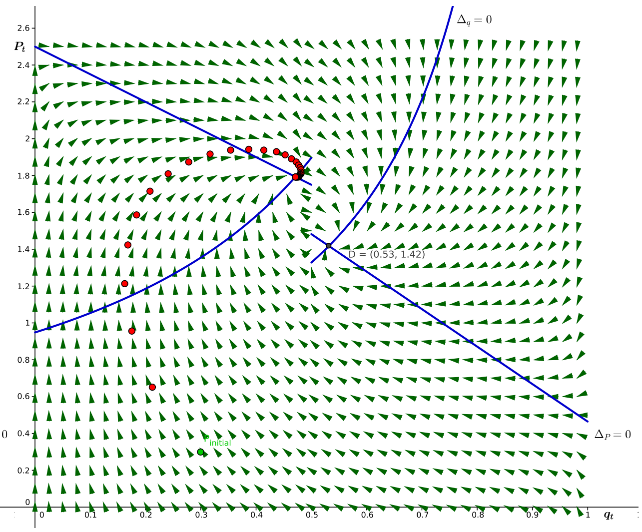


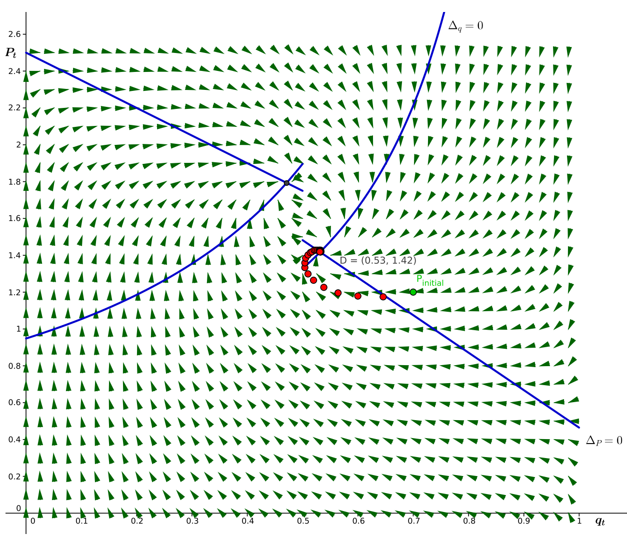
Figure 11: Starting green and with a low level of pollution.



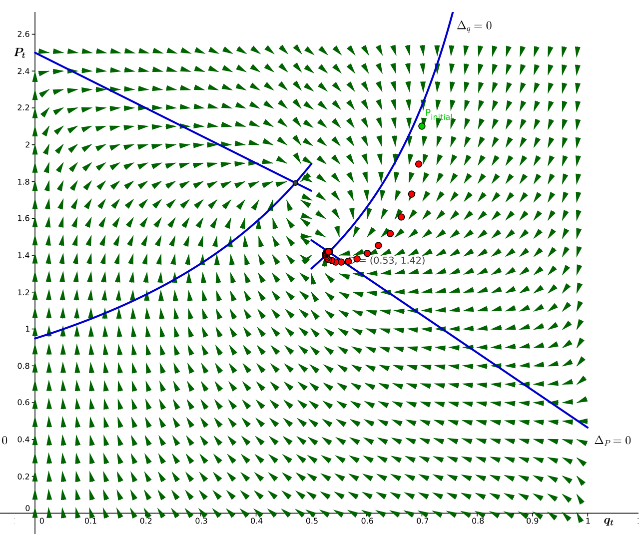
(a) Low q_0 and high P_0.



(b) Low q_0 and low P_0.



(c) High q_0 and low P_0.



(d) High q_0 and high P_0.

Figure 12: Cases starting and ending in the same side.

4.1 Some last comments

Summing-up, I show that using the pollution as a stock, instead of having the pollution as a flux, will not change the form of the equilibria found before, but it will allow us to switch from the brown to the green side, and vice-versa. This may not be very interesting from a mathematical point of view, but it can have some compelling implications from a social point of view. Let us imagine the case displayed in Figure 13. This case seems more 'real', since we are starting with a low proportion of 'green' people.[18] In this case, the pollution will start to raise, while making the society greener, although not green enough to have a green government. It could be the case that the pollution inertia could allow us to cross this threshold and actually end up with a green government (as the figure shows). This is due to the fact that since the moral gain is a positive function of the pollution level, and the pollution is a stock now (it has inertia), then even though they are more and more green people, this is not enough to produce a decline in pollution in such a way that we reach the brown equilibrium (point B). We hence cross the threshold and end up in the green equilibria (point D). From a pollution perspective, we can clearly observe that the second equilibria (point D) is better than the first one (point B). Therefore it is interesting to note that a small change in the initial conditions (or a small change during the first part of the path) could make the system end up in the brown equilibria. These (little) shocks or small changes in the initial conditions are of the same type of ideas explained in Malcolm Gladwell's book "The Tipping Point" [15]. Essentially, there are some situation where a small change (at a precise moment, in the correct 'place') can make a big difference, therefore what is called a *tipping point*.

[18]Since the pollution problem is a recent one, we can expect that the initial proportion of green people starts from a point close to zero. It is only with the recent raise of the pollution levels, that some portion of society has developed an environmental consciousness.

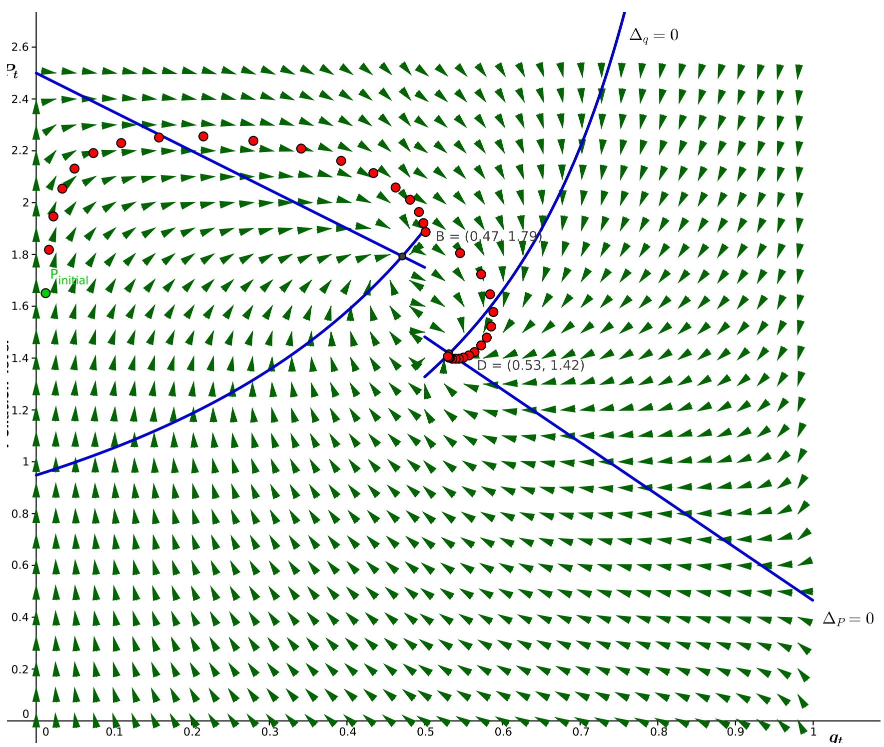


Figure 13: Example of a limit trajectory.

5 Conclusions

We have seen a model of evolution with some type of preferences, in this case to be 'green', where 'success' or 'failure' (in an evolutionary way) has to do with moral satisfaction, as in Nyborg et al.(2003) [28]. Therefore the number of green people will increase if being green has a positive pay-off, and the contrary if not. Taking this natural idea into consideration and translating this into a green/brown consuming behavior, I have been able to analyze how the environment and society could evolve.

The main idea has been to check if attaining a sustainable (green) environment is politically possible. In my model, 'politically' just means checking if there is a sufficient number of green people in order to move forward with green policies. In this case the policy has been the implementation of a lump sum tax (imposed on the whole population) in order to subsidize the consumption proportion of the green product.

In doing so, I have been able to study the behavior of the evolution of this society and to find some equilibria points, i.e. where the proportion of green and brown people stays the same. It has been found that, according to some sound assumptions, there could exist two such points: a brown equilibrium and a green one. This means that we can end up with different type of societies, even though they have the same original characteristics, and are only differentiated by their starting point (the initial proportion of green and brown people). Or they could actually be two originally identical countries, but they could have a small perturbation during the evolution process, which could make both countries differ and end up in quite different equilibria.

Having that type of result could actually explain some known facts. For example, how two (theoretically) similar countries could actually have quite different behaviors with respect to the treatment of pollution (as with the U.S. and Nordic countries). This type of model can also explain why countries with low income do actually care about the environment, even in some cases more that their developed counterparts.

Furthermore, if we use the pollution as a stock, which is a realistic and standard way to model it, I find that in some particular, but interesting cases, the society can evolve from a brown point, into a green equilibrium (and vice-versa). These last remarks could be of interest when applying them into a political arena. Environmental NGO's and governments could be interested in understand how the preferences (social movements) could actually evolve and some possible insights concerning this transformation could take place. In other words a NGO, for example, could 'exploit' the idea that modifying a little the perceived mean contribution could help their goals and work along this concept in order to incentive a 'switch' in the society behavior.

It seems also interesting to study the behavior of the system with the use of different government systems as, for example, a probabilistic political system. It would be also compelling to study the sensitivity of the different parameters on the behavior of the system, in order to see if we observe any 'tipping point' behaviors, as in [15] and [18]. This last remark can be applied to the case of Ω, the distorting factor of perception. The same idea can be developed for the impact of different shocks (cultural, environmental) in the system and how this could be used for different purposes. Finally some possible extension could be the use of a multi-type society (more than two types of people). It would be interesting to study the evolution of, for example, a society composed by a green group, with a high absolute moral incentive, a brown group and another green group with a high relative moral moral. It is not quite clear if all the previous results will hold and therefore it could be an interesting matter of research.

References

[1] Akin, E. (1979). (book) *"The Geometry of Population Genetics"*. Springer, Berlin.

[2] F. Alvarez-Cuadrado, G. Monteiro, S. Turnovsky, *"Habit Formation, Catching Up with the Joneses, and Economic Growth"*, Journal or Economic Growth (2004), 9, 47-80

[3] Andreoni, James, 1990. *"Impure Altruism and Donations to Public Goods: A Theory of Warm-Glow Giving?"*, Economic Journal, Royal Economic Society, vol. 100(401), pages 464-77, June.

[4] J. Baron, D. Cobb-Clark, N. Erkal, *"Cultural Transmission of Work-Welfare Attitudes and the Inter-generational Correlation in Welfare Receipt"*, The Australian National University (2009), Discussion Paper No. 594.

[5] Jeroen C.J.M. van den Bergh, 2007. *"Evolutionary Thinking in Environmental Economics"*, Tinbergen Institute Discussion Papers 07-018/3, Tinbergen Institute.

[6] Pierre Bernhard (2011), *"ESS, Population Games, Replicator Dynamics: Dynamics and Games if not Dynamic Games"*, Advances in Dynamic Games, Vol. 11. pp.291-311.

[7] A. Bisin, T. Verdier, *"A model of cultural transmission, voting and political ideology"*, European Journal of Political Economy (2000), Vol.16, 5-29.

[8] Immanuel Bomze (1983). *"Lotka-Volterra equation and replicator dynamics: A two-dimensional classification"*, Biological Cybernetics, Vol. 48, No. 3. (7 October 1983), pp. 201-211.

[9] Brekke K.A., S. Kverndokk and K. Nyborg (2003): *"An Economic model of Moral Motivation"*, Journal of Public Economics, Elsevier, vol. 87(9-10), pages 1967-1983, September.

[10] Jon Elster, 1998: *"Emotions and Economic Theory"*, Journal of Economic Literature, American Economic Association, vol. 36(1), pages 47-74, March.

[11] Albert Faber & Koen Frenken, 2008. *"Models in evolutionary economics and environmental policy: Towards an evolutionary environmental economics"*, Innovation Studies Utrecht (ISU) working paper series 08-15, Utrecht University, Department of Innovation Studies, revised Apr 2008.

[12] Frey B.S., 1999. *"Morality and Rationality in Environmental Policy"*, Journal of Consumer Policy, Volume 22, Number 4, December 1999 , pp. 395-417(23).

[13] Daniel Friedman (1991), *"Evolutionary Games in Economics"*, Econometrica, Econometric Society, vol. 59(3), pages 637-66, May.

[14] Daniel Friedman (1998), *"On economic applications of evolutionary game theory"*, Journal of Evolutionary Economics, Springer, vol. 8(1), pages 15-43.

[15] Malcolm Gladwell, Book: *"The Tipping Point"*, Little and Brown (2000).

[16] Osborne Groves, M.A. (2005), *"Personality and the intergenerational transmission of economic status"*, in S. Bowles, H. Gentis and M.A. Osborne Groves (Eds.), Unequal Chances: Family Background and Economic Success, Princeton: Princeton University Press, 208-231.

[17] Marc Harper (2009), *"Information geometry and evolutionary game theory"*, 2009.

[18] Geoffrey Heal & Howard Kunreuther, 2011. *"Tipping Climate Negotiations,"* NBER Working Papers 16954, National Bureau of Economic Research, Inc.

[19] Josef Hofbauer & Karl Sigmund, (book) *"Evolutionary Games and Population Dynamics"*, Cambridge University Press.

[20] Josef Hofbauer & Karl Sigmund (2003), *"Evolutionary Game Dynamics"*, Bulletin (New Series) of the American Mathematical Society, Volume 40, Number 4, Pages 479-519. Article electronically published on July 10, 2003.

[21] Holländer, Heinz, 1990. *"A Social Exchange Approach to Voluntary Cooperation"*, American Economic Review, American Economic Association, vol. 80(5), pages 1157-67, December.

[22] P-A Jouvet, P.Michel, J-P Vidal, *"Intergenerational Altruism and the Environment"*, Scandinavian Journal of Economics (2000), 102(1), 135-150.

[23] Mariani, F.; Prez-Barahona, A.; Raffin, N.; 2009. *"Life Expectancy and the Environment"*, IZA Discussion Papers 4564, Institute for the Study of Labor (IZA).

[24] P. Michel, P. Pestieau, *"Fiscal Policy with Agents Differing in Altruism and in Ability"*, Centre of Economic Research (2004), Discussion Paper No. 4254

[25] Martin Nowak (1989). *"An Evolutionarily Stable Strategy May Be Inaccessible"*, Journal of Theoretical Biology (1990), 142, 237-241.

[26] Nyborg, Karine, 2000. *"Homo Economicus and Homo Politicus: interpretation and aggregation of environmental values"*, Journal of Economic Behavior & Organization, Elsevier, vol. 42(3), pages 305-322, July.

[27] Nyborg, Karine & Rege, Mari, 2003. *"Does Public Policy Crowd Out Private Contributions to Public Goods"*, Public Choice, Springer, vol. 115(3-4), pages 397-418, June.

[28] Nyborg, Karine & Howarth, Richard B. & Brekke, Kjell Arne, 2003. *"Green consumers and public policy: On socially contingent moral motivation"*, Memorandum 31/2003, Oslo University, Department of Economics.

[29] Mari Rege, 2004. *"Social Norms and Private Provision of Public Goods"*, Journal of Public Economic Theory, Association for Public Economic Theory, vol. 6(1), pages 65-77, 02.

[30] M. Saez-Marti, Y. Zenou, *"Cultural Transmission and Discrimination"*, Institute of Study and Labor (IZA, Germany) (2005), Discussion Paper No. 1880.

[31] Sandholm, William H. & DokumacI, Emin & Lahkar, Ratul (2008), *"The projection dynamic and the replicator dynamic"*, Games and Economic Behavior, Elsevier, vol. 64(2), pages 666-683, November.

[32] Karl H. Schlag (1995), *"Why Imitate, and if so, How? A Bounded Rational Approach to Multi-Armed Bandits"*, Discussion Paper Serie B 361, University of Bonn, Germany, revised Mar 1996.

[33] P. Schuster, K. Sigmund and R. Wolff (1978), *"Dynamical systems under constant organization I. Topological analysis of a family of non-linear differential equations"*, Bulletin of Mathematical Biology. Vol. 40. pp.743-769. Pergamon Press Ltd. 1978.

[34] J. Hofbauer, P. Schuster, K. Sigmund, R. Wolff (1980), *"Dynamical Systems Under Constant Organization II: Homogeneous Growth Functions of Degree $p = 2$"*, SIAM Journal on Applied Mathematics, Vol. 38, No. 2 (Apr., 1980), pp. 282-304.

[35] S. Shahshahani (1979), *"A new mathematical framework for the study of linkage and selection"*, Memoirs American Mathematical Society, 17(211), 1979.

[36] Karl Sigmund (1984), *"A survey of Replicator Equations"*, Institute of Mathematics, University of Vienna, Austria.

[37] Karl Sigmund (1984). *"A maximum principle for frequency dependent selection"*, Mathematical Biosciences, Volume 84, Issue 2, June 1987, Pages 189–195.

[38] Gerald Silverberg (1997), *"Evolutionary modeling in economics : recent history and immediate prospects"*, Research Memoranda 008, Maastricht : MERIT, Maastricht Economic Research Institute on Innovation and Technology.

[39] P. Taylor & L. Jonker (1978), *"Evolutionarily Stable Strategies and Game Dynamics"*, Mathematical Biosciences 40(2), 145-156 (1978).

[40] E. Zeeman (1980). *"Population dynamics from game theory"*, Global Theory of Dynamical Systems, Volume 819, Pages 471 - 497.

[41] International Energy Agency. *http://www.iea.org/*

[42] World Bank's: CO2 emissions (metric tons per capita).
http://data.worldbank.org/indicator/EN.ATM.CO2E.PC

A Cost of being green under other consumption utility functions

At this point, I will use two different utility functions, which will have different effects. As with the moral satisfaction function, β will represent the weight of the economic part in the overall utility function of the agent:

$$\begin{aligned} f_1(n) &= \beta \ln(n) &\text{(A.1)}\\ f_2(n) &= \beta\sqrt{n} &\text{(A.2)}\end{aligned}$$

Replacing the previous functions in ΔU_e, we get:

$$\begin{aligned}\Delta U_e^1 &= \beta\ln(n_1) - \beta\ln(n_2)\\ &= \beta(\ln(n_1) - \ln(n_2))\\ &= \beta\ln\left(\frac{n_1}{n_2}\right)\\ &= \beta\ln\left(\frac{(1-\delta)w}{w}\right)\\ \Delta U_e^1 &= \beta\ln(1-\delta) &\text{(A.3)}\end{aligned}$$

$$\begin{aligned}\Delta U_e^2 &= \beta\sqrt{n_1} - \beta\sqrt{n_2}\\ &= \beta(\sqrt{n_1} - \sqrt{n_2})\\ &= \beta\left(\sqrt{(1-\delta)w} - \sqrt{w}\right)\\ \Delta U_e^2 &= \beta\sqrt{w}\left(\sqrt{(1-\delta)} - 1\right) &\text{(A.4)}\end{aligned}$$

As it can be seen from the previous two equations, the choice of the utility function can have some interesting impacts (to be seen in the equilibria analysis). In the meantime, we can see that in equation 2.6 the value of ΔU_e^1 is independent of the wage level w, whereas in equation A.4 the difference ΔU_e^2 is actually increasing with w, proportional to $\sqrt{w}$.

This means that, in the latter case, the 'cost' of being green increases with the level of wage. Analyzing if there is a 'sound' utility function which could have the inverse effect, we can see that if we choose $f(n) = \ln(n+K)$, where K is a fixed positive constant ($K > 0$):

$$\frac{\partial \Delta U_e}{\partial w} = (1-\delta)f'((1-\delta)w) - f'(w)$$

$$\frac{\partial \Delta U_e}{\partial w} = \frac{(1-\delta)}{(1-\delta)w+K} - \frac{1}{w+K}$$

$$\frac{\partial \Delta U_e}{\partial w} = \frac{-\delta K}{((1-\delta)w+K)(w+K)} < 0$$

Even though $\frac{\partial \Delta U_e}{\partial w} < 0$, we can see that if w grows, then $\frac{\partial \Delta U_e}{\partial w} \to 0$. The other possibility is to choose an utility function with a ceiling, as for example $f(n) = \beta\frac{(n-1)}{n}$. Therefore $\Delta U_e^3 = \beta(\frac{n_1-1}{n_1} - \frac{n_2-1}{n_2}) = -\beta\delta w/(1-\delta)$. In that case, we find that the value of ΔU_e^3 decreases as w increases.

B Analysis on the equilibrium point q^*

Now I will calculate the equilibrium point q^* and then see how a green tax/subsidy can impact on it. Recalling the motion equation of q_t, and setting the last term to zero, we have that the equilibrium point q^* must satisfy:

$$\alpha\gamma r(1-q)(1-q(\delta+r-r\delta)) + \beta\ln(1-\delta) = 0 \tag{B.1}$$

or

$$\alpha\gamma r(1-q)(1-q(\delta+r-r\delta)) = -\beta\ln(1-\delta)$$

Knowing the fact that, for small values of δ, we have that $\ln(1-\delta) \approx -\delta$ and replacing r with its value, we get the following second degree equation for q^*:

$$\delta(1+\rho)q^2 - (\delta+\delta\rho+\rho)q + \rho\left[1 - \frac{(1-\delta)\rho\beta}{\alpha\gamma}\right] \approx 0 \tag{B.2}$$

which yields to:

$$q^* \approx \frac{1}{2} + \frac{\rho \pm \sqrt{(\delta+\delta\rho-\rho)^2 + \frac{4\beta\delta\rho^2(1+\rho)(1-\delta)}{\alpha\gamma}}}{2\delta(1+\rho)} \tag{B.3}$$

Although we get a the 'complex' form for the equilibrium point, we can deduce some interesting things from it. First, we will see that one of the two roots of q^* is actually greater than 1, therefore leaving us only to deal with the second one.

For the bigger root to be less or equal to 1, we should have:

$$\frac{1}{2} + \frac{\rho + \sqrt{(\delta + \delta\rho - \rho)^2 + \frac{4\beta\delta\rho^2(1+\rho)(1-\delta)}{\alpha\gamma}}}{2\delta(1+\rho)} \leq 1$$

which would imply that:

$$\frac{\rho + \sqrt{(\delta + \delta\rho - \rho)^2 + \frac{4\beta\delta\rho^2(1+\rho)(1-\delta)}{\alpha\gamma}}}{2\delta(1+\rho)} \leq \frac{1}{2}$$

$$\rho + \sqrt{(\delta + \delta\rho - \rho)^2 + \frac{4\beta\delta\rho^2(1+\rho)(1-\delta)}{\alpha\gamma}} \leq \delta(1+\rho)$$

$$\sqrt{(\delta + \delta\rho - \rho)^2 + \frac{4\beta\delta\rho^2(1+\rho)(1-\delta)}{\alpha\gamma}} \leq \delta(1+\rho) - \rho$$

Since the square root has to be greater or equal to zero and recalling that $\frac{\delta}{(1-\delta)} = r\rho$, we have:

$$\begin{aligned}
\delta(1+\rho) - \rho &\geq 0 \\
\delta &\geq \rho - \delta\rho \\
\delta &\geq \rho(1-\delta) \\
\frac{\delta}{(1-\delta)\rho} &\geq 1 \\
r &\geq 1
\end{aligned}$$

Which would mean that $q^* = 1$, $r^* = 1$ and that the value inside the square root is also equal to zero. I will assume that these extremely specific conditions (regarding α, β, γ and ρ) do not hold and, therefore, we are left with only one solution for q^*:

$$q^* \approx \frac{1}{2} + \frac{\rho - \sqrt{(\delta + \delta\rho - \rho)^2 + \frac{4\beta\delta\rho^2(1+\rho)(1-\delta)}{\alpha\gamma}}}{2\delta(1+\rho)} \tag{B.4}$$

So now we can study whether the equilibrium point is a brown or green one. Following a similar reasoning from the previous one, we find that, to get a green equilibrium, we need:

$$\frac{1}{2} + \frac{\rho - \sqrt{(\delta + \delta\rho - \rho)^2 + \frac{4\beta\delta\rho^2(1+\rho)(1-\delta)}{\alpha\gamma}}}{2\delta(1+\rho)} \geq \frac{1}{2}$$

which gives us the following condition for a green equilibrium:

$$(4\beta(1+\rho)(1-\delta) - \alpha\gamma(2-\delta))\rho^2 - 2\alpha\gamma(1-\delta)\rho + \alpha\gamma\delta < 0$$

which finally gives us ρ^*, which much lie between these two values:

$$\rho^* = \frac{\alpha\gamma(1-\delta) \pm \sqrt{\alpha^2\gamma^2 - 4\alpha\beta\gamma\delta(1+\rho)(1-\delta)}}{4\beta(1+\rho)(1-\delta) - \alpha\gamma(2-\delta)} \tag{B.5}$$

It is important to notice that for a green equilibrium to exist (a real solution of ρ), we need for the discriminant to be greater or equal to zero, giving us the following condition:

$$\frac{\alpha\gamma}{4\beta\delta} \geq (1+\rho)(1-\delta) \tag{B.6}$$

Now we will go back to the analysis of the solution of q^* and how it changes with δ and ρ. Due to the complexity of the formulation (given in equation 2.10), I will rely on simulation. For this matter, I will use the previous values of α, β and γ, as shown in figure 14. Nevertheless and observing previous equation 2.10, we can see that the effect of α (and in the same manner γ) will be a shift up and down of the curve (with some deformations). In the same fashion, but with opposite direction, will be the effect of β on the curve $q^*(\delta)$.

It is important to notice that depending on the values taken by δ and ρ, r will reach its maximum value of 1. For example, let's check for the case of $\rho = 0.3$ (Figure 14). As δ grows, so does its corresponding value of r (dotted orange line), reaching its maximum value, shown with the dotted vertical red line. From this point on, $r = 1$ (for this particular value of ρ) and therefore the change in the shape of the curve (kink). For value of $\rho \geq 0.45$ this kink becomes much more evident. The idea here is that when r reaches its maximum, any new green 'effort', which is an increase in δ, only adds more 'cost' to the green agent, but no more satisfaction, making q^* decrease (even though it was increasing at that point, before reaching $r = 1$).

One interesting thing to point out is the fact that when γ decreases, q^* will decrease too. This causality could have some curious effect. For example, let's suppose that we are in a green equilibrium ($q^* > 1/2$) and, due to an external cause (such as a technological improvement), γ decreases, meaning that the brown products now pollutes less. In principle, this should be 'good' news. However, we will also end up with a lower equilibrium, maybe now in a brown point ($q^* < 1/2$), therefore maybe having a counter effect.

Also two cases have been highlighted: $\rho = 0.2$, giving a green equilibrium and $\rho = 0.3$, leading to a brown one. These two cases were also analyzed in previous figures 1 and 2 and are highlighted here for comparison matters.

For high values of ρ, the mark up of green products, we find that the equilibrium point is increasing with δ, the green willingness (lower curves of the graph). But, in this case (high ρ) we always end up with a brown equilibrium. On the contrary, for 'milder' values of ρ, we get the opposite effect, meaning that the equilibrium point moves to a more brown state, for a higher value of δ, the green willingness.

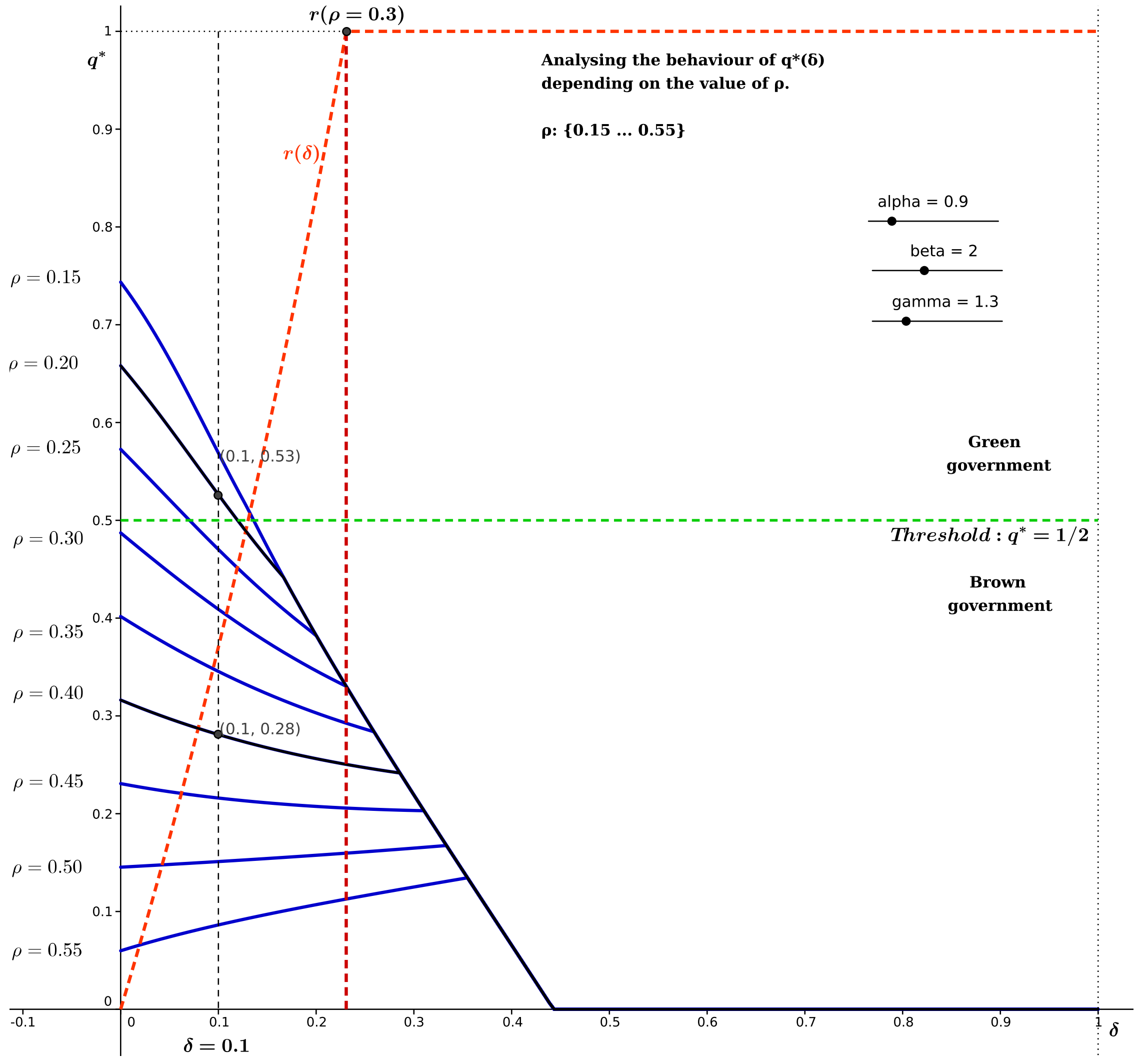


Figure 14: Analysis q*

The intuition here is that when the mark up for green products is high ($\rho \approx 0.5$), we will be in a range of q^* which will be low (brown). We will also have a low value of r, the green proportion of green people. Therefore, an increase in δ, which will translate in an certain increase in r, will have a high impact in the moral gain (since q is low[19]). This increase in moral gain will overcome the 'lost' in utility units due to the increase in δ , and therefore will make q^* grow.

On the contrary, when the mark up is in a lower level ($\rho \approx 0.25$), an increase in δ will have a different impact. Even though the same increase in δ will imply a greater increase in r (with respect to the previous case), a lower level of ρ will have us in a higher equilibrium point q^*. The later will translate as follows: the increase in r will not be followed by the corresponding increase in the moral benefit, since the fact that q has a higher value (see footnote). Therefore, the increase

[19]It might be useful to recall that $\Delta U = \alpha\gamma r(1-q)(1-rq) + \beta \ln(1-\delta)$

in moral gain will be completely thwarted by the given 'cost' due to the initial increase in δ, making q^* drop.

Now focusing on the changes due to ρ, we can see that as ρ decreases, the equilibrium point q^* will increase, and depending on the value of δ could actually cross the brown/green government threshold.

C The green government

In the following section I will introduce the difference between having a brown or a green government. As was explained in the introduction and in the first paragraphs of section 2.2, the type of government elected will respond to a simple majority rule, meaning that if $q_t < 1/2$ we will have a brown government, where if $q_t \geq 1/2$ we will get a green one.

In the first case, the government will just leave the market work, i.e. it will not change the prices of the brown or green goods, nor implement any kind of incentive to pollute less. It is just not in the best direct interests of its constituents, the brown people.[20]

But in the second instance, the government will actually make some changes. In this model, when in the presence of a green government, it will be introduced a lump sum tax τ on their income, to everybody. Therefore the available income will change from w to $(1-\tau)w$. With this tax, the government will subsidy the green goods, changing it price from $(1+\rho)$ to $(1+\rho-\epsilon)$, where ϵ is the value of the subsidy.[21]

C.0.1 Subsidizing the green products

The idea here is that the government will levy a tax, with a rate τ on the income, in order to subsidy the green good. I will call ρ' the new value of the mark up of the green good, where $\rho' = \rho - \epsilon$. With this, we will get a new value of r, which will be called r'. Obviously the government has no intention to overcharge the population with an excessive tax rate, therefore we have two direct constraints:

$$\begin{aligned} \rho' &\geq 0 \\ r' &\leq 1 \end{aligned}$$

Meaning that we neither want to make green products cheaper than the brown ones (the same price will actually be enough), nor have a proportion (of green buying among the green people) higher than one. We also will assume that the government budget constraint is level, therefore all

[20]From a direct economic point of view.

[21]Some other options could be added to the model, as for example a method to change the value of γ. Mechanisms such as these are not considered, since they get out of the scope of the present model, although it could be thought as extensions of the present work.

the money collected in taxes is used in the same period for subsidies. Then,

$$\begin{aligned} \textit{Total subsidy} &= \textit{Total tax collected} \\ \epsilon r' q n_1' &= \tau w \\ \epsilon &= \frac{\tau w}{r' q n_1'} \end{aligned} \quad (C.1)$$

We also have the new (total) amount of goods bought by each type of person is now:

$$n_1' = \frac{(1-\tau)w}{1+r'\rho'}$$

$$n_2' = (1-\tau)w$$

So, replacing $\rho' = \rho - \epsilon$ on C.1 and doing some math, we get:

$$r' = r + \frac{\tau}{q\rho(1-\tau)(1-\delta)} \quad (C.2)$$

As can been seen in the relationship C.2, $r' > r$, which was something one would expect. The difference among them is increasing with τ (as expected) and decreasing with q and ρ. The first relationship, of the last two, is also quite intuitive. The fewer green people in society, the stronger the effect of the tax on the price and therefore the increase of r, since the tax is levied on all the population. The second effect, coming from ρ, has to do with the fact that if the green price is too close to the brown one ($\rho \approx 0$), then a small subsidy will make all green people consume only green products. As always, we have to remind that $r' \leq 1$.

Replacing r on C.2 and using the last inequality ($r' \leq 1$), we have:

$$r' = r + \frac{\tau}{q\rho(1-\tau)(1-\delta)} \quad (C.3)$$

$$\tau \leq \frac{(\rho(1-\delta)-\delta)q}{1+(\rho(1-\delta)-\delta)q} \quad (C.4)$$

In other words, the government will not use a green tax higher than what is stated in equation C.4, due to the arguments previously explained.

In order to know how q^* moves with respect to r (or δ), I will analyze the derivative of q^* with respect to r. Observing the figure 15, we can see that for 'usual' range of r ($r \leq 0.84$), the derivative is always positive. This means that a rise in r will actually increase the equilibrium point. This is important due to the fact that when we are changing from a brown into a green government, we want to see if this transition will remain, or if it will turn out unstable (due to the fact that the green government implements the green tax and therefore increases r). Since the derivative is positive, we can conclude that the transition is actually 'stable'.

In the example below, we have $\rho = 0.3$, which gives us a corresponding value of $r = 0.37$, watching the dotted green curve (with $\delta = 0.1$ as usual). At this level of r, we can see that q^* will almost grow as much as r.

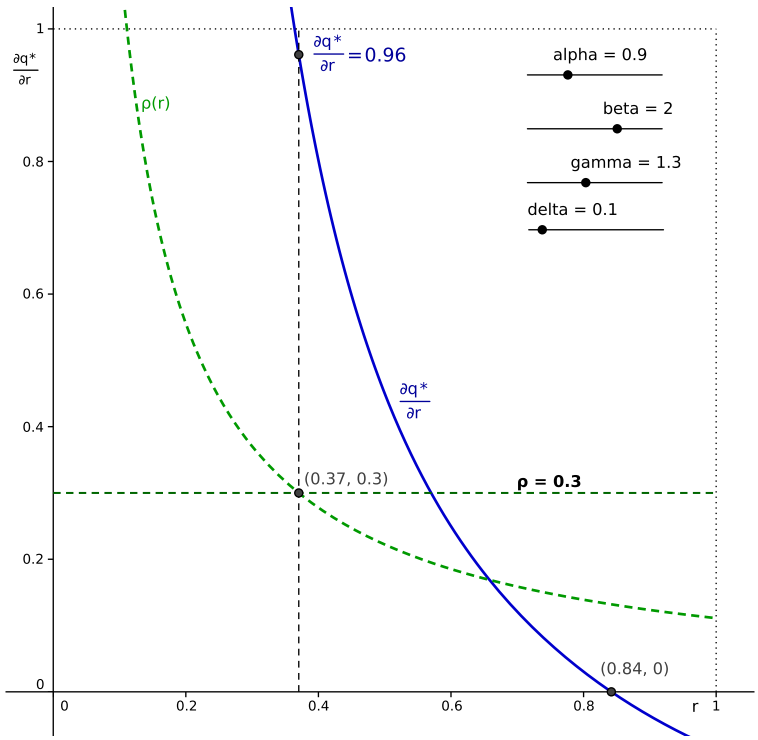


Figure 15: Analysing q* with respect to r

C.1 Equilibrium Points and Total Welfare

Now that we have the different relationships between the variables and possible equilibria (brown or green), we will first analyze what would happen if we were in a green equilibrium. In order to do this, I will rely in the following figure 16. Hoping that the graph is not too tangled to the reader, I will start describing its construction.

To start with, I would like to remind that the tax rate τ has a limit, which was seen in the conditions stated in equation C.4. Since this constraint depends on q, it was plotted in order to verify that we are actually moving on the allowed areas of the graph. In this case, at the selected value of $q = 0.56$, the maximal value of τ would be 0.09, which is much bigger than the actual one of 0.03 used.

Given this, we can now notice that we have two curves $r'(q)$ (the black ones). The 'first' one (the decreasing one), is given by equation C.2, which has to do with the effect of τ, the tax rate, on r'. The 'second' one is the inverse function of the solution of q^*, as in equation 2.10. The point where they meet [0.56 , 0.57] is the solution for that given value of τ. In this case we got $q^* = 0.56$ and $r' = 0.57$.

Now, by changing the value of τ, we will shift the first curve to the right. This will move the intersection point to the right (not too much) and mostly upwards. This means that the 'effort' on augmenting τ (at this level) mainly translates into the green people consuming more green products, but not in a bigger change in the society composition.

Now let's observe the different welfares. First, I will focus on the 'apparent' welfare, that is, the agent's welfare without the pollution disutility. These two curves are represented by the brown and green dotted lines on the graph.[22] As we can see, the brown 'apparent' utility is constant in each type of government, having a lower value in the green government, due to the payment of the tax rate τ. The green one, which has the same 'jump' in $q = 1/2$, is a decreasing one, as we saw previously in figures 1 and 2.

Therefore, if the green government chooses a tax level τ in order to stay in office, it will choose a τ such as q is just greater than $1/2$.

Now, let's focus on the pollution disutility. As we saw in the beginning of this work, we called $h(P_{t-1})$ the disutility function for certain pollution level P_{t-1}. We know that this disutility function is increasing with the level of pollution (which could be, for example, in a linear or exponential way). We will just focus on a linear form (with a constant factor of ϕ). Since we are analyzing the equilibrium states, then $P_t = P_{t-1}$. Recalling that $P_t = \gamma(1 - rq)$, we can plot the pollution level in our graph, which is represented by the red curve. Therefore, as the green population grows, the

[22] A 'zoom' factor has been used in order to place all the curves in the same range of the *y axis*.

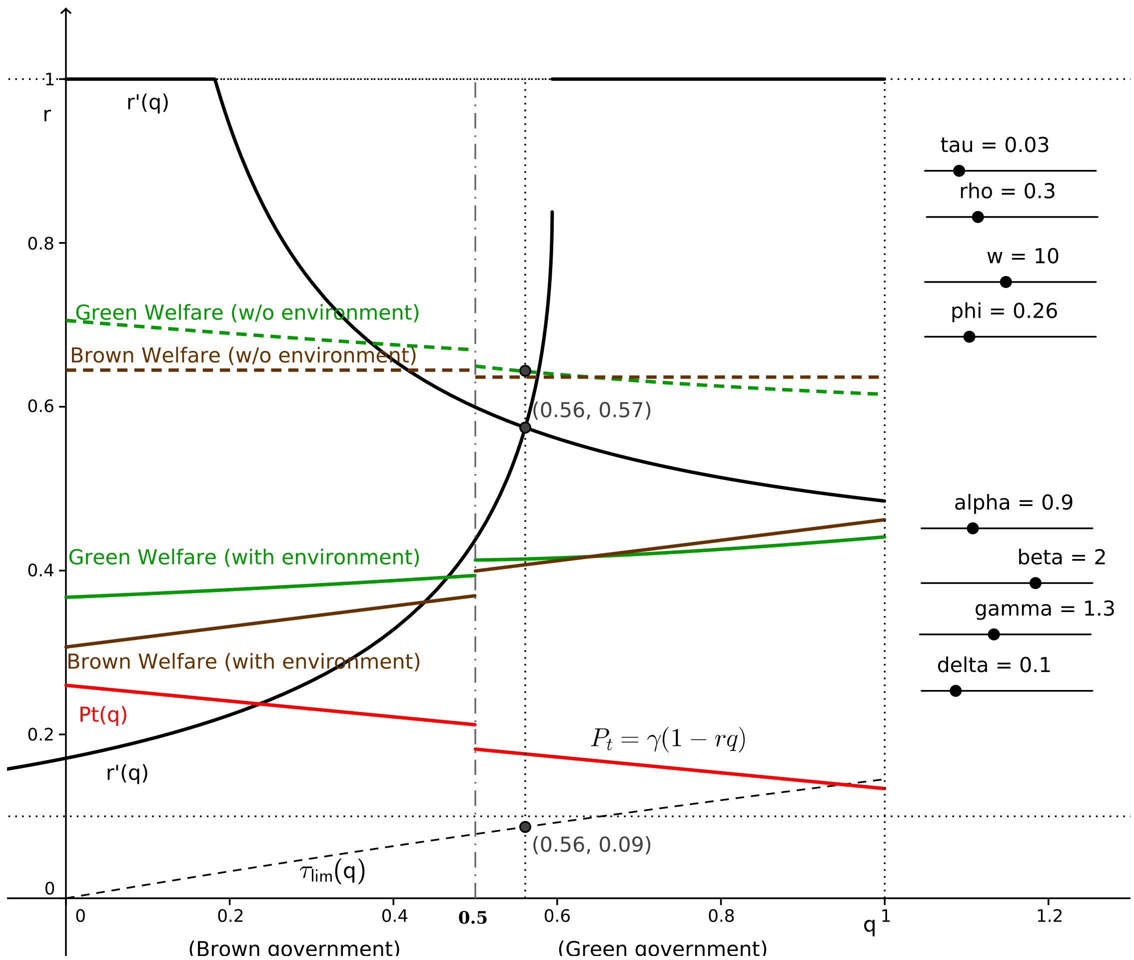


Figure 16: Analysing τ and welfares

pollution decreases and, with it, its disutility, adding up to a higher utility when $q \to 1$.[23]

Hence, returning to our green government choice of the tax rate τ, now if it actually incorporates the pollution effect upon the green people (or for this matter, on everybody's welfare), the government would like to have a higher tax rate. This will be the case until the intersection point goes all the way up to $r = 1$ (around $q^* = 0.6$), since there is no gain to overtaxing after this point. In the present figure, I have chosen a value of $\tau = 0.03$. This has been done with two simple academic objectives. First, to allow the graph to be in a simpler point to study it; and, secondly, to have an intermediate point (between the two previous assumptions).

Although the Total Welfare was not graphed in figure 16 (to avoid the graph becoming any more complex) we can make similar deductions to the ones made earlier. Since the Total Welfare should 'treat' brown and green people in the same way, we consequently would have their utility

[23]For simplicity, I have assume that ϕ is high enough to have this effect. It could be the case that ϕ is too low and the overall welfare still decreases with q.

functions (per capita) be the same weight. Therefore, the total welfare (TW) would be:

$$TW = q \cdot W_{green} + (1 - q) \cdot W_{brown} \tag{C.5}$$

In other words, the Total Welfare is a weighed average (proportional to the share of each type) of each specific welfare. Looking again at the figure, we can see that both curves (brown and green ones) move in the same fashion, and, therefore, conserve the previously analyzed behavior.