# All you need is loan

## Credit market frictions and the exit of firms in recessions*

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**Abstract**

This paper investigates how credit market frictions may alter business cycle dynamics by modifying the exit behavior of firms. We show that the extensive margin yields a significant amplification mechanism as credit frictions increase the number of firms vulnerable to a fall in aggregate productivity. Unlike the standard financial accelerator, this amplification channel does not hinge on the sensitivity of firms' net worth to aggregate shocks. Moreover, though credit market frictions distort the selection of exiting firms, the average idiosyncratic productivity of firms during recessions rises more than in a frictionless economy.

**Keywords**: business cycles, firm dynamics, credit frictions.
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# 1 Introduction

The interplay between macroeconomic shocks and financial conditions has gained renewed interest after the global recession of 2008-2009. Several empirical studies document the role of credit market frictions on the severity and duration of recessions. Over a sample covering 21 OECD countries during the period 1960-2007, Claessens et al. (2009) show that recessions associated with credit crunches tend to be deeper and longer than other recessions. Moreover, a series of papers (Kroszner et al. (2007), Dell'Ariccia et al. (2008), Bricongne et al. (2009)) emphasize that during banking crises, and in particular during the recent financial crisis, firms in financially vulnerable sectors were hit harder. As shown by Braun and Larrain (2005), industries highly dependent on external finance are more sensitive not only to banking crises, but also to recessions caused by non-financial factors. Braun and Larrain (2005) also show that the number of establishments is more severely affected in those industries, suggesting that financial conditions affect not only the response of firms' output to business cycle shocks (the intensive margin) but also the response of the number of firms (the extensive margin). The extensive margin is an important channel for aggregate fluctuations. Data on the US private sector provided by the *Bureau of Labor Statistics* from 1994 to 2010 show that entering and exiting establishments account for 20% of the volatility of annual employment growth[1]. However, very little research has been devoted to understanding the firms' exit decision under financial frictions and its implication for business cycle fluctuations. Following Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), the theoretical literature has focused on the implication of credit frictions on the intensive margin and leaves aside the consequences of credit frictions on the extensive margin.

The objective of this paper is to analyze how credit market frictions may alter business cycle dynamics by modifying the exit behavior of firms. Our main contribution is to show that the extensive margin yields a significant amplification mechanism. Credit market frictions amplify the fluctuations in the exit rate as they make the industry more vulnerable to negative aggregate shocks, thereby leading to a substantially larger output loss. In the presence of credit market frictions, the fall in aggregate productivity induces the exit of firms that would have survived in a frictionless economy. Unlike the standard financial accelerator, the mechanism emphasized in this paper does not depend on the sensitivity of

[1] We use the data from the Business Employment Dynamics program which are constructed from state unemployment insurance records.

firms' net worth to aggregate shocks. We show that when the balance sheet channel is shut down, credit market frictions amplify aggregate fluctuations by increasing the number of firms vulnerable to the adverse productivity shock.

We explore this mechanism in a model of firm dynamics with credit constraints. We analytically characterize the exit decision of firms that face credit constraints and incur a fixed cost of production. This is a contribution to the existing literature on the exit decision of firms ((McDonald and Siegel, 1985; Dixit, 1989; Hopenhayn, 1992)).[2] In the model, as in Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997), credit constraints endogenously arise from asymmetric information and costly state verification. The financial contract determines the amount the firm can borrow and the interest rate charged by the financial intermediary as a function of the firm's levels of productivity and net worth. When firms are hit by an adverse productivity shock, they may be unable to repay their debt, they default and are left with zero net worth. After default, most firms are excluded from the credit market and are therefore forced to exit. However, default is not the only motive for exit as firms also decide to leave the market when their expected profits are too low. This happens when firms are not sufficiently productive, as in the frictionless economy, but also when their balance sheets are too weak. Firms with a low net worth face tighter credit constraints and higher borrowing costs, which raise their probability to exit. As the firms' exit decision depends on their net worth, firms that exit are not necessarily the least productive ones. Credit market frictions therefore distort the selection of exiting firms: some high productivity firms are forced to exit in case of financial distress while some low productivity firms may survive. Despite this imperfect selection, we find that average idiosyncratic productivity rises more after a fall in aggregate productivity when firms face credit constraints. Credit market frictions magnify the increase in the exit rate as they increase the number of firms vulnerable to the fall in aggregate productivity. Though productivity is not the only determinant of survival, these exiting firms are, on average, less productive than surviving firms. Therefore, by amplifying the number of exiting firms, credit market frictions also amplify the increase in average idiosyncratic productivity.

We then investigate the consequences of the fluctuations in the exit rate on aggregate output. We focus on the effects of credit frictions on the vulnerability of the industry to a fall in aggregate productivity, and leave out the consequences of the endogenous change

[2] All these papers consider perfect financial markets, and to the best of our knowledge, the properties of the exit decision under credit constraints have not been derived analytically.

in net worth. In fact, the consequences of the fall in net worth are well-known at the intensive margin, and are likely to generate a similar mechanism at the extensive margin. In the economy with credit frictions, the decline in aggregate productivity leads to a significantly larger output loss. We disentangle the output loss induced by the intensive margin from that induced by the exit of firms. We show that, when the impact of the fall in aggregate productivity on the firms' net worth is shut down, credit frictions amplify the output loss at the exit margin while dampening aggregate fluctuations at the intensive margin. This amplification channel at the exit margin appears then more robust than the financial accelerator mechanism as it does not hinge on the sensitivity of firms' net worth to aggregate shocks. These results suggest that the exit margin is an important channel for understanding the aggregate implications of credit market frictions.

This paper contributes to the debate on the importance of the extensive margin for business cycle dynamics. While Samaniego (2008) finds that entry and exit respond very little to aggregate productivity shocks, Lee and Mukoyama (2008), Bilbiie et al. (2007), and Clementi and Palazzo (2010) show that the extensive margin plays a crucial role for aggregate fluctuations. These papers focus on the behavior of the entry rate, and highlight the effects of the procyclicality of the entry rate on the propagation of aggregate shocks. By contrast, we emphasize the role of the exit margin and assume a constant mass of potential entrants. Despite this assumption, the actual number of entrants is endogenous as firms enter the market only when their expected profits are sufficiently high. We show that the output loss due to the exit of incumbents and potential entrants accounts for a large fraction of the output drop, in particular when firms are subject to credit constraints. Our results therefore suggest that the exit of firms is an important dimension of aggregate fluctuations in less financially developed countries.

This paper is also related to the literature on firm dynamics in a credit constrained environment (Cooley and Quadrini, 2001; Miao, 2005; Arellano et al., 2009). The theoretical model is related to Cooley and Quadrini (2001). In their paper, they show how financial frictions can account for the size and age dependance of firm growth. Miao (2005) analyzes how the interaction between financing and production decisions affects firm turnover. Arellano et al. (2009) examine the link between financial development and firm growth.[3]

[3]Our model differs from these papers both in its focus and in its modeling choices, in particular with respect to exit. In these papers, exit is either exogenous or induced by default and financial imperfections. By contrast, in our model, firms endogenously exit when their expected profits are too low. As firms would exit even in the absence of financial frictions, our modeling assumption allows us to compare the exit behaviors in the credit constrained economy and in the frictionless economy.

This paper is also related to the literature on the cleansing effect of recession (Caballero and Hammour, 1994; Barlevy, 2003; Ouyang, 2009). Barlevy (2003) studies how credit market frictions distort the selection of exiting firms. In his model, credit frictions may reverse the cleansing effect of recessions as they lead resources to flow from high to low productivity firms. This contrasts with our results as, in our setup, a higher productivity facilitates the firm's access to credit. We show that though some high productivity firms may exit during the recession, credit frictions mainly increase the exit rate of low productivity firms. Therefore, credit frictions lead average productivity to rise further during recessions, somewhat exacerbating the cleansing effect of recessions.

This paper is organized as follows. Section 2 describes the model of firm dynamics and credit constraints. In Section 3, we show analytically how the exit decision differs from the frictionless economy. In Section 4, we first analyze numerically the properties of the steady state economy. We then show how the distorted exit decision may amplify the consequences of a fall in aggregate productivity on average productivity and aggregate output. Section 5 concludes.

## 2 A model of firm dynamics and credit market frictions

In this section, we describe the model of firm dynamics with credit market imperfections. In what follows, we define the production technology and the timing of the firms' decisions, then present the frictionless economy and the economy with credit market frictions.

### 2.1 Technology and timing of decisions

The economy is constituted of risk neutral firms with a constant discount factor $0 < \beta < 1$. Firms are heterogenous with respect to their productivity and their net worth, and have access to a production technology with capital as the only input and decreasing returns to scale. Each period, firms incur a fixed operating cost to start production. After production, firms determine the amount of dividends to distribute and the amount of profits to reinvest. The firm can decide to stay in the market and reinvest its profit in production or invest in a risk-free asset. When the value from investing in the safe asset is higher than the value from producing, the firm chooses to exit and never enters again. Exiting firms lose the opportunity to receive future profits from production, but also avoid to pay the fixed cost. Firms therefore exit when their expected income from production is not sufficiently high to compensate the fixed cost.

After paying the fixed operating cost $c$, the firm produces output: $Z(\theta + \epsilon)k^{\alpha}$ with $0 < \alpha < 1$. The capital $k$ used for production depreciates at rate $0 < \delta < 1$. $Z$ denotes the value of aggregate productivity. Every period, firms are hit by a persistent idiosyncratic productivity shock $\theta \in [\theta_{\min}, \theta_{\max}]$, and a non-persistent idiosyncratic productivity shock $\epsilon \in [\epsilon_{\min}, \epsilon_{\max}]$. The non-persistent component $\epsilon$ is independently and identically distributed (i.i.d) across time and across firms, with distribution $\Phi$, zero mean, and standard deviation $\sigma$. We impose $\epsilon_{\min} > -\theta_{\min}$, which ensures a non negative production whatever the value of the shock $\epsilon$. The persistent component $\theta$ follows a Markov process independent across firms with conditional distribution $F(\theta'|\theta)$. The conditional distribution $F(\theta'|\theta)$ is assumed to be strictly decreasing in $\theta$ : the higher is the productivity shock at time $t$, the more likely are high shocks in period $t + 1$. This assumption ensures that the value of the firm is an increasing function of the current productivity $\theta$. In what follows, for any generic variable $x$, we adopt the notation $x'$ to define the next period value of the variable $x$.

The value of the persistent idiosyncratic shock is revealed one period in advance. Therefore, at the beginning of the period, firms choose their capital knowing their persistent idiosyncratic shock $\theta$, the value of aggregate productivity $Z$, and their net worth $e$. At the beginning of the period, firms do not know their idiosyncratic shock $\epsilon$. They observe the realization of $\epsilon$ after production, and must then reimburse their debt over the capital borrowed and the fixed operating cost $(c + k - e)$. They are left with the end-of-period net worth $q$. At the end of the period, a firm with net worth $q$ observes its productivity shock $\theta'$, and decides its next period net worth $e'$ (or equivalently the amount of dividends $(q - e')$ to distribute), and whether to exit or stay in the market. A firm decides to exit when its value from producing is lower than the value from investing in the safe asset, which is equal to $q_t + \Sigma_{s=0}^{+\infty}\beta^s[\beta(1 + r) - 1]e_{t+s+1}$. Note that if $\beta(1 + r) \leq 1$, the value from investing in the safe asset simplifies to $q_t$. In that case, either the firm is indifferent about the timing of dividends ($\beta(1 + r) = 1$) or it prefers to distribute its end-of-period net worth as dividends ($\beta(1 + r) < 1$).

## 2.2 The frictionless economy

In the frictionless economy, firms borrow $(c+k-e)$ at the risk-free interest rate $r = 1/\beta - 1$. The value of a firm at the beginning of the period is:

$$V_{FL}(e, \theta, Z) = \max_k \mathbb{E}_\theta \int_{\epsilon_{\min}}^{\epsilon_{\max}} \max\left[q, \max_{e'}(q - e' + \beta V_{FL}(e', \theta', Z))\right] d\Phi(\epsilon),$$

where the end-of-period net worth is equal to

$$q = Z(\theta + \epsilon)k^\alpha + (1 - \delta)k - (1 + r)(c + k - e),$$

and $\mathbb{E}_\theta$ denotes expectations conditional on the current value of $\theta$. The value of the firm depends on the expected outcome of its investment. Firm exit when the value from investing in the safe asset is higher than the value from investing in production. As $r = 1/\beta - 1$, the firm is indifferent about the timing of dividends and the value from investing in the safe asset is then equal to its end-of-period net worth $q$. Furthermore, the Modigliani-Miller theorem holds and the value of the firm is independent of its financing decision. In particular, the exit and capital decisions of the firm do not depend on its level of equity. It can be shown that, conditional on surviving, the program of the firm is equivalent to maximizing its expected profits:

$$\widehat{V}_{FL}(\theta, Z) = \max_k \mathbb{E}_\theta \int_{\epsilon_{\min}}^{\epsilon_{\max}} [Z(\theta + \epsilon)k^\alpha - (r + \delta)k - (1 + r)c] d\Phi(\epsilon) + \beta \max\left[0, \widehat{V}_{FL}(\theta', Z)\right].$$

When credit markets are perfect, firms exit when they are not productive enough: they exit if $\theta' < \underline{\theta}_{FL}(Z)$, where $\underline{\theta}_{FL}(Z)$ is defined by $\widehat{V}_{FL}(\underline{\theta}_{FL}, Z) = 0$.

## 2.3 The economy with credit market frictions

As in Cooley and Quadrini (2001), we embed a one-period financial contract *à la* Bernanke and Gertler (1989) into a dynamic framework. Credit constraints arise from asymmetric information between the firm and the financial intermediary. After production, the non-persistent idiosyncratic shock $\epsilon$ is privately observed by the firm, whereas the financial intermediary can observe $\epsilon$ only at a cost $\mu k^\alpha$. We consider a one-period debt contract in which the firm defaults when the shock is too low, and the financial intermediary monitors the firm's income only when the firm defaults. The terms of the financial contract depend on the value of the firm's net worth $e$, on its current productivity $\theta$, and on the value of aggregate productivity $Z$, all observable by the financial intermediary and the firm at zero cost.

Assumption 1. The risk-free interest rate is such that: $\beta < \frac{1}{1+r}$.

As in Cooley and Quadrini (2001), this assumption implies that the risk-free rate is lower in the economy with credit frictions than in the frictionless economy, and guarantees that firms will not always choose to reinvest all their profits, thus giving an upper bound to their net worth. This condition can be interpreted as a general equilibrium property of economies with financial constraints. As it goes beyond the scope of this paper to analyze the impact of credit frictions on the risk-free rate, note that we choose to leave aside this general equilibrium effect when comparing the results in the credit constrained economy with the frictionless case. In the following, we therefore compare the credit constrained economy with the same economy without credit frictions but with the same risk-free rate $r$.

The capital chosen by the firm is financed by its equity $e$, and if $k+c>e$, the firm borrows $(k+c-e)$ at rate $\widetilde{r}$ from the financial intermediary. When a firm is not able to reimburse its debt, it defaults. In this case, the financial intermediary pays a cost to verify the firm's income and confiscates all the firm's income. The default threshold $\bar{\epsilon}$ is given by:[4]

$$Z(\theta+\bar{\epsilon})k^{\alpha}+(1-\delta)k \geq (1+\widetilde{r})(k+c-e). \tag{1}$$

Note that default leads to a zero net worth but does not necessarily lead to the exit of the firm, as observed empirically. Depending on its persistent productivity component $\theta$, the firm could find profitable to stay in the market with zero net worth.
The financial intermediary lends $(k+c-e)$ to the firm only if its expected income from the loan is equal to the opportunity cost of the funds. The break even condition reads:

$$(1+\widetilde{r})(k+c-e)[1-\Phi(\bar{\epsilon})]+\int_{\epsilon_{\min}}^{\bar{\epsilon}}[Z(\theta+\epsilon)k^{\alpha}+(1-\delta)k-\mu k^{\alpha}]\,d\Phi(\epsilon) \geq (1+r)(k+c-e).$$

The expected income of the financial intermediary is equal to the repayment of the loan if the firm does not default ($\epsilon \geq \bar{\epsilon}$) and to the firm's income net of monitoring costs when the firm defaults ($\epsilon < \bar{\epsilon}$). Using the default condition (Equation 1), we can rewrite the participation constraint of the financial intermediary as:

[4] Note that the debt is never renegotiated after default. The financial intermediary could agree to reduce the debt to $(1+\widetilde{r})(k+c-e)-D$, with $0 \leq D \leq (1+\widetilde{r})(k+c-e)-(Z(\theta+\epsilon)k^{\alpha}+(1-\delta)k)$. This would leave the firm with end-of-period net worth $q=-D$. However, the renegotiation is never mutually profitable. Since there are no additional cost related to default, the firm always prefers to default and start the next period with zero net worth ($q=0$) than to renegotiate the debt and have a negative net worth ($q=-D$).

$$Z[\theta + G(\bar{\epsilon})]k^{\alpha} + (1-\delta)k - \mu k^{\alpha}\Phi(\bar{\epsilon}) \geq (1+r)(k+c-e),$$

with $G(\bar{\epsilon}) \equiv [1-\Phi(\bar{\epsilon})]\bar{\epsilon} + \int_{\epsilon_{\min}}^{\bar{\epsilon}} \epsilon d\Phi(\epsilon)$.

As it is more convenient to write the problem of the firm as a function of the default threshold $\bar{\epsilon}$, we characterize the financial contract by the couple $(k, \bar{\epsilon})$ and then derive the implied interest rate $\widetilde{r}$ charged by the financial intermediary from the default condition. Given $Z$, $\theta$ and $e$, the participation constraint indicates the amount the firm can borrow and the associated default threshold $\bar{\epsilon}$ required by the financial intermediary. A higher level of net worth relaxes the financial intermediary's participation constraint and allows the firm to borrow more capital. The credit constraint is tighter if the firm incurs a higher default rate when borrowing a larger amount. Assumption 2 gives the regularity condition on the distribution $\Phi$ that ensures a positive correlation between the amount the firm can borrow and the default threshold $\bar{\epsilon}$[5].

Assumption 2. The distribution function of the transitory shock is such that $\Phi'(\epsilon_{\min}) < \frac{Z}{\mu}$ and $\frac{\Phi'(\epsilon)}{1-\Phi(\epsilon)}$ is monotone in $\epsilon$.

For some firms, the income of the financial intermediary is too low for its participation constraint to be satisfied. In fact, given $\theta$ and $Z$, there is a unique threshold $\underline{e}_b(\theta, Z)$ below which the financial intermediary refuses to lend any fund (see Appendix A). This threshold is defined as:

$$Z[\theta + G(\bar{\epsilon}_b)]k_b^{\alpha} + (1-\delta)k_b - \mu k_b^{\alpha}\Phi(\bar{\epsilon}_b) = (1+r)(k_b + c - \underline{e}_b), \tag{2}$$

where $(\bar{\epsilon}_b, k_b)$ maximize the expected income of the financial intermediary. When the net worth of the firm is below $\underline{e}_b(\theta, Z)$, the financial intermediary would rather invest in the safe asset than lend to the firm. Proposition 1 shows that in that case the net worth of the firm is not sufficiently high to cover the fixed cost of production, and the firm is therefore forced to exit the market.

[5]This condition will be necessary to prove the continuity of the value function. It implies that the income of the financial intermediary is either increasing in $\bar{\epsilon}$, or is an inverted U-shaped curve.

PROPOSITION 1. **Exit of firms due to credit rationing**

Firms that are not financed by the financial intermediary cannot cover their fixed cost of production:

$$\underline{e}_b(\theta, Z) \leq c, \quad \forall \theta \in [\theta_{\min}, \theta_{\max}], \quad \forall Z.$$

Proof: see Appendix A.

After production, the firm's end-of-period net worth is equal to:

$$q = \begin{cases} Z(\theta+\epsilon)k^{\alpha} + (1-\delta)k - (1+\tilde{r})(c+k-e) & \text{if } \epsilon > \bar{\epsilon} \\ 0 & \text{if } \epsilon \leq \bar{\epsilon}. \end{cases}$$

Using again the definition of the default threshold (Equation 1), the end-of-period net worth reads:

$$q = \max\{Zk^{\alpha}(\epsilon - \bar{\epsilon}); 0\}.$$

### 2.3.1 The firm's problem

Define $V$ as the value of the continuing firm at the beginning of the period, before choosing its level of capital. The value of the firm depends on the outcome of its investment and on its exit decision. At the end of the period, the firm learns its next period productivity $\theta'$ and, depending on its end-of-period net worth, decides which fraction of its profit to distribute as dividends, and whether to stay or exit the market. When its end-of-period net worth is too low $q < \underline{e}_b(\theta', Z)$, the participation constraint of the financial intermediary is not satisfied. As stated in Proposition 1, in that case the firm cannot finance the fixed cost of production and must therefore exit the market. On the other hand, when $q \geq \underline{e}_b(\theta', Z)$, the firm decides whether to stay in the market or exit by comparing the value from producing with the outside opportunity. As the discount rate is higher than the safe asset return $r$, the firm always prefers to distribute its end-of-period net worth as dividends rather than invest it in the safe asset. The firm therefore exits when its continuing value is lower than its end-of-period net worth $q$. We prove in Appendix A that the value function of the firm exists and is unique. The problem of the firm reads:

$$V(e,\theta,Z) = \max_{(k,\bar{\epsilon})} \mathbb{E}_\theta \left\{ \int_{\epsilon_{\min}}^{\epsilon_{\max}} I(q)q + (1-I(q)) \max \left[ q, \max_{e'} \left( q - e' + \beta V(e',\theta',Z) \right) \right] d\Phi(\epsilon) \right\}$$

with:

$$I(q) = \begin{cases} 0 & \text{if } q \geq \underline{e}_b(\theta',Z) \\ 1 & \text{if } q < \underline{e}_b(\theta',Z) \end{cases}$$

subject to:

$$Z[\theta + G(\bar{\epsilon})]k^\alpha + (1-\delta)k - \mu k^\alpha \Phi(\bar{\epsilon}) \geq (1+r)(k+c-e) \tag{3}$$

$$q = \max\left\{ Zk^\alpha(\epsilon - \bar{\epsilon}); 0 \right\} \tag{4}$$

$$\underline{e}_b(\theta',Z) \leq e' \leq q. \tag{5}$$

The firm maximizes its expected dividends subject to the participation constraint of the financial intermediary defined by Equation (3). Equation (4) describes the end-of-period net worth $q$, while equation (5) imposes that the firm cannot issue new shares[6] and can then increase its net worth only by reinvesting its profits. The firm faces a trade-off when deciding its level of capital. On the one hand, if the firm is solvent, a higher level of capital increases its next period level of production. On the other hand, it increases its probability to default as the default threshold required by the financial intermediary increases with the amount borrowed (Assumption 2).

We assume that the value function is differentiable. This allows us to derive analytical results on the exit decision of the firms. It also permits to characterize the dividend decision of the firm. Because the discount rate is higher than the risk-free rate (Assumption 1), the firm will not always choose to reinvest all its profits. It will distribute dividends if its end-of-period net worth $q \geq \bar{e}(\theta,Z)$ with $\bar{e}(\theta,Z)$ defined by $\beta \frac{\partial V(\bar{e},\theta,Z)}{\partial e} = 1$.

[6] Allowing $e' > q$ makes the financial constraints irrelevant as firms would finance all their investment with equity.

### 2.3.2 Exit conditions

By contrast with the frictionless economy, productivity is not the only determinant of firms' survival. Firms exit if they are not sufficiently productive, but they may also exit because their balance sheet position is too weak. In fact, in the presence of credit frictions, two additional motives for exit arise. At the end of the period, firms may exit because their level of net worth is not high enough for their participation constraint ($\beta V(q, \theta, Z) < q$) or for the participation constraint of the financial intermediary to be satisfied ($q < \underline{e}_b(\theta, Z)$). These exit conditions are described in the following proposition.

PROPOSITION 2. **Exit conditions**
For a given level of aggregate productivity $Z$, there exist three thresholds $\underline{\theta}(Z) < \theta^*(Z) < \theta^{**}(Z)$ that characterize the exit decision of the firm. These productivity thresholds[7] delimit four exit regions:

**A.** The firm exits when $\theta < \underline{\theta}(Z)$ whatever its level of net worth.

**B.** The firm exits when $\underline{\theta}(Z) \leq \theta < \theta^*(Z)$ if its end-of-period net worth is too low for its participation constraint to be satisfied: $q < \underline{e}_f(\theta, Z)$, where $\underline{e}_f(\theta, Z)$ is defined by $\underline{e}_f = \beta V(\underline{e}_f, \theta, Z)$.

**C.** The firm exits when $\theta^*(Z) \leq \theta < \theta^{**}(Z)$ if its end-of-period net worth is too low for the participation constraint of the financial intermediary to be satisfied: $q < \underline{e}_b(\theta, Z)$ where $\underline{e}_b(\theta, Z)$ is defined by equation (2).

**D.** The firm never exits when $\theta \geq \theta^{**}(Z)$ whatever its level of net worth.

PROOF: see Appendix A.

Figure 1 represents the four exit regions defined in Proposition 2. All firms with a couple $(\theta, e)$ below the downward sloping frontier (solid line) exit the market, whereas all firms with a couple $(\theta, e)$ above the exit frontier produce and stay in the market. Firms with

[7] The productivity thresholds $\underline{\theta}(Z)$, $\theta^*(Z)$ and $\theta^{**}(Z)$ are defined by the following equations:

$$
\begin{aligned}
\bar{e}(\underline{\theta}, Z) &= \beta V(\bar{e}(\underline{\theta}, Z), \theta, Z) \\
\underline{e}_b(\theta^*, Z) &= \beta V(\underline{e}_b(\theta^*, Z), \theta^*, Z) \\
(1+r)(k_b + c) &= Z[\theta^{**} + G(\bar{\epsilon}_b)]k_b^\alpha - \mu k_b^\alpha \Phi(\bar{\epsilon}_b) + (1-\delta)k_b
\end{aligned}
$$

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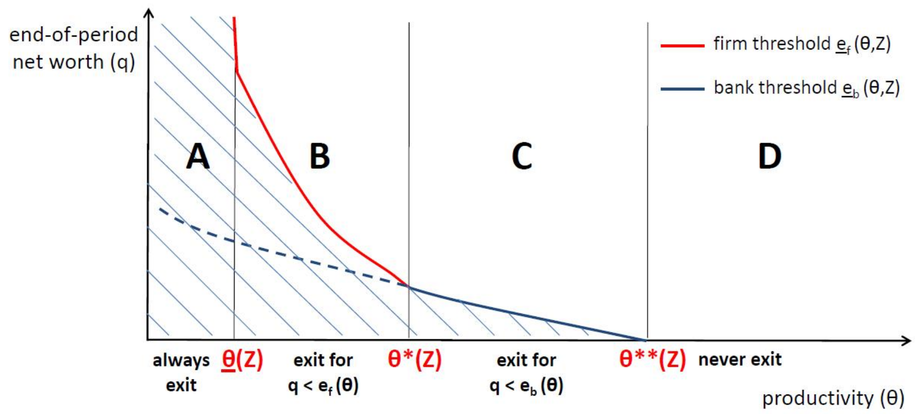


Figure 1: Exit frontier

productivity $\underline{\theta}(Z) \leq \theta < \theta^*(Z)$ (region B) exit when their net worth is below $\underline{e}_f$. A low level of net worth raises the borrowing costs of the firm, which may then not be sufficiently profitable to stay in the market. Figure 2 shows how the firm's net worth threshold $\underline{e}_f$ is determined. This threshold is the value of net worth for which the firm is indifferent between investing this value into production or distributing it as dividends. Firms with a higher level of productivity always find it profitable to stay in the market. However, firms with productivity $\theta^*(Z) \leq \theta < \theta^{**}(Z)$ (region C) can be forced to exit the market when their net worth is too low for the participation constraint of the financial intermediary to be satisfied. In contrast with the frictionless economy, the firm's decision to exit is sensitive to the non-persistent idiosyncratic shock. An adverse shock $\epsilon$ may lead some firms to exit as it may lower their net worth below the thresholds $\underline{e}_b(\theta, Z)$ or $\underline{e}_f(\theta, Z)$. In particular, firms with $\theta < \theta^{**}(Z)$ necessarily exit after defaulting on their debt. However, this is not the case for high productivity firms $\theta \geq \theta^{**}(Z)$ (region D) as the financial intermediary accepts to lend to those firms even after they defaulted. Whatever their level of net worth, firms with $\theta \geq \theta^{**}(Z)$ never exit the market.

The exit thresholds $\underline{e}_b(\theta, Z)$ and $\underline{e}_f(\theta, Z)$ are both decreasing functions of the persistent component of productivity $\theta$ (see proof in Appendix A). This implies that high productivity firms have a lower probability to exit the market. This negative correlation between exit and productivity is likely to be reinforced once the endogenous distribution of net worth is accounted for. In fact, high productivity firms have a lower probability to exit also because they tend to accumulate more net worth.
Finally, for all firms with $\theta \geq \underline{\theta}(Z)$, the exit, default and dividend decisions restrict the net

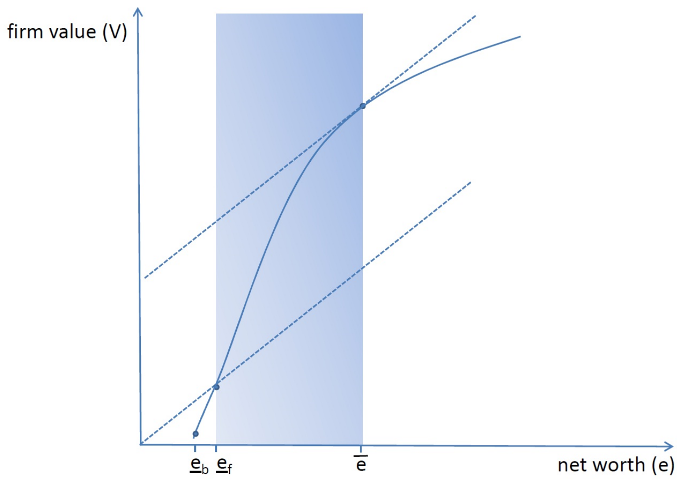


Figure 2: Firm value

worth of continuing firms to $[\underline{e}(\theta, Z), \bar{e}(\theta, Z)]$ with $\underline{e}(\theta, Z) = \max\{0, \underline{e}_b(\theta, Z)\}$ if $\theta \geq \theta^*(Z)$ and $\underline{e}(\theta, Z) = \underline{e}_f(\theta, Z)$ if $\underline{\theta}(Z) \leq \theta < \theta^*(Z)$. The set $[\underline{e}(\theta, Z), \bar{e}(\theta, Z)]$ for firms with $\underline{\theta}(Z) \leq \theta < \theta^*(Z)$ is represented by the shaded area in Figure 2.

## 2.4 Entry, stationary distribution and aggregate output

This paper focuses on the consequences of credit market frictions on the exit rate and leaves aside the implications of credit frictions at the entry margin. We assume that the mass of potential entrants is constant. Despite this assumption, the actual number of entrants is endogenous as firms enter the market only when their expected profits are sufficiently high. The net worth $e$ and productivity $\theta$ of potential entrants are characterized by the joint distribution $\nu$. The distribution $\nu$ of potential entrants, the distributions $\Phi$ and $F$ of the productivity shocks, together with the endogenous decision rules of the firms on capital, default, dividends and exit generate an endogenous joint distribution of productivity and net worth $\xi$. More specifically, these conditions give rise to a mapping $\Omega$ that indicates the next period joint distribution of net worth and productivity given the current distribution:

$\xi' = \Omega(\xi)$. The stationary joint distribution is the fixed point of the mapping $\xi^* = \Omega(\xi^*)$[8]. We can now use the joint distribution of firms to write the aggregate production in the industry[9]:

$$Y(Z,\xi) = Z \int_{\underline{\theta}(Z)}^{\theta_{\max}} \int_{\underline{e}(\theta,Z)}^{\bar{e}(\theta,Z)} \theta \left[k(e,\theta,Z)\right]^{\alpha} d\xi(e,\theta). \tag{6}$$

# 3 Distortion of the exit decision

In this section, we show analytically how the exit decision of the firm is modified once credit market frictions are taken into account. We first show that credit market frictions distort the productivity distribution of exiting firms. In the frictionless economy, all exiting firms are less productive than surviving firms. In the credit constrained economy, since exit also depends on firms' net worth, some high productivity but financially fragile firms are forced to exit, while less productive firms may survive. In order to show this distortion, we measure the quality of the selection at the exit margin by the productivity gap between the most productive exiting firm $\theta^{**}$ and the less productive surviving firm $\underline{\theta}$. The larger the productivity gap, the lower is the quality of the selection at the exit margin.

PROPOSITION 3. **Imperfect selection**
The quality of the selection at the exit margin is negatively related to the degree of credit market frictions :

$$\frac{d(\theta^{**} - \underline{\theta})}{d\mu} > 0.$$

PROOF: See Appendix A.

Figure 3 provides an illustration of Proposition 3. An increase in the productivity gap between the most productive exiting firm $\theta^{**}$ and the least productive surviving firm

[8] The distribution reads:

$$\begin{aligned}\xi'(\mathcal{E}\times\Lambda) \quad = \quad & \int_{\underline{\theta}(Z)}^{\theta_{\max}} \int_{\underline{e}(\theta,Z)}^{\bar{e}(\theta,Z)} \left( \int_{\theta'\in\Lambda\cap[\underline{\theta}(Z),\theta_{\max}]} Prob(e' \in \mathcal{E}\cap[\underline{e}(\theta',Z),\bar{e}(\theta',Z)]|\theta,e)dF(\theta'|\theta) \right) d\xi(\theta,e) \\ & +\psi \int_{\Lambda\cap[\underline{\theta}(Z),\theta_{\max}]} \int_{\mathcal{E}\cap[\underline{e}(\theta,Z),\bar{e}(\theta,Z)]} d\nu(\theta,e),\end{aligned}$$

where $\psi$ is the mass of potential entrants. The associated stationary probability measure exists and is unique as the associated transition function is monotone, has the Feller property and satisfies a mixing condition. The proof is similar to Cooley and Quadrini (2001).

[9] We assume that the fraction of output required for the monitoring process is not destroyed.

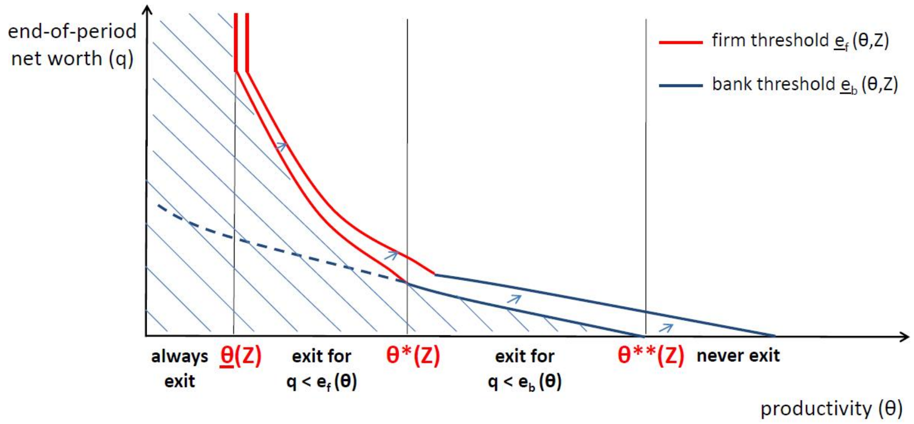


Figure 3: Distortion of the exit frontier due to an increase in credit frictions ($\Delta\mu > 0$)

$\underline{\theta}$ reveals larger distortions in the selection mechanism. This result implies that credit frictions increase the misallocation of resources at the exit margin. In fact, credit market frictions may generate an inefficient reallocation of resources as financial funds flow from high to low productivity firms. Besides modifying the distribution of exiting firms, credit market frictions also affect the mass of firms that exit the market. Proposition 4 describes the impact of credit frictions on the exit thresholds.

PROPOSITION 4. **Firm destruction**
The firms' probability to exit increases with the degree of credit market frictions:

$$\frac{d\underline{\theta}}{d\mu} > 0 \; ; \; \frac{d\underline{e}_f}{d\mu} > 0 \; ; \; \frac{d\underline{e}_b}{d\mu} > 0.$$

PROOF: See Appendix A.

Credit market frictions lead to the exit of firms with low productivity and low net worth that would have been economically viable absent any credit frictions. A higher degree of credit frictions raises firms' borrowing costs, which then increases their probability to exit.

So far, we have highlighted the distortions caused by credit market frictions in the absence of aggregate productivity shocks. Our main objective is to understand the consequences of these distortions on the propagation of aggregate shocks. The complexity of the model prevents us from solving analytically the response of output to a decline in aggregate productivity. However, the distortion in the firms' exit behavior suggests that the fall in output may be amplified by the fluctuations in the exit rate. In fact, higher credit frictions increase the set of firms vulnerable to shocks and may then magnify the impact of an aggregate productivity drop by leading more firms to exit the market. Recall that only firms with productivity $\theta < \theta^{**}$ may exit after an adverse idiosyncratic shock. Similarly, a fall in aggregate productivity $\Delta Z = Z' - Z < 0$ only raises the exit probability of firms with productivity $\theta < \theta^{**}(Z')$. More precisely, the aggregate productivity drop increases the mass of exiting firms by $\Delta N_X$, which is an increasing function of $\theta^{**}$:

$$\Delta N_X = \int_{\underline{\theta}(Z')}^{\theta^{**}(Z')} \int_{\underline{e}(\theta,Z)}^{\underline{e}(\theta,Z')} d\xi^*(e,\theta) + \int_{\underline{\theta}(Z)}^{\underline{\theta}(Z')} \int_{\underline{e}(\theta,Z)}^{\bar{e}(\theta,Z)} d\xi^*(e,\theta).$$

We therefore interpret $\theta^{**}$ to be a measure of the industry vulnerability to aggregate shocks.

PROPOSITION 5. **Industry vulnerability**
The industry vulnerability to aggregate shocks increases with the degree of credit market frictions:

$$\frac{d\theta^{**}}{d\mu} > 0.$$

PROOF: See Appendix A.

Proposition 5 suggests that aggregate productivity shocks are likely to generate a stronger response of the exit rate when credit frictions are taken into account. As the response of the economy to a fall in aggregate productivity cannot be derived analytically, we solve the model numerically and further investigate these questions in Section 4.

# 4 Aggregate implications of firm exit under credit market frictions

In this section, we analyze numerically how the exit of firms under credit market frictions modifies the impact of a fall in aggregate productivity. We solve the model using value function iteration. The method is described in Appendix B. We first present the benchmark calibration and describe the firm capital and exit decisions in steady state. Then, we illustrate how credit constraints distort the exit decision of the firm, and analyze its implication on average productivity and on output[10]. Finally, we show how the idiosyncratic volatility $\sigma$ affects the results.

### 4.1 Benchmark calibration

The model period is one year. We calibrate the parameters on the frictionless economy and then analyze how the introduction of credit market frictions modifies the impact of a fall in aggregate productivity with respect to this benchmark. We normalize the value of aggregate productivity $Z$ to one in the steady state and then consider the impact of a permanent decline in $Z$ of one standard deviation. We use the standard deviation of the innovation to the Solow residual estimated by King and Rebelo (1999) to be 0.0072 for 1947-1996.

Consistently with the business cycle literature, we set the risk-free rate $r$ to 4%, the discount rate $\beta$ to 0.9606, and the depreciation rate $\delta$ to 7%. Following the estimates of Hennessy and Whited (2007), the returns to scale parameter of the production function $\alpha$ is set to 0.7.

We assume a Pareto steady state distribution of productivity $\theta$ for incumbents firms, normalize its mean to 0.35 and set the shape parameter to 4 which yields an interquartile ratio of 1.32. This value is line with Del Gatto et al. (2008) who estimate the intra-industry shape parameter of total factor productivity to be between 3 and 4 in the Italian economy, as well as with the estimates of Syverson (2004) who reports an average interquartile ratio that ranges between 1.3 and 1.6 in the US manufacturing sector. We then discretize this steady state Pareto distribution into 5 levels of productivity $\{\theta_1, .., \theta_5\}$ and their associated distribution $\xi^*_{FL}$[11]. To capture the non linear effects of credit frictions, $\xi^*_{FL}(\theta_i)$ is set so that the discretized productivity levels $\theta_i$ are concentrated at the lower end of the

[10] Recall that we leave aside the implications of credit market frictions on the risk-free interest rate and therefore compare the economy with credit frictions with a frictionless economy characterized by the same risk-free rate.

[11] $\xi^*_{FL}$ is the fixed point of the mapping:

$$\xi'_{FL}(\Lambda) = \int_{\theta \in [\theta_{\min}, \theta_{\max}]} \int_{\theta' \in \Lambda \cap [\underline{\theta}_{FL}, \theta_{\max}]} dF(\theta'|\theta) d\xi_{FL}(\theta) + \psi_{FL} \int_{\theta \in \Lambda \cap [\underline{\theta}_{FL}, \theta_{\max}]} d\nu_\theta(\theta),$$

where $\nu_\theta$ is the marginal productivity distribution of potential entrants and $\psi_{FL}$ is the mass of potential entrants, normalized to have a unit mass of incumbents firms. $\theta_i$ is computed as the expectation of the truncated Pareto distribution in the interval $[m_{i-1}, m_i]$, with $Prob(m_{i-1} < \theta < m_i) = \xi^*_{FL}(\theta_i)$, $i = 1..5$ where $m_0$ is the scale of the Pareto distribution.

distribution.

In the frictionless steady state economy, firms exit when their productivity is equal to $\theta_0$, with $\theta_0$ set to be 30 percent lower than the average productivity of incumbents firms. Together with the fixed cost $c$, the conditional distribution of productivity shocks $F(\theta'|\theta)$ is then chosen to match the stationary distribution of productivity $\xi^*_{FL}$ and a steady state exit rate of 5%. This value is below usual estimates of the exit rate as we believe traditional measures are likely to overstate the number of exiting firms[12]. Dunne et al. (1989) report a 5-year exit rate of 36% in the US manufacturing sector, which induces a 7.2% annual exit rate, assuming that the number of firms remains constant during these 5 years. OECD firm level data (Scarpetta et al., 2002) exhibit a 8.4% exit rate for the US private sector. We also constrain $F(\theta'|\theta)$ to be decreasing in $\theta$. Because of the discretization, we need to ensure that the aggregate shock yields a realistic increase in the exit rate. This was achieved by limiting the size of $\xi^*_{FL}(\theta_1)$[13] in order to generate a 6% exit rate after the adverse productivity shock. See Appendix C for more details on the calibration procedure.

The remaining parameters ($\underline{e}_0$, $\overline{e}_0$, $\mu$, $\sigma$) pertain to the credit constrained economy. Entrants are assumed to be uniformly distributed on $\{\theta_1, ..\theta_5\} \times [\underline{e}_0, \overline{e}_0]$. We choose $\underline{e}_0 = 0$ and $\overline{e}_0 = 5$ and verify that $\overline{e}_0$ is close to the highest level of net worth among incumbents firms. We report in Appendix D the sensitivity of the results to this assumption. We set the monitoring costs $\mu$ to match an average bankruptcy cost equal to 10% of capital. This value includes direct costs such as administrative and legal fees, but also indirect costs of bankruptcy linked to the efficiency of debt enforcement. Andrade and Kaplan (1998) estimate these costs to be between 10% and 20% of the firm's capital value. Concerning the dispersion of idiosyncratic shocks, we assume that these shocks are drawn from a normal distribution of mean zero and standard deviation $\sigma$ truncated on $[-\theta_0, +\theta_0]$ to avoid a negative production for very low levels of the idiosyncratic shock. Since we could not pin down the dispersion of idiosyncratic shocks from standard targets[14], we choose $\sigma = 0.2$ as the benchmark and present in Section 4.5 the results for alternative values of $\sigma$. We report the set of values of the benchmark calibration in Table 1.

[12] The exit rate is usually measured as the number of firms disappearing from a given dataset. The disappearance could be due to reasons unrelated to exit such as mergers, acquisition, restructuring, etc.

[13] $\xi^*_{FL}(\theta_1) = exit(Z') - 4/5 exit(Z)$ where $exit(Z)$ is the exit rate for a Z level of aggregate productivity.

[14] In particular, the default rate and risk premium appear to be weakly sensitive to $\sigma$: the increase in idiosyncratic risk raises the default probability of firms, but it also increases the exit rate of firms with a high default probability, which tends to reduce the average default rate.

Table 1: Benchmark calibration

| Parameter | Symbol | Value |
|---|---|---|
| Discount factor | $\beta$ | 0.9606 |
| Risk-free rate | $r$ | 0.04 |
| Depreciation rate | $\delta$ | 0.07 |
| Returns to scale | $\alpha$ | 0.70 |
| Aggregate productivity | $Z$ | 1 |
| Aggregate shock | $\Delta Z$ | -0.72% |
| Persistent productivity | $\theta_1, .., \theta_5$ | $0.2632, 0.2655, 0.2711, 0.2898, 0.4019$ |
| Exit productivity | $\theta_0$ | 0.2450 |
| Productivity distribution | $\xi^*_{FL}$ | $0.02, 0.05, 0.10, 0.30, 0.53$ |
| Fixed cost | $c$ | 0.49 |
| Idiosyncratic volatility | $\sigma$ | 0.20 |
| Monitoring cost | $\mu$ | 0.2127 |
| Net worth of entrants | $[\underline{e}_0, \overline{e}_0]$ | [0,5] |

## 4.2 Steady state capital and exit behavior

Figure 4 displays how credit constraints link the firms' capital choice to their level of net worth. A higher level of net worth relaxes the financial intermediary's participation constraint and allows the firm to expand its production scale[15]. Firms with a high level of net worth are not subject to credit constraints and can invest as much as in the frictionless case. Moreover, a creditworthy firm has a lower probability to default on its debt as well as a lower probability to exit the market. An increase in the persistent productivity $\theta$ produces similar effects on the exit rate, but barely affects the default rate. As expected, productivity and creditworthiness are highly correlated: an increase in the persistent level of productivity $\theta$ shifts the cumulative distribution of net worth to the right and raises the average net worth.

## 4.3 Imperfect selection and average productivity

We now consider a one standard deviation permanent fall in aggregate productivity ($\Delta Z = -0.72\%$) and simulate the impact response of the economy to this shock. In this section,

[15] For a low net worth, the capital function is not necessarily monotone. This is due to the non concavity of the firm's income (and hence of the value function) around the default threshold.

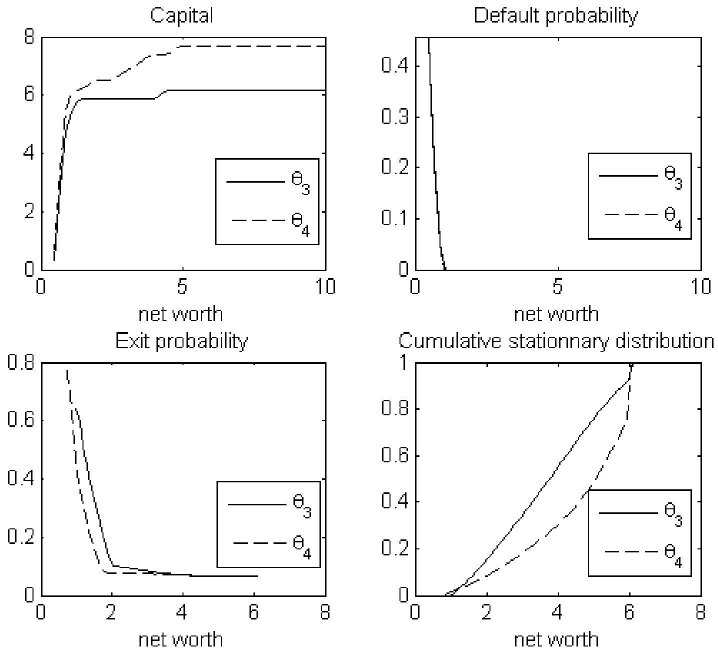


Figure 4: The firm's decision rules

we illustrate how credit constraints distort the selection at the exit margin and analyze its implication on average productivity.

As shown in Proposition 3, credit market frictions distort the productivity distribution of exiting firms. We capture the degree of the imperfect selection by computing the productivity gap between the most productive exiting firm and the less productive surviving firm. In the economy with credit market frictions, the productivity gap is equal to $\theta^{**}/\underline{\theta} - 1$. As this gap widens, the quality of the selection process deteriorates.

Table 2: Credit frictions and imperfect selection

| | Productivity gap | Average productivity |
|---|---|---|
| Frictionless | -0.9% | +0.49% |
| Credit frictions | 51.3% | +0.95% |

Note: the average productivity refers to the average of the productivity component $\theta$ among incumbents firms and is expressed in deviation from steady state.

Table 2 illustrates how the productivity gap is affected by credit market frictions. With credit frictions, some firms are forced to exit though their productivity is 51% higher than some surviving firms. In the frictionless economy, the productivity gap is slightly negative as exiting firms are strictly less productive than surviving firms. In a frictionless economy, the selection effect is usually associated with an increase in average productivity, as recessions reallocate resources from the least productive firms towards more productive ones. This is the intuition behind the cleansing effect of recessions, as emphasized in Caballero and Hammour (1994). Since credit frictions distort the selection mechanism, one could think that this distortion dampens the increase in average productivity. Nevertheless, the results reported in Table 2 indicate that this is not the case, as credit market frictions lead to a larger increase in productivity. Though credit market frictions lead to the exit of some high productivity firms, low productivity firms have a higher probability to exit. Since firms that exit are, on average, less productive than surviving firms, the fall in aggregate productivity tends to increase the average productivity of incumbent firms.

Table 3: Exit rate

| | Z=1 | Z =0.9928 | Change |
|---|---|---|---|
| Frictionless | 5.00% | 6.00% | 1.00 p.p. |
| Credit frictions | 5.51% | 8.00% | 2.49 p.p. |

The variation in the exit rate, shown in table 3, then explains why credit frictions induce a higher increase in idiosyncratic productivity than in the frictionless economy. The results reported in table 3 illustrate the increased destruction highlighted in Proposition 4 : the number of exiting firms in the credit constrained economy is higher than in the frictionless economy. More importantly, these results also indicate that credit market frictions amplify the fluctuations in the exit rate as suggested by Proposition 5. Credit market frictions increase the number of firms vulnerable to the aggregate productivity drop. In fact, the fall in aggregate productivity raises the exit rate by 2.49 percentage points in the economy with credit frictions versus a 1 percentage point increase in the frictionless economy. As credit market frictions amplify the increase in the number of exiting firms which are, on average less productive than surviving firms, they also amplify the increase in average idiosyncratic productivity.

### 4.4 Amplification at the exit margin

In this section, we investigate the consequences of the fluctuations in the exit rate on aggregate output. Our objective is to analyze how credit market frictions make the industry more vulnerable to a fall in aggregate productivity. In order to focus on this new mechanism, we leave out the effects of the endogenous change in net worth. In fact, the consequences of the fall in net worth are well known at the intensive margin and are likely to generate a similar mechanism at the extensive margin. We therefore consider the impact response of aggregate output, when the fall in aggregate productivity has not produced yet any effect on the firms' net worth. We decompose the impact of the decline in aggregate productivity ($Z$ drops to $Z'$) on aggregate output as follows:

$$\Delta Y = \underbrace{\Delta Z \int_{\underline{\theta}'}^{\theta_{\max}} \int_{\underline{e}'}^{\bar{e}'} \theta k'^{\alpha} d\xi^* +}_{\text{Direct effect}} \underbrace{Z \left[ \int_{\underline{\theta}'}^{\theta_{\max}} \int_{\underline{e}'}^{\bar{e}'} \theta k'^{\alpha} d\xi^* - \int_{\underline{\theta}'}^{\theta_{\max}} \int_{\underline{e}'}^{\bar{e}} \theta k^{\alpha} d\xi^* \right]}_{\text{Intensive margin}}$$
$$\underbrace{- Z \left[ \int_{\underline{\theta}'}^{\theta^{**}} \int_{\underline{e}}^{\underline{e}'} \theta k^{\alpha} d\xi^* + \int_{\underline{\theta}}^{\underline{\theta}'} \int_{\underline{e}}^{\bar{e}} \theta k^{\alpha} d\xi^* \right]}_{\text{Exit margin}}$$

The fall in aggregate productivity lowers the productivity of incumbents firms (*direct effect*), leading to a reduction in their investment (*intensive margin*), and inducing some firms that have become unprofitable to exit (*exit margin*)[16].

Table 4: A decomposition of the output loss upon impact

| | Aggregate production | Direct effect | Intensive margin | Exit margin |
|---|---|---|---|---|
| Frictionless | 3.09% | 0.70% | 1.66% | 0.70% |
| Credit frictions | 3.63% | 0.70% | 1.54% | 1.39% |

Table 4 reports the result of this output decomposition for the economy with credit frictions and the frictionless economy. The aggregate productivity drop causes a decline in output about 20% larger in the economy with credit frictions. The decomposition shows that the overall impact on output masks a larger effect at the exit margin. In the economy with credit frictions, the output loss induced by business shutdowns is twice as big as in the

[16]Note that the exit margin also includes the exit of potential entrants. The number of actual entrants also declines as fewer potential entrants find it profitable to stay in the market and immediately exit.

frictionless economy. In the latter, the exit margin generates a 0.7% loss in output while inducing a 1.39% loss when credit frictions are accounted for. By contrast, the output decomposition points to a dampening effect along the intensive margin. The intensive margin reduces output by 1.54% in the economy with credit frictions and by 1.66% in the frictionless economy. Firms reduce less their capital when they face tighter credit constraints. This is the outcome of two counteracting effects. On the one hand, the fall in aggregate productivity raises the financing costs of credit constrained firms, which tends to exacerbate the decline in investment. On the other hand, the marginal cost of financing also decreases as firms reduce the amount borrowed, which tends to dampen the impact of the aggregate shock. As reported in Table 4, the second effect dominates, and therefore leads to an overall dampening effect[17]. Following the fall in aggregate productivity, creditworthy firms decrease their investment further than firms with low net worth. This result extends to the comparison of the frictionless economy and the economy with credit frictions. For a given distribution of net worth, a fall in aggregate productivity reduces capital further in the frictionless economy. As highlighted by the standard financial accelerator, the intensive margin may however amplify aggregate fluctuations once the fall in net worth is accounted for. In that respect, the amplification at the exit margin is more robust than the financial accelerator as it does not hinge on the sensitivity of firms' net worth to aggregate shocks. We show that, when the balance sheet effect is shut down, credit frictions lead to a substantially larger output loss at the exit margin. Moreover, table 4 indicates that the exit margin accounts for a significant part of output fluctuations, especially in presence of credit frictions. In that case, the exit margin contributes as much as the intensive margin to the decline in aggregate output. All in all, these results suggest that the exit margin is an important channel for understanding the aggregate implications of credit market frictions.

### 4.5 The role of idiosyncratic productivity

In this section we show how the volatility of the non-persistent component of productivity $\epsilon$ affects the amplification mechanism at the exit margin. We use the benchmark parameters presented in section 4 and study the response of the economy to a fall in aggregate productivity when the idiosyncratic volatility $\sigma$ varies from 0.10 to 0.40. Table 5 shows that the fluctuations in aggregate output and average productivity are amplified when idiosyn-

[17]A similar dampening effect is at work in Carlstrom and Fuerst (1997). On impact, output respond less to an aggregate productivity fall in the economy with credit frictions. However, after the initial period, the decline in net worth raises the cost of borrowing and may reduce output further.

cratic volatility is high. An increase in idiosyncratic volatility exacerbates the sensitivity of the exit rate to aggregate productivity. Following the aggregate productivity drop, the negative output response is 10% larger than the frictionless economy when $\sigma = 0.10$, and 28% larger when $\sigma = 0.40$. For high levels of idiosyncratic volatility, the contribution of the exit margin to the output loss is more important than the contribution of the intensive margin. These results show that the interaction between credit market frictions and the volatility of idiosyncratic shocks is crucial for the amplification of aggregate shocks.

These results also illustrate the importance of idiosyncratic volatility for aggregate dynamics in the economy with credit market frictions. In the frictionless economy, the volatility of the non-persistent idiosyncratic productivity $\epsilon$ does not matter for aggregate fluctuations. The firm exit and capital decisions are driven by expected profits which are not affected by the non-persistent component of productivity. By contrast, the volatility of $\epsilon$ plays a key role when firms face credit constraints. A high idiosyncratic volatility reduces the expected income of the financial intermediary and raises the firm's financing costs and exit probability. The amplification at the exit margin therefore results from the interplay of credit market frictions and idiosyncratic volatility.

Table 5: The role of idiosyncratic volatility

| | Aggregate production | Direct effect | Intensive margin | Exit margin | Average productivity |
|---|---|---|---|---|---|
| $\sigma = 0.10$ | 3.39% | 0.70 % | 1.61% | 1.08% | 0.74 % |
| $\sigma = 0.15$ | 3.48% | 0.70 % | 1.54% | 1.24% | 0.85 % |
| $\sigma = 0.20$ | 3.63% | 0.70 % | 1.54% | 1.39% | 0.95 % |
| $\sigma = 0.25$ | 3.78% | 0.70 % | 1.53% | 1.55% | 1.06 % |
| $\sigma = 0.30$ | 3.81% | 0.70% | 1.42% | 1.69% | 1.16 % |
| $\sigma = 0.40$ | 3.92% | 0.70% | 1.55% | 1.67% | 1.14 % |

## 5 Conclusion

In this paper, we analyze the exit decision of firms subject to credit constraints, and then investigate its implication for the propagation of business cycle fluctuations. We show that, in the presence of credit frictions, the extensive margin is an important channel for the amplification of aggregate fluctuations: credit market frictions exacerbate the fluctuations in the exit rate by increasing the number of firms vulnerable to aggregate shocks. Unlike the standard financial accelerator, this new amplification mechanism does not depend on the sensitivity of firms' net worth to aggregate shocks. Note that the industry vulnerability and the financial accelerator mechanisms are complementary and an accurate estimate of the business cycle implications of credit frictions should take into account both mechanisms. This work is left for further research.

This paper also emphasizes how credit frictions distort the selection of exiting firms: high productivity but financial fragile firms may exit during recessions while low productivity and high net worth firms may survive. This distortion suggests that credit frictions lead to a inefficient reallocation of resources during recessions, as resources may flow from high to low productivity firms. Interestingly, this imperfect selection does not weaken the cleansing effect of recessions. On the contrary, the presence of credit frictions contributes to a higher increase in average idiosyncratic productivity during recessions. Finally, our results support the recent line of research (Bilbiie et al., 2007; Clementi and Palazzo, 2010) which takes into account firm dynamics in the study of business cycles. Our results suggest that the exit behavior of firms plays an important role in explaining the response of average productivity and aggregate output to business cycle shocks.

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# Appendix A

### Financial intermediary net worth threshold $\underline{e}_b$

Let us define the net income of the financial intermediary as $B(e, k, \bar{\epsilon})$ where

$$B(e, k, \bar{\epsilon}) = Z[\theta + G(\bar{\epsilon})]k^{\alpha} + (1 - \delta)k - \mu k^{\alpha}\Phi(\bar{\epsilon}) - (1 + r)(k + c - e).$$

The participation constraint of the financial intermediary is not satisfied when the firm's net worth is below $\underline{e}_b(\theta, Z)$, where this threshold is defined by:

$$B(\underline{e}_b, k_b, \bar{\epsilon}_b) = 0,$$

with $(k_b, \bar{\epsilon}_b)$ being the values of capital and default threshold that maximize the income of the financial intermediary. Note that $\bar{\epsilon}_b$ can be a corner solution depending on the shape of $\Phi$:

$$\bar{\epsilon}_b = \begin{cases} \epsilon_{\max} & \text{if } \frac{\Phi'}{\Phi}(\epsilon) \text{ is decreasing in } \epsilon \\ \text{such that } Z(1 - \Phi(\bar{\epsilon})) = \mu\Phi'(\bar{\epsilon}) & \text{otherwise.} \end{cases}$$

As we assume $\epsilon_{\min} > -\theta_{\min}$, $Z(G(\bar{\epsilon}_b) + \theta) - \mu\Phi(\bar{\epsilon}_b) > 0$ and the financial intermediary's income is a concave function of capital. Therefore $k_b$ is an interior solution:

$$k_b = \left( \frac{\alpha \left[ Z(G(\bar{\epsilon}_b) + \theta) - \mu\Phi(\bar{\epsilon}_b) \right]}{\delta + r} \right)^{\frac{1}{1-\alpha}}.$$

Since the income of the financial intermediary $B(e, k, \bar{\epsilon})$ is strictly increasing in the net worth $e$, there is a unique net worth threshold $\underline{e}_b$ such that $B(\underline{e}_b, k_b, \bar{\epsilon}_b) = 0$.

### PROOF of Proposition 1. Exit when the financial intermediary refuses the loan

The firm exits the market when the participation constraint of the financial intermediary is not satisfied, that is when the end-of-period net worth is too low $q < \underline{e}_b(\theta, Z)$. Indeed, we show that $e_b(\theta, Z) \leq c$, $\forall\theta \in [\theta_{\min}, \theta_{\max}]$ and therefore the firm which is rationed from the credit market cannot self finance its fixed operating cost.
Recall that $\underline{e}_b(\theta, Z)$ is defined by the following equation:

$$\max_{(k,\bar{\epsilon})} \left( Z[\theta + G(\bar{\epsilon})]k^{\alpha} - (\delta + r)k - \mu k^{\alpha}\Phi(\bar{\epsilon}) \right) = (1 + r)(c - \underline{e}_b)$$

Notice that $\max_{(k,\bar{\epsilon})}\left(Z[\theta+G(\bar{\epsilon})]k^{\alpha}-(\delta+r)k-\mu k^{\alpha}\Phi(\bar{\epsilon})\right)\geq 0$ since the financial intermediary can always choose $k=0$ and have 0. It follows that $c-\underline{e}_b(\theta,Z)\geq 0$.

## PROOF of existence and uniqueness of the value function

Consider the problem of the firm defined as:

$$T(V)(e,\theta,Z)=\max_{(k,\bar{\epsilon})\in\Gamma(e)}\mathbb{E}_{\theta}\left\{\int I(q)q+(1-I(q))\max\left[q,\max_{e'\in\Upsilon(q)}\left(q-e'+\beta V(e',\theta,Z)\right)\right]d\Phi(\epsilon)\right\}$$

with:

$$I(q)=\begin{cases}0 & \text{if } q\geq \underline{e}_b\\ 1 & \text{if } q<\underline{e}_b.\end{cases}$$

$$\Gamma(e)=\{(k,\bar{\epsilon})\in\mathbb{R}^{+}\times[\epsilon_{\min},\epsilon_{\max}]:Z[\theta+G(\bar{\epsilon})]k^{\alpha}+(1-\delta)k-\mu k^{\alpha}\Phi(\bar{\epsilon})\geq(1+r)(k+c-e)\}$$

$$\Upsilon(q)=\{e'\in\mathbb{R}:\underline{e}_b\leq e'\leq q\}$$

$$q=\max\left\{Zk^{\alpha}(\epsilon-\bar{\epsilon});0\right\}.$$

In the following, we prove that there exist a unique function $V$ that satisfies the functional equation $V=T(V)$ assuming the idiosyncratic productivity level $\theta$ is constant. The proof extends to non-permanent level of $\theta\in[\theta_{\min},\theta_{\max}]$.

First note that the participation constraint of the financial intermediary and Assumption 1 ($\beta(1+r)<1$) limits the space for net worth of continuing firms to $X=[\underline{e}_b(\theta,Z),\bar{e}(\theta,Z)]$. Because $\beta(1+r)<1$, entrepreneurs will not always reinvest their net worth in the firm, and will start distributing dividend when their net worth is sufficiently high. In particular, there exists a threshold $\bar{e}(\theta,Z)$ above which the firm will stop accumulating net worth.

We then show that the value of the continuing firm $V:X\rightarrow\mathbb{R}^{+}$, is necessarily bounded. The value of the firm is the discounted sum of the income from production and/or investing in the safe asset. As the decreasing returns to scale technology put an upper bound on the profits of the firm and Assumption 1 limits net worth accumulation, the value of the firm is bounded. This also means that the function resulting from the mapping $TV$ is bounded and $T$ maps the space of bounded functions $B(X)$ into itself. Then, we observe that the operator $T$ is a contraction since it satisfies the Blackwell conditions of monotonicity and discounting. The condition for discounting is verified as $\forall V \in B(X)$,

$$\begin{aligned} T(V+c) &= \max_{(k,\bar{\epsilon})\in\Gamma(e)} \left\{ \int I(q)q + (1-I(q)) \max\left[q, \max_{e'\in\Upsilon(q)} (q - e' + \beta V(e',\theta,Z) + \beta c)\right] d\Phi(\epsilon) \right\} \\ &\leq \max_{(k,\bar{\epsilon})\in\Gamma(e)} \left\{ \int I(q)q + (1-I(q)) \max\left[q + \beta c, \max_{e'\in\Upsilon(q)} (q - e' + \beta V(e',\theta,Z) + \beta c)\right] d\Phi(\epsilon) \right\} \\ &\leq TV + \beta c, \end{aligned}$$

where $c > 0$, and $0 < \beta < 1$ by definition.

Since $B(X)$ is a complete metric space (see for example, Godement (2001)), the Contraction Mapping Theorem applies and the operator $T$ has a unique fixed point $V$, which is bounded.

Let us show that $V$ is a continuous function. The participation constraint of the bank generates a discontinuity in the firm's end-of-period outcome. For $q < \underline{e}_b$, the value of the firm is simply $q$ and if $q = \underline{e}_b$, the firm can invest in production and obtains the value $\beta V(\underline{e}_b)$ which has no reason to coincide with $q$. However, we can show that, though the end-of period value of the firm is discontinuous, the expectation of this value is a continuous function of the threshold $\frac{\underline{e}_b}{Zk^\alpha} + \bar{\epsilon}$ below which the firm cannot borrow from the financial intermediary. This appears clearly when rewriting the value function as follows:

$$V(e,\theta,Z) = \max_{(k,\bar{\epsilon})\in\Gamma(e)} \left\{ \int_{\epsilon_{\min}}^{\frac{\underline{e}_b}{Zk^\alpha}+\bar{\epsilon}} q d\Phi(\epsilon) + \int_{\frac{\underline{e}_b}{Zk^\alpha}+\bar{\epsilon}}^{\epsilon_{\max}} \max\left[q, \max_{e'\in\Upsilon(q)} \left(q - e' + \beta V(e',\theta,Z)\right)\right] d\Phi(\epsilon) \right\}.$$

Despite the discontinuity at $\underline{e}_b$ in the end-of-period value of the firm, the discontinuity disappears in the objective function of the continuing firm. Furthermore, note that if $V$ is a continuous function, then the objective function of the firm deciding its next period net worth $e'$ is also continuous. Because the correspondence $\Upsilon(q)$ that describes the feasibility constraint for $e'$ is non-empty, continuous and compact-valued, the theorem of the maximum ensures that the maximum exists and the function resulting from this dividend choice is continuous. As the threshold $\frac{\underline{e}_b}{Zk^\alpha} + \bar{\epsilon}$ below which the firm cannot borrow from the financial intermediary is a continuous function of $e$, the objective function of the firm deciding its capital level is the sum of two continuous functions and is therefore a continuous function of $e$. Using again the theorem of the maximum, we can finally show that the function resulting from the mapping $T(V)(e)$ is continuous since the correspondence $\Gamma$ that describe the feasibility constraint for $k$ and $\bar{\epsilon}$ is non-empty, continuous[18]. and compact-valued. This means that $T$ maps the space of continuous and bounded functions into itself, $T : C(X) \to C(X)$. As $C(X)$ is a closed subset of the complete metric space of bounded functions $B(X)$, the fixed point $V$ is a continuous function by the corollary of the contraction mapping theorem[19].

Let us now characterize more precisely the value function $V$. Notice that $\Upsilon$ and $\Gamma$ are increasing correspondences: $q_1 \leq q_2$ implies $\Upsilon(q_1) \subseteq \Upsilon(q_2)$ and $e_1 \leq e_2$ implies $\Gamma(e_1) \subseteq \Gamma(e_2)$. A higher net worth $e$ relaxes the credit constraint of the firm and allows the firm to reach a higher end-of-period net worth $q$. This means that the period return function $q$ is strictly increasing in e. This implies that $T$ maps the space of bounded continuous and strictly increasing functions into itself. As this is a closed subset of the space of bounded functions $B(X)$, $V$ is a strictly increasing function of $e$. By the same reasoning, we can show that $V$ is also strictly increasing in $\theta$[20].

To further characterize the value function, let us write the Lagrangian of the firm's problem:

$$\mathcal{L}(k, \bar{\epsilon}, \lambda) = \int_{\epsilon_{\min}}^{\frac{e_b}{Zk^\alpha}+\bar{\epsilon}} [Zk^\alpha(\epsilon - \bar{\epsilon})] \, d\Phi(\epsilon)$$

$$+ \int_{\frac{e_b}{Zk^\alpha}+\bar{\epsilon}}^{\epsilon_{\max}} \max\left\{Zk^\alpha(\epsilon - \bar{\epsilon}), \max_{e' \in \Upsilon(q)} Zk^\alpha(\epsilon - \bar{\epsilon}) - e' + \beta V(e', \theta, Z)\right\} d\Phi(\epsilon)$$

$$+\lambda g(k, \epsilon, e)$$

with $g(k, \bar{\epsilon}, e) = \big(Z[\theta + G(\bar{\epsilon})]k^\alpha + (1-\delta)k - \mu k^\alpha \Phi(\bar{\epsilon}) - (1+r)(k + c - e)\big)$.

In the following, we assume that $V$ is differentiable. We can then compute its second order derivative to show that $V$ is a concave function. Using the envelop theorem, we can write:

$$\frac{\partial^2 V}{\partial e^2} = \frac{\partial \lambda}{\partial e}(1 + r)$$

[18]The continuity of $\Gamma$ derives from the continuity of the participation constraint of the bank $B(e, k, \bar{\epsilon})$ and from the uniqueness of $\bar{\epsilon}_b$ guaranteed by Assumption 2.

[19]See Corollary 1 of Theorem 3.2 in Stokey et al. (1989).

[20]In the case of stochastic productivity $\theta$, we further assume that the transition function $F(\theta'|\theta)$ is strictly decreasing in $\theta$.

We then take the total differential of the financial intermediary's participation constraint with respect to $\lambda$ and $e$:

$$\frac{d\lambda}{de} = -\frac{1+r}{\frac{\partial}{\partial\lambda} g(k(\lambda), \epsilon(\lambda), e)}$$

Applying the implicit function theorem to the first order optimality conditions, we can write:

$$\frac{\partial}{\partial\lambda} g(k(\lambda), \bar{\epsilon}(\lambda), e) = -\begin{pmatrix} \frac{\partial g}{\partial \bar{\epsilon}} \\ -\frac{\partial g}{\partial k} \end{pmatrix}' H \begin{pmatrix} \frac{\partial g}{\partial \bar{\epsilon}} \\ -\frac{\partial g}{\partial k} \end{pmatrix},$$

where $H$ is the hessian matrix of the Lagrangian function[21]. As long as the second order optimality conditions of the firm problem are satisfied, $H$ is negative definite. Then $\frac{\partial}{\partial\lambda} g(k(\lambda), \bar{\epsilon}(\lambda), e) > 0$ and $V$ is a concave function of $e$.

The concavity of the firm value function also allows us to characterize the dividend decision of the firm. There exists a unique threshold $\bar{e}(\theta, Z)$ above which the firm decides to distribute some dividends. This optimal threshold is given by:

$$\beta \frac{\partial V}{\partial e}(\bar{e}, \theta, Z) = 1. \tag{A.1}$$

Therefore, the optimal next period net worth is $e' = \min[q, \bar{e}(\theta, Z)]$.

**PROOF of Proposition 2. Exit conditions**

Under Assumptions 1 and 2, let us show that for a given $Z$, the thresholds $\underline{\theta}(Z)$, $\theta^*(Z)$ and $\theta^{**}(Z)$ exist and are unique.

- No exit threshold $\theta^{**}$:

  We have already shown that firms exit the market when the participation constraint of the financial intermediary is not satisfied. However, for some high productivity firms the participation constraint of the financial intermediary is always satisfied. The financial intermediary accepts to lend to firms with $\theta \geq \theta^{**}$ whatever their level of net worth. Therefore, these firms never exit because they are not sufficiently creditworthy. The no exit threshold $\theta^{**}$ is characterized by:

$$Z\left[\theta^{**} + G(\bar{\epsilon}_b)\right] k_b^{\alpha} - \mu k_b^{\alpha} \Phi(\bar{\epsilon}_b) - (\delta + r) k_b - (1+r)c = 0. \tag{A.2}$$

[21] $H = \begin{pmatrix} \frac{\partial \mathcal{L}^2}{\partial \bar{\epsilon}^2} & \frac{\partial \mathcal{L}^2}{\partial \bar{\epsilon} \partial k} \\ \frac{\partial \mathcal{L}^2}{\partial \bar{\epsilon} \partial k} & \frac{\partial \mathcal{L}^2}{\partial k^2} \end{pmatrix}$

Let us show that this threshold is unique. Denote $\hat{\theta} = \frac{\mu}{Z}\Phi(\bar{\epsilon}_b) - G(\bar{\epsilon}_b)$, the level of productivity below which the net income of the financial intermediary is decreasing in $k$. The left hand side of the no exit threshold condition is strictly increasing in $\theta$ for all $\theta > \hat{\theta}$. Furthermore, as this expression is negative for $\theta = \hat{\theta}$, this implies that the threshold $\theta^{**}$ is unique.

- Credit market exit threshold $\theta^*$ :

  Firms also exit if their participation constraint is not satisfied. Using the optimal dividend decision (Equation A.1), firms exit when:

$$e' > \beta V(e', \theta, Z), \qquad \text{with } e' = \min[q, \bar{e}(\theta, Z)].$$

  Note that if the firm is sufficiently productive, their participation constraint is always satisfied. Recall $V(e, \theta, Z)$ is defined on $[\underline{e}_b(\theta, Z), \bar{e}(\theta, Z)]$. Then, if $\underline{e}_b(\theta, Z) < \beta V(\underline{e}_b, \theta, Z)$, the firm finds it profitable to stay in the market whatever its level of net worth. The productivity threshold $\theta^*$ above which firms always satisfy their participation constraint is characterized by:

$$\underline{e}_b(\theta^*, Z) = \beta V(\underline{e}_b(\theta^*, Z), \theta^*, Z). \tag{A.3}$$

  Above this threshold, firms exit only when the participation constraint of the financial intermediary is not satisfied. To show that this threshold is unique, we start with the fact that a firm with productivity $\theta_0 \leq \underline{\theta}_{FL}$ is not profitable and therefore exits the market in the frictionless economy but also in the credit constrained economy (as the financing cost faced by the firm is higher in this case). Therefore: $\underline{e}_b(\theta_0, Z) > \beta V(\underline{e}_b(\theta_0, Z), \theta_0, Z)$. We now need to show that $\underline{e}_b(\theta, Z)$ is a decreasing function and $V(\underline{e}_b(\theta, Z), \theta, Z)$ a strictly increasing function of $\theta$.

  Recall that the threshold $\underline{e}_b(\theta, Z)$ below which the financial intermediary refuses to loan any funds is defined by:

$$Z[\theta + G(\bar{\epsilon}_b)]k_b^\alpha + (1-\delta)k_b - \mu k_b^\alpha \Phi(\bar{\epsilon}_b) = (1+r)(k_b + c - \underline{e}_b).$$

  Taking the total differential of this equation indicates that firms with a high productivity are less frequently rationed from the market: $d\underline{e}_b/d\theta \leq 0$.

  We now need to show that $V(\underline{e}_b(\theta), \theta, Z)$ is increasing in $\theta$.

Using the envelop theorem, it comes:

$$\begin{aligned}
\frac{\partial V}{\partial e} = \frac{\partial \mathcal{L}}{\partial e} &= \lambda(1+r) \\
\frac{\partial V}{\partial \theta} = \frac{\partial \mathcal{L}}{\partial \theta} &= \lambda Z k^{\alpha} + \beta \int_{\frac{e_b}{Zk^{\alpha}}+\bar{\epsilon}}^{\epsilon_{\max}} \frac{\partial V}{\partial \theta}(e', \theta, Z) \mathbb{1}_{e' < \beta V(e', \theta, Z)} d\Phi(\epsilon) \\
&\quad + \frac{1}{Zk^{\alpha}} \frac{d\underline{e}_b}{d\theta} \left(\underline{e}_b - \beta V(\underline{e}_b, \theta, Z)\right) \Phi'\left(\frac{\underline{e}_b}{Zk^{\alpha}} + \bar{\epsilon}\right) \mathbb{1}_{\underline{e}_b < \beta V(\underline{e}_b, \theta, Z)}.
\end{aligned}$$

Then, it follows that $V(\underline{e}_b(\theta), \theta, Z)$ is strictly increasing in $\theta$,

$$\begin{aligned}
\frac{dV}{d\theta}(\underline{e}_b(\theta, Z), \theta, Z) &= \frac{\partial V}{\partial e}(\underline{e}_b, \theta, Z) \frac{d\underline{e}_b}{d\theta} + \frac{\partial V}{\partial \theta}(\underline{e}_b, \theta, Z) \\
&= -\lambda(1+r) \frac{Zk_b^{\alpha}}{1+r} + \lambda Z k_b^{\alpha} + \beta \int_{\frac{e_b}{Zk^{\alpha}}+\bar{\epsilon}}^{\epsilon_{\max}} \frac{\partial V}{\partial \theta}(e', \theta, Z) \mathbb{1}_{e' < \beta V(e', \theta, Z)} d\Phi(\epsilon) \\
&\quad + \frac{1}{Zk_b^{\alpha}} \frac{Zk_b^{\alpha}}{1+r} \left(\beta V(\underline{e}_b, \theta, Z) - \underline{e}_b\right) \Phi'\left(\frac{\underline{e}_b}{Zk_b^{\alpha}} + \bar{\epsilon}\right) \mathbb{1}_{\underline{e}_b < \beta V(\underline{e}_b, \theta, Z)} \\
&> 0,
\end{aligned}$$

where the last line follows from the fact that $V$ is strictly increasing in $\theta$.

- Full exit threshold $\underline{\theta}$ :

  Low productivity firms always exit whatever their level of net worth. We can find a productivity threshold $\underline{\theta}$ below which the participation constraint of the firm is never satisfied. As $e' \leq \bar{e}(\underline{\theta}, Z)$ if $\bar{e}(\underline{\theta}, Z) = \beta V(\bar{e}(\underline{\theta}, Z), \underline{\theta}, Z)$ the firm never finds it profitable to stay in the market. The threshold $\underline{\theta}$ is therefore defined as:

$$\bar{e}(\underline{\theta}, Z) = \beta V(\bar{e}(\underline{\theta}, Z), \underline{\theta}, Z). \tag{A.4}$$

  Firms with productivity $\theta_0 \leq \underline{\theta}_{FL}$ are not profitable and therefore exit the market : $\bar{e}(\theta_0, Z) > \beta V(\bar{e}(\theta_0, Z), \theta_0, Z)$. We complete the proof by showing that the dividend threshold $\bar{e}$ increases with $\theta$ less than the value function. Using the dividend decision condition (Equation A.1), we can show:

$$\begin{aligned}
\beta \frac{dV}{d\theta}(\bar{e}(\theta, Z), \theta, Z) &= \beta \frac{\partial V}{\partial e}(\bar{e}, \theta) \frac{d\bar{e}}{d\theta} + \beta \frac{\partial V}{\partial \theta}(\bar{e}, \theta) \\
&= \frac{d\bar{e}}{d\theta} + \beta \frac{\partial V}{\partial \theta}(\bar{e}, \theta) \\
&> \frac{d\bar{e}}{d\theta}.
\end{aligned}$$

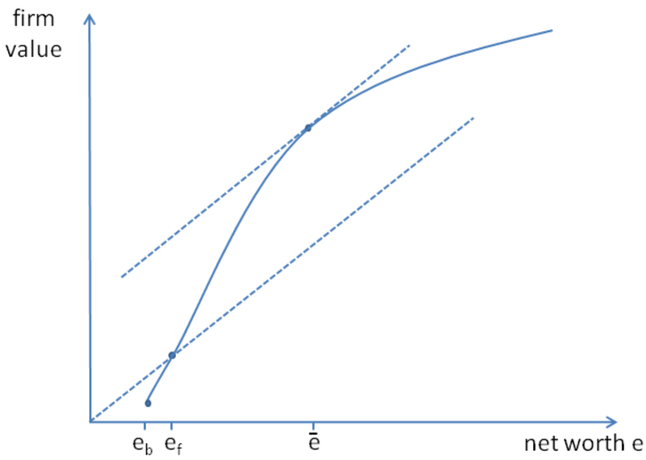


Figure A.1: Firms' net worth threshold $\underline{e}_f$

**Firms' net worth threshold $\underline{e}_f$**

For $\underline{\theta} \leq \theta < \theta^*$, firms exit if their participation constraint is not satisfied: they exit if $e < \underline{e}_f(\theta, Z)$ with $\underline{e}_f(\theta, Z)$ defined by:

$$\underline{e}_f = \beta V(\underline{e}_f, \theta, Z). \tag{A.5}$$

Given $\theta$, we show that this threshold is unique by observing that $\beta \frac{dV(e,\theta,Z)}{de} > 1$ as long as $e < \bar{e}(\theta, Z)$ and $\underline{e}_b(\theta, Z) > \beta V(\underline{e}_b(\theta), \theta, Z)$ for any $\underline{\theta} \leq \theta < \theta^*$.

Furthermore we can show that the exit threshold $\underline{e}_f$ is decreasing in $\theta$ as $\frac{d\underline{e}_f}{d\theta} = \frac{\beta \frac{\partial V}{\partial \theta}}{1 - \beta \frac{\partial V}{\partial e}}$.

Figures A.1, A.2 and A.3 illustrate how the thresholds $\underline{e}_f$, $\underline{\theta}$ and $\theta^*$ are determined.

**PROOF of Propositions 3, 4 and 5**:

The no exit threshold $\theta^{**}(Z)$, the credit market exit threshold $\theta^*(Z)$ and the full exit threshold $\underline{\theta}(Z)$ are defined respectively by Equations (A.2), (A.3) and (A.4). Taking the total differential of Equations (A.2), (A.3), (A.4) and (A.5) yields the following results:

$$\frac{d\underline{\theta}}{d\mu} = -\frac{\frac{\partial V}{\partial \mu}(\bar{e}, \underline{\theta}, Z)}{\frac{\partial V}{\partial \theta}(\bar{e}, \underline{\theta}, Z)} > 0;$$

$$\frac{d\underline{e}_b}{d\mu} = \frac{\Phi(\bar{\epsilon}_b) k_b^\alpha}{1 + r} > 0;$$

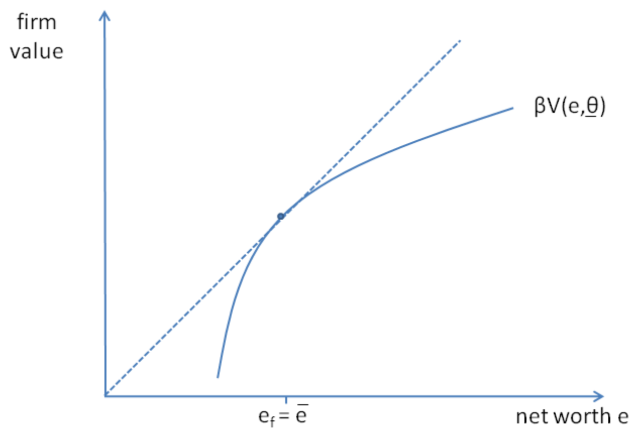


Figure A.2: Productivity threshold $\underline{\theta}$

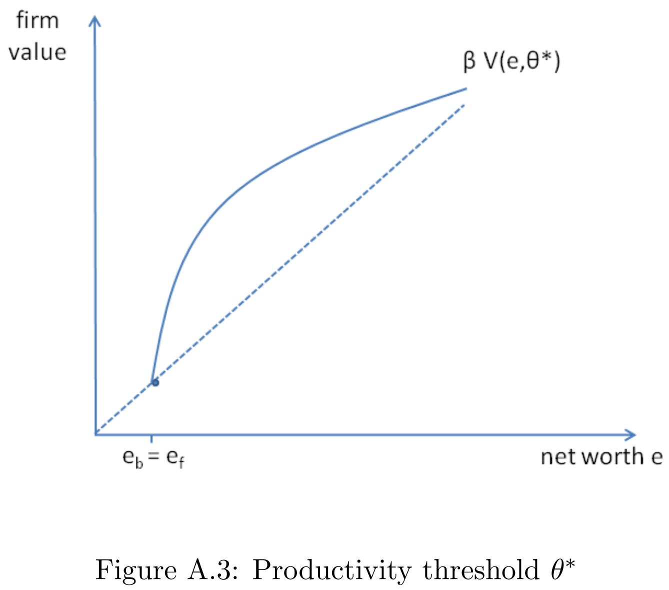


Figure A.3: Productivity threshold $\theta^*$

$$\frac{d\theta^{**}}{d\mu} = \Phi(\bar{\epsilon}_b) > 0;$$

$$\frac{d\underline{e}_f}{d\mu} = \beta \frac{\frac{\partial V}{\partial \mu}(\underline{e}_f, \theta, Z)}{1 - \beta \frac{\partial V}{\partial e}(\underline{e}_f, \theta, Z)} > 0.$$

## Appendix B: Numerical procedure

The model is solved using value function iteration on the discretized state space, using splines to approximate between grid points.

1. Discretize the shocks $\epsilon$.
2. Start by choosing a grid for the net worth of firms with productivity $\theta_1$. The lower point is initialized at $\max\{\underline{e}_b(Z,\theta_1), 0\}$ and the upper bound at a guess $\bar{e}$.
3. Choose a grid for $k$. The lower point is initialized at 0 and the upper bound at the frictionless level.
4. Compute for each $e$, the maximum level of capital that the firm can borrow $k_{\max}(e)$, with $k_{\max}(e)$ defined by:

   $$Z[\theta + G(\bar{\epsilon}_b)]k_{\max}^{\alpha} + (1-\delta)k_{\max} - \mu k_{\max}^{\alpha}\Phi(\bar{\epsilon}_b) = (1+r)(k_{\max} + c - e)$$

   where $\bar{\epsilon}_b$ maximizes the income of the financial intermediary.
5. Compute the default threshold $\bar{\epsilon}(k,e)$ for $k < k_{\max}(e)$
6. For each $\epsilon$, compute the end-of-period net worth $q = (\epsilon - \bar{\epsilon}(k,e))Zk^{\alpha}$
7. Compute the solution of the static problem to be used as the initial point for the value function $V^0(e,\theta,Z)$.
8. Repeat steps 2 to 7, for the other levels of productivity $\theta$.
9. For each $\epsilon$, each $\theta'$ and each $k$, solve for the optimal exit and dividend decision $d$:

   $$\max_d d + \beta\left\{\max(1+r)(q-d); V^0(q-d, Z, \theta')\right\}$$

10. Repeat the last step for the other levels of productivity $\theta$.
11. Solve for the optimal capital and compute the corresponding value function :

    $$V^1(e,\theta,Z) = \max_k \mathbb{E}_\theta \left\{ \int_{\epsilon_{\min}}^{\epsilon_{\max}} \max_d \left[d + \beta \max\left\{(1+r)(q-d), V^0(q-d,\theta',Z)\right\}\right] d\Phi(\epsilon) \right\}$$

12. Update the guess with this new found value function. Iterate step 9 to 11 until convergence.

## Appendix C: Calibration of the transition matrix

The transition matrix Q is chosen to match the distribution of steady state productivity $\xi^*_{FL}$ and a steady state exit rate of 5%. We also constrain $F(\theta'|\theta)$ to be decreasing in $\theta$. Note that these conditions do not pin down a unique value for Q.

The transition matrix is obtained as follows:

1. Find the values that solve $\xi^*_{FL} = Q\xi^*_{FL} + \psi_{FL}e$ where $e$ is a column vector with unit elements and $\psi_{FL} = 0.05/5$ consistent with the stationarity (exit rate=entry rate=0.05) and the uniform entry assumptions. The solution to this system is not unique, and the set of solutions can be characterized by adding the linear combination of the basis vector of the homogenous equation to a particular solution of the full equation.

2. The set of solution found in step 1 is then reduced to keep only the solutions for which:

- $F(\theta'|\theta) \geq 0$
- $\sum_{\theta'} F(\theta'|\theta) = 1$
- $F(\theta'|\theta)$ is decreasing in $\theta$

## Appendix D: Alternative calibration

In this appendix, we check the sensitivity of the results to an alternative assumption about the distribution of the potential entrants. In particular, we report the case in which potential entrants have a higher average net worth, and assume that they are uniformly distributed over $[\theta_1, \theta_5] \times [0, 10]$.

Table A.1: Net worth of entrants uniformly distributed over$[\theta_1, \theta_5] \times [0, 10]$

| | Aggregate production | Direct effect | Intensive margin | Exit margin | Productivity increase |
|---|---|---|---|---|---|
| $\sigma = 0.05$ | 3.21% | 0.70 % | 1.65% | 0.85% | 0.44 % |
| $\sigma = 0.10$ | 3.25% | 0.70 % | 1.63% | 0.91% | 0.47 % |
| $\sigma = 0.15$ | 3.27% | 0.70 % | 1.58% | 0.99% | 0.66 % |
| $\sigma = 0.20$ | 3.38% | 0.70 % | 1.59% | 1.09% | 0.73 % |
| $\sigma = 0.25$ | 3.54% | 0.70 % | 1.59% | 1.24% | 0.83 % |
| $\sigma = 0.30$ | 4.38% | 0.69% | 1.45% | 2.25% | 1.53 % |
| $\sigma = 0.35$ | 4.50% | 0.69% | 1.56% | 2.24% | 1.52 % |
| $\sigma = 0.40$ | 4.53% | 0.69% | 1.60% | 2.24% | 1.52 % |