Education Economics

Education, life expectancy and family bargaining: the Ben-Porath effect revisited

Laura Leker\textsuperscript{a} & Gregory Ponthiere\textsuperscript{a}

\textsuperscript{a} Paris School of Economics, 48 Boulevard Jourdan, Paris 75014, France

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Education, life expectancy and family bargaining: the Ben-Porath effect revisited

Laura Leker and Gregory Ponthiere*

Paris School of Economics, 48 Boulevard Jourdan, Paris 75014, France

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Following Ben-Porath [1967. “The Production of Human Capital and the Life-Cycle of Earnings.” *Journal of Political Economy* 75 (3): 352–365], the influence of life expectancy on education and on human capital has attracted much attention among growth theorists. Whereas existing growth models rely on an education decision made either by the child or by his parent, we revisit the Ben-Porath effect by modelling education as the outcome of bargaining between the parent and the child. We develop a three-period overlapping generations (OLG) model, where human capital increases life expectancy and shows that as a result of the unequal remaining lifetimes faced by parents and children, the form of the Ben-Porath effect depends on how bargaining power is distributed within the family, which in turn affects long-run economic dynamics. Using data on 16 OECD countries (1940–1980), we show that introducing family bargaining helps to rationalize the observed education patterns across countries.

**Keywords:** education; life expectancy; family bargaining; OLG model

**JEL Classification:** D13; J10; O41

1. **Introduction**

Following the pioneer works by Lucas (1988) and Romer (1990), human capital accumulation is now regarded as a major determinant of economic growth. As it is widely acknowledged, the human capital accumulation process is strongly related to demographic trends, concerning both mortality and fertility (Ehrlich and Lui 1991; Boucekkine, de la Croix, and Licandro 2002). On the mortality side, a major link was emphasized by Ben-Porath (1967). The so-called Ben-Porath effect states that, when life expectancy increases, lifetime returns on education investment tend, in general, to increase, leading to a rise in the education level chosen by individuals.

In accordance with the Ben-Porath hypothesis, we observe, for most countries, a positive correlation between life expectancy and education. To illustrate this, Figure 1 presents, for five cohorts (born between 1940 and 1980) and five countries, period life expectancies at age 20 (source: Human Mortality Database 2012) and average years of education (per cohort) (source: Cohen and Leker 2013). The correlation between the two variables is unambiguously positive.

Note that, although the Ben-Porath mechanism is a simple way to rationalize the observed patterns, alternative explanations exist. For instance, the positive correlation

*Corresponding author. Email: gregory.ponthiere@ens.fr

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between education and life expectancy may result from a reverse causal chain: better education can trigger higher longevity. There may also be a third, omitted variable, determining jointly education and health outcomes. But even if one abstracts from those identification problems, the observed relationship between education and longevity is far from trivial. Indeed, Figure 1 displays increasing relationships between life expectancy and education, but with various slopes. A given gain in life expectancy can be associated with education gains of various sizes. The education–longevity relationship, although monotonic, turns out to exhibit various patterns, depending on the country and the period under study. All this explains why the Ben-Porath effect, although widely used by growth theorists, finds mitigated empirical support, and as such, invites some refinements on the theoretical side.

We propose to revisit the Ben-Porath effect, by making alternative assumptions on the education decision. Existing models suppose that the education decision is made by a single agent: either the parent (Ehrlich and Lui 1991) or the child (de la Croix and Licandro 2013). However, the family is a collective decision unit, and education is not the outcome of a single individual’s decision. An abundant literature has pointed out the impact of family bargaining on various outcomes, such as time allocation and education in Konrad and Lommerud (2000), or education and fertility in de la Croix and Vander Donkt (2010). Following these works, we propose to construct a model where education results from intrafamily bargaining, and we examine the effect of the distribution of bargaining power on the Ben-Porath effect. More precisely, the education decision is modelled here as the outcome of intergenerational bargaining, i.e. bargaining between parents and children.
In this paper, we develop a three-period overlapping generations (OLG) model, where human capital accumulation results from an education investment decided through a bargaining process between parents and children. In this framework, agents educate themselves to benefit from higher wages in the future, while parents enjoy coexistence with educated children. We first characterize the optimal education from the point of view, respectively, of the child and of his parent, and, then, derive the education level resulting from family bargaining. We show how education varies with the distribution of bargaining power in the family. Then, we analyse the long-run dynamics, when mortality is endogenized, in order to take into account the double causal link between longevity and human capital. In our model, agents do not directly choose their own life expectancy, but human lifetime is endogenously determined by the human capital accumulation process, to which all past cohorts contributed through their education investments.

Our model shares with Cervellati and Sunde (2005) and de la Croix and Licandro (2013) the refining of the Ben-Porath mechanism by endogenizing mortality, which allows a positive feedback loop between human capital and longevity. But these models are based on the simple Ben-Porath mechanism, where individuals decide alone on their own education, contrary to our model where parent’s preferences affect the education decision. Our model shares with Ehrlich and Lui (1991) the time-horizon effect of parents’ longevity on children’s education, in the context of egoistic parents’ decisions for children’s education. But contrary to us, Ehrlich and Lui (1991) consider that the education decision is made only by the parent. Finally, Soares (2005) takes into account both the parent’s and the child’s decisions with respect to human capital investment by distinguishing early education, which is within the parent’s province, and high education, which is within the child’s. We differ from his approach by considering education as a collective decision, resulting from intrafamily bargaining.

Anticipating our results, we first study the determinants of the disagreement between children and parents as far as education investment is concerned. We show that the disagreement is due to: (i) differences in the (remaining) time horizons between parents and children, and (ii) differences in the motivation for children’s education between parents and children. In a second stage, we study the effects of the distribution of bargaining power in the family on long-run dynamics, and show that, if children want more education than what their parents are willing to invest, economies with high parental bargaining power are more likely to be trapped in poverty. We also consider an extended model, where the distribution of bargaining power in the family depends on human capital accumulation, and consider two cases: children emancipation thanks to human capital accumulation and parental authority reinforced. We show how the relation between knowledge and power affects the long-run dynamics of the economy. Finally, we propose an empirical application of the model on 16 OECD countries (1940–1980), and show how the introduction of family bargaining among agents heterogeneous in terms of age – i.e. children and parents – helps to rationalize the various observed patterns of education and life expectancy across countries.

The rest of the paper is organized as follows. Section 2 presents the baseline model and describes the education decision as the outcome of family bargaining. Section 3 examines the long-run dynamics of the economy. Section 4 endogenizes the distribution of bargaining power. Section 5 illustrates, by means of data on 16 OECD countries (1940–1980), how the family bargaining model can replicate patterns of education and life expectancy. Section 6 concludes.
2. The basic model

2.1. Environment

Let us consider a three-period OLG model. All periods are of unitary length. Each cohort is a continuum of agents, with a measure normalized to 1. There is an implicit period of childhood not presented in the model, so that the first period is a period of young adulthood. Reproduction is asexual, and individuals give birth to one child at the beginning of the first period.

All agents live the first period of life (young adulthood). This consists of a period during which individuals divide their time between work and education for themselves, with the help of the previous generation. All agents live the second period of life (old adulthood). This is a period during which individuals work, consume and devote time to educate their child.

However, not all agents will reach the third period: only a proportion $\pi_{t+2}$ of a cohort of young adults at $t$ will enjoy the third period of life. During this third period, agents work, consume and enjoy the companionship of their – more or less educated – children.

The survival probability to the third period $\pi_{t+2}$ is increasing in the human capital agents enjoyed when being educated adults (second period). The probability of survival to the third period of life of a person who is a young adult at $t$, denoted $\pi_{t+2}$, depends on the stock of human capital $h_{t+1}$ by means of the survival function

$$\pi_{t+2} = \pi(h_{t+1}),$$  \hspace{1cm} (1)

where $\pi(\cdot)$ exhibits the following properties: $\pi(\cdot) > 0$, $\pi'(\cdot) > 0$ and $\pi''(\cdot) < 0$. We also assume that $\pi(\cdot)$ is bounded from below and from above: $\lim_{h \to 0} \pi(h) = \pi$, $0 < \pi < 1$, and $\lim_{h \to \infty} \pi(h) = \bar{\pi}$, $0 < \pi < \bar{\pi} < 1$.

2.2. Production and human capital accumulation

For simplicity, production is assumed to be linear in human capital

$$y_t = wh_t,$$  \hspace{1cm} (2)

where $y_t$ denotes the output, $w$ is the wage per unit of human capital and $h_t$ the stock of human capital. For the sake of the presentation, we will normalize $w$ to 1.

The human capital of an individual who is a young adult at time $t$ equals $h_t$, i.e. the human capital inherited from his parent. Then, at old adulthood, he enjoys a human capital level $h_{t+1}$, which depends on past human capital $h_t$ and on the time investment in education $e_t$. The returns on education investment take the following form:

$$h_{t+1} = h(e_t) = Ah_t e_t^\alpha,$$  \hspace{1cm} (3)

where $e_t$ is the education investment, $A$ a productivity parameter ($A > 0$), while $\alpha$ the elasticity of future human capital to education. Following the literature, we assume decreasing marginal returns to education ($0 < \alpha < 1$).
2.3. Education decision

Whereas existing models assume that either the child or the parent chooses the education investment, we assume in this model that both the parent and the child take part in the education decision. For simplicity, the education investment has a temporal form, and involves both the parent and the child: they must spend together a fraction of their life period to improve the child’s human capital.

The expected lifetime welfare of a young adult agent at time $t$ takes the following form:

$$EU_t = \log(c_t) + \log(c_{t+1}) + \pi_{t+2} \log(c_{t+2}) + \pi_{t+2} \gamma \log(e_{t+1}),$$

where $c_t$ is the consumption at time $t$, while $e_{t+1}$ the education investment of the agent’s child. The parameter $\gamma$ captures the parental taste for his child’s education ($\gamma > 0$). That kind of parental taste for having educated children is widespread in the existing literature. For instance, Ehrlich and Lui (1991) allow, within what they call ‘companionship functions’, parents to derive utility from coexisting with highly educated children rather than uneducated children. Our assumption is in line with such ‘companionship functions’. It explains why that term is weighted, in parent’s lifetime utility function, by the survival probability $\pi_{t+2}$, since coexistence is possible only if the parent is still alive at that time.9

Note that there is a priori no reason why the parent and the child would like to choose the same education investment for the child, since the parental valuation of the child’s education lies in the companionship with an educated child, while, for the child himself, the value of education comes from the higher wage he will get in return. The reasons for a potential disagreement within the family will be studied in detail below. As we will see, a major source of disagreement lies in the difference between the remaining lifetime horizons of the parent and the child. Education investment concerns the future, but the future lifetime is much shorter for the parent than for the child, and this causes a disagreement on the choice of education.

A parent young adult at $t$ and a child young adult at $t+1$ will reach an agreement on the time to devote to the child’s education thanks to bargaining at the beginning of the $t+1$ period. Hence, formally, the education investment is assumed to be the outcome of bargaining, with a bargaining power $\varepsilon$ to the parent, and $1 - \varepsilon$ to the child. Thus the education investment resulting from the bargaining process is the solution of the following maximization problem:

$$\max_{e_{t+1}} \varepsilon EU_t + (1 - \varepsilon)EU_{t+1},$$

where $EU_t$ is the expected lifetime welfare of the parent, who was a young adult at $t$, and $EU_{t+1}$ the expected lifetime welfare of the child, who will become young adult at $t+1$.

2.3.1. The disagreement between parents and children

Before considering the intrafamily bargaining problem, we will first explain why and to what extent the parent and the child disagree about the fraction of time to devote to education. We will, therefore, look at what the parent would have chosen to invest in his child’s education if he was the only one to decide. Then, in a second stage, we will look at what the child would have chosen to invest in his own education if he could decide alone.
The parent’s optimum: When choosing the optimal education for his child, the parent compares, on the one hand, the welfare loss caused by educating his child, which is increasing in the foregone income \( h(e_t) e_{t+1} \) due to time spent to educating the child (instead of working), with, on the other hand, the expected future welfare gains from having an educated child. Formally, the young parent at \( t \) chooses his child’s education \( e_{t+1} \) such as to maximize his own expected lifetime welfare:

\[
\max_{e_{t+1}} \log(h_t(1 - e_t)) + \log(h(e_t)(1 - e_{t+1})) + E_{t+1}(\pi_{t+2}) \log(h(e_t)) \\
+ \gamma E_{t+1}(\pi_{t+2}) \log(h(e_{t+1})),
\]

(6)

where \( E_{t+1}(\pi_{t+2}) \) is the expected level of the survival probability to period 3. Assuming that the parent has perfect foresight, i.e. \( E_{t+1}(\pi_{t+2}) = \pi_{t+2} \), the parent’s problem becomes

\[
\max_{e_{t+1}} \log(h_t(1 - e_t)) + \log(h(e_t)(1 - e_{t+1})) + \pi_{t+2} \log(h(e_t)) + \gamma \pi_{t+2} \log(h(e_{t+1})).
\]

The first-order condition (FOC) yields

\[
\frac{h'(e^*_{t+1})}{h(e^*_{t+1})} = \frac{1}{\gamma \pi_{t+2}(1 - e^*_{t+1})},
\]

where \( e^*_{t+1} \) is the optimal education for the parent. As \( h(e_{t+1}) = A h_{t+1} e^\alpha_{t+1} \), we have

\[
e^*_{t+1} = \frac{\alpha \gamma \pi_{t+2}}{1 + \alpha \gamma \pi_{t+2}}.
\]

(7)

Let us now study the influence of the parent’s expected lifetime on the optimal education level. We obtain

\[
\frac{\partial e^*_{t+1}}{\partial \pi_{t+2}} = \frac{\alpha \gamma}{(1 + \alpha \gamma \pi_{t+2})^2} > 0,
\]

\[
\frac{\partial^2 e^*_{t+1}}{\partial \pi_{t+2}^2} = \frac{-2 \alpha^2 \gamma^2}{(1 + \alpha \gamma \pi_{t+2})^3} < 0.
\]

There is a positive time-horizon effect. The higher the life expectancy of the parent is, the higher the education investment in his child is \( \text{ceteris paribus} \). The intuition goes as follows. Investing time in the child’s education involves costs now, and gains in the future. Hence a higher chance to enjoy those future gains gives parents motivations to invest more in the education of the child. Note that this parental horizon effect is \( \text{concave} \). The positive effect of the rise in parental expected lifetime on optimal education for his child is decreasing with the parent’s remaining life expectancy.
This educational investment depends also positively on the parent’s taste for child’s education $\gamma$ and on the elasticity $\alpha$

\[
\frac{\partial e^*_{t+1}}{\partial \gamma} = \frac{\alpha \pi_{t+2}}{(1 + \alpha \gamma \pi_{t+2})^2} > 0
\]

\[
\frac{\partial e^*_{t+1}}{\partial \alpha} = \frac{\alpha \pi_{t+2}}{(1 + \alpha \gamma \pi_{t+2})^2} > 0
\]

The child’s optimum: When choosing the best education investment for himself, the child compares, on the one hand, the current welfare loss from being educated, which depends on the foregone income $h_{t+1}e_{t+1}$ due to time spent to educating himself, with, on the other hand, the expected welfare gain from being educated, through larger future wages. Formally, the young adult at $t + 1$ maximizes his expected utility over his own education $e_{t+1}$

\[
\max_{e_{t+1}} \log(h_{t+1}(1 - e_{t+1})) + \log(h(e_{t+1})(1 - e_{t+2})) + \pi_{t+3} log(h(e_{t+1})) \\
+ \gamma \pi_{t+3} log(h(e_{t+2})).
\]  

(8)

Assuming that the child fails to perfectly anticipate his lifetime horizon, i.e. $E_{t+1}(\pi_{t+3}) = \pi_{t+2}$, the child’s problem becomes\textsuperscript{12}

\[
\max_{e_{t+1}} \log(h_{t+1}(1 - e_{t+1})) + \log(h(e_{t+1})(1 - e_{t+2})) + \pi_{t+2} log(h(e_{t+1})) \\
+ \gamma \pi_{t+2} log(h(e_{t+2})).
\]

Note that the child’s expected utility differs from the parent’s forwarded by one period, since the child does not perfectly anticipate his longevity.\textsuperscript{13} The FOC yields

\[
\frac{h'(e^*_{t+1})}{h'(e^*_{t+1})} = \frac{1}{(1 - e^*_{t+1})(1 + \pi_{t+2})},
\]

where $e^*_{t+1}$ is the optimal time investment for the child in his own education. As $h(e_{t+1}) = Ah_{t+1}e_{t+1}$, we have

\[
e_{t+1}^* = \frac{\alpha(1 + \pi_{t+2})}{1 + \alpha(1 + \pi_{t+2})}.
\]  

(9)
Let us now study the impact of lifetime horizon on the education decision, in line with Ben-Porath (1967). We obtain

$$\frac{\partial e^{**}_{t+1}}{\partial \pi_{t+2}} = \frac{\alpha}{[1 + \alpha(1 + \pi_{t+2})]^2} > 0,$$

$$\frac{\partial^2 e^{**}_{t+1}}{\partial \pi_{t+2}^2} = -\frac{2\alpha}{[1 + \alpha(1 + \pi_{t+2})]^3} < 0.$$

There is a positive time-horizon effect. This is the Ben-Porath mechanism. A higher life expectancy makes young adults invest more in their education \textit{ceteris paribus}. Note that this positive effect of the child’s life horizon is concave. Moreover, as \(\lim_{t \to +\infty} \pi_t = \bar{\pi} < 1\) and as \(\pi(\cdot) > 0\), we have

$$0 < \frac{\alpha}{1 + \alpha} < e^{* *}_{t+1} < \frac{\alpha(1 + \bar{\pi})}{1 + \alpha(1 + \bar{\pi})} < \frac{2}{3}.$$

The disagreement between the parent and the child: As shown above, the parent would like his child to receive an education equal to \((\alpha \gamma p_{t+2})/(1 + \alpha \gamma p_{t+2})\), whereas the child would like to receive an education equal to \((\alpha(1 + p_{t+2}))/(1 + \alpha(1 + p_{t+2}))\). There is no obvious reason why those two optimal education levels would necessarily coincide.

The goal of this section is precisely to study the causes of the disagreement between the parent and the child on the child’s optimal education. The following proposition sums up some important results obtained by comparing the parent’s and the child’s optimal education, i.e. \(e^*_{t+1}\) and \(e^{**}_{t+1}\).

**Proposition 1**

- The child wants more or less education than the parent depending on

$$e^{**}_{t+1} \geq e^*_{t+1} \iff \frac{1 + \pi_{t+2}}{\pi_{t+2}} \geq \gamma.$$

- The parent’s and the child’s optimal educations display a positive time-horizon effect

$$\frac{\partial e^*_{t+1}}{\partial \pi_{t+2}} > 0 \text{ and } \frac{\partial e^{**}_{t+1}}{\partial \pi_{t+2}} > 0.$$

**Proof** See the comparison of FOCs for the child and the parent’s choices of optimal \(e^*\) and \(e^{**}\). 

The disagreement between the parent and the child depends on two factors.

The first determinant is the difference between the parent’s and the child’s (expected) \textit{lifetime horizons} at the time when they choose the child’s education. The parent’s (remaining) life expectancy is equal to \(1 + \pi_{t+2}\), while the child’s (remaining) life expectancy is \(2 + \pi_{t+2}\). That difference in terms of remaining lifespan explains, to some extent, the differential between the optimal education chosen by the child and
the parent. Education investments bring some benefit in the future, but the child will enjoy those benefits during a period of expected duration \(1 + \pi_{t+2}\), whereas his parent will enjoy these during a period of expected length \(\pi_{t+2}\). That difference explains a large part of the gap between \(e^*_t+1\) and \(e^{**}_t+1\).

The second factor explaining the disagreement lies in the intrinsic motivations for children’s education. That second source of disagreement is reflected by the parameter \(\gamma\), i.e. the parent’s taste for an educated child’s companionship. When the parental taste for education is low (i.e. \(\gamma \leq ((1 + \pi_{t+2})/(\pi_{t+2}))\), the child wants more education than the parent. If, on the contrary, the parent has a strong taste for children’s education (i.e. \(\gamma > ((1 + \pi_{t+2})/(\pi_{t+2}))\), the child wants less education than the parent.

Note also that, despite the disagreement, both the parent and the child’s optimal education level are increasing with the survival probability \(\pi_{t+2}\). This has strong consequences when considering how the disagreement evolves over time. Clearly, if the human capital stock is increased over time, this contributes to raise \(\pi_{t+2}\), with some effect on the size of the disagreement between parents and children, as we will discuss below.

Finally, it should be stressed that the size of the disagreement between the child and the parent depends on the conjunction of those two factors: horizon effects and parental taste for education. It is only in a special case, where \(\gamma = ((1 + \pi_{t+2})/(\pi_{t+2}))\), that the child and the parent’s optimal educations are equal: \(e^{*}_{t+1} = e^{**}_{t+1}\). That case is rare: inter-generational disagreement on education is the norm rather than the exception.

2.3.2. Family bargaining

As shown above, the child and the parent tend, under general conditions, to disagree on the right education investment for the child. However, there must be some agreement between the parent and the child on some amount of education, since the child cannot educate himself without his parent’s effort, and the parent cannot have an educated child without the participation of his child to the education process. Hence some agreement is to be found between children and parents. In this section, we modelize the family as a collective decision unit, where both the parent and the child can affect the chosen education level. It is assumed that the parent and the child are engaged in a bargaining process. As a consequence, the education level \(e^{***}_{t+1}\) that is resulting from the family bargaining process is the solution to the problem

\[
\max_{e^{***}_{t+1}} eEU_t + (1 - e)EU_{t+1},
\]

where \(EU_t\) and \(EU_{t+1}\) denote the objective functions of, respectively, the parent and the child.

Solving that maximization problem, we obtain that the optimal child’s education \(e^{***}_{t+1}\) is

\[
e^{***}_{t+1} = \frac{\varepsilon \alpha \gamma \pi_{t+2} + \alpha (1 - \varepsilon) (1 + \pi_{t+2})}{1 + \varepsilon \alpha \gamma \pi_{t+2} + \alpha (1 - \varepsilon) (1 + \pi_{t+2})}.
\]

That formula is a mixture of the determinants of optimal education from the perspective of both the parent and the child, those determinants being weighted with the
bargaining power weights $\varepsilon$ for the parent and $(1 - \varepsilon)$ for the child. The weight $\varepsilon$ influences negatively the child’s education when the child wants more education than the parent

$$\frac{\partial e_{t+1}^{***}}{\partial \varepsilon} = \frac{\alpha \gamma t_{t+2} - \alpha (1 + t_{t+2})}{[1 + \varepsilon \alpha \gamma t_{t+2} + \alpha (1 - \varepsilon)(1 + t_{t+2})]^2} \leq 0 \iff \gamma \leq \frac{1 + t_{t+2}}{t_{t+2}},$$

while the elasticity $\alpha$ influences positively the child’s education $e_{t+1}^{**}$

$$\frac{\partial e_{t+1}^{**}}{\partial \alpha} = \frac{\varepsilon \gamma t_{t+2} + (1 - \varepsilon)(1 + t_{t+2})}{[1 + \varepsilon \alpha \gamma t_{t+2} + \alpha (1 - \varepsilon)(1 + t_{t+2})]^2} > 0,$$

As both the parent’s and the child’s optimal investment in education are subject to a lifetime-horizon effect (see above), the fraction of time $e_{t+1}^{**}$ resulting from the bargaining process is also subject to a time-horizon effect

$$\frac{\partial e_{t+1}^{**}}{\partial t_{t+2}} = \frac{\varepsilon \gamma + \alpha (1 - \varepsilon)}{[1 + \varepsilon \alpha \gamma t_{t+2} + \alpha (1 - \varepsilon)(1 + t_{t+2})]^2} > 0,$$

$$\frac{\partial^2 e_{t+1}^{**}}{\partial t_{t+2}^2} = \frac{-2 [\varepsilon \gamma + \alpha (1 - \varepsilon)]^2}{[1 + \varepsilon \alpha \gamma t_{t+2} + (1 - \varepsilon)(1 + t_{t+2})]^3} < 0.$$

The education investment chosen by the family as a whole is increasing in the survival probability $t_{t+2}$. Yet, it does not result from a pure Ben-Porath effect, nor from a pure time-horizon effect for the parent due to the companionship of his child, but from a combination of these two lifetime-horizon effects. Hence, in some sense, it could be argued that the modelling of the education decision as a collective decision, which takes the form of a family bargaining process, tends to qualify the standard Ben-Porath effect. There still exists, within our framework, a horizon effect, but it takes a quite different form: both the lifetime horizons of the child and the parent affect the education level, with weights that depend on how bargaining power is distributed within the family.

The following proposition summarizes our results.

**Proposition 2** The education investment determined by the intrafamily bargaining process is

$$e_{t+1}^{**} = \frac{\varepsilon \alpha \gamma t_{t+2} + \alpha (1 - \varepsilon)(1 + t_{t+2})}{1 + \varepsilon \alpha \gamma t_{t+2} + \alpha (1 - \varepsilon)(1 + t_{t+2})},$$

with

$$\frac{\partial e_{t+1}^{**}}{\partial \varepsilon} \leq 0, \quad \frac{\partial e_{t+1}^{**}}{\partial t_{t+2}} > 0 \quad \text{and} \quad \frac{\partial e_{t+1}^{**}}{\partial \alpha} > 0.$$

**Proof** See supra the FOC under family bargaining and the derivative of the family’s optimum with respect to $t_{t+2}$ and $\varepsilon$. \hfill \blacksquare
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This section showed that the differences in age — and thus time horizon — between the parent and the child can lead to a disagreement on the child’s education, and introduced a family bargaining process as a solution to that disagreement. The main conclusion from that exploration is the derivation of a qualified Ben-Porath effect: the influence of lifetime horizon on education can be decomposed into two distinct horizon effects (one for the child and one for the parent), with distinct weights reflecting bargaining power within the family. Hence the longevity/education relationship will also depend on family structures, and, in particular, their structure in terms of decision-making. The next section explores the implications of this on long-run economic dynamics.

3. Long-run dynamics

Let us now characterize the long-run dynamics of the economy. Given that the survival probability \( \pi_{t+1} \) and the output \( y_t \) are functions of the human capital stock \( h_t \), it follows that education investment \( \epsilon_t \) is also a function of \( h_t \). Hence the constancy of the human capital stock \( h_t \) over time brings the constancy of all variables: \( y_t, \pi_{t+1} \) and \( \epsilon_t \).

Substituting for the level of education resulting from the family bargaining in the human capital accumulation equation yields

\[
h_{t+1} = A \left( \frac{\epsilon a \gamma \pi(h_t) + \alpha(1 - \epsilon)(1 + \pi(h_t))}{1 + \epsilon a \gamma \pi(h_t) + \alpha(1 - \epsilon)(1 + \pi(h_t))} \right) \alpha h_t \equiv G(h_t). \tag{11}
\]

The issue of the existence of a stationary equilibrium amounts to studying whether the transition function \( G(h_t) \) admits a fixed point, that is, a value \( h_t \) such that \( G(h_t) = h_t \). The following proposition summarizes our results.

**Proposition 3** The long-run dynamics of the economy belongs to one of the three following cases:

- **Case 1:** If
  
  \[
  \frac{(\epsilon a \gamma \pi + \alpha(1 - \epsilon)(1 + \pi))}{(1 + \epsilon a \gamma \pi + \alpha(1 - \epsilon)(1 + \pi))} < \left( \frac{1}{A} \right)^{1/\alpha} \and \frac{(\epsilon a \gamma \pi + \alpha(1 - \epsilon)(1 + \pi))}{(1 + \epsilon a \gamma \pi + \alpha(1 - \epsilon)(1 + \pi))} < \left( \frac{1}{A} \right)^{1/\alpha},
  \]
  
  then \( h^* = 0 \) is the unique stationary equilibrium: any economy with \( h_0 > 0 \) will converge towards \( h^* = 0 \). That equilibrium is stable.

- **Case 2:** If
  
  \[
  \frac{(\epsilon a \gamma \pi + \alpha(1 - \epsilon)(1 + \pi))}{(1 + \epsilon a \gamma \pi + \alpha(1 - \epsilon)(1 + \pi))} < \left( \frac{1}{A} \right)^{1/\alpha} \and \frac{(\epsilon a \gamma \pi + \alpha(1 - \epsilon)(1 + \pi))}{(1 + \epsilon a \gamma \pi + \alpha(1 - \epsilon)(1 + \pi))} > \left( \frac{1}{A} \right)^{1/\alpha},
  \]
  
  there exists two stationary equilibria: \( h^* = 0 \) and \( h^{**} > 0 \). \( h^* \) is locally stable, while \( h^{**} \) is unstable. Any economy with \( h_0 < h^{**} \) will converge to \( h^* = 0 \) while any economy with \( h_0 > h^{**} \) will exhibit perpetual growth of human capital.

- **Case 3:** If
  
  \[
  \frac{(\epsilon a \gamma \pi + \alpha(1 - \epsilon)(1 + \pi))}{(1 + \epsilon a \gamma \pi + \alpha(1 - \epsilon)(1 + \pi))} > \left( \frac{1}{A} \right)^{1/\alpha} \and \frac{(\epsilon a \gamma \pi + \alpha(1 - \epsilon)(1 + \pi))}{(1 + \epsilon a \gamma \pi + \alpha(1 - \epsilon)(1 + \pi))} > \left( \frac{1}{A} \right)^{1/\alpha},
  \]
  
  \( h^* = 0 \) is the unique stationary equilibrium. That equilibrium is unstable. Any economy with \( h_0 > 0 \) will exhibit perpetual growth of human capital.

**Proof** See the appendix.
The results of Proposition 3 are expressed in terms of two expressions, which are
\[
\frac{a g \tilde{p} + a (1 - \varepsilon)}{1 + a g \tilde{p} + a (1 - \varepsilon)} \text{ and } \frac{a g \widetilde{p} + a (1 - \varepsilon)}{1 + a g \widetilde{p} + a (1 - \varepsilon)}.
\]
The first expression consists of the chosen education level when the level of human capital is zero, whereas the second expression consists of the chosen education level when the human capital stock tends to infinity. In each case, the conditions express whether the chosen education level is sufficiently large or not so as to allow for the growth of the human capital stock.

In Case 1, the productivity parameter \(A\) and the parameters \(\{a, \gamma, \varepsilon\}\) determining the education investment are such that no strictly positive level of human capital can be reproduced and sustained over time, whatever the level of \(h_t\) is. Human capital must necessarily vanish in the long-run. The economy converges towards a zero human capital and the lowest life expectancy \(2 + \tilde{\pi}\), whatever the initial human capital is.

Case 3 is the opposite case: the productivity parameter \(A\) and the parameters \(\{a, \gamma, \varepsilon\}\) are such that the chosen education level is sufficiently high, so as to allow for human capital growth whatever the level of \(h_t\) is. In that case, whatever the initial level of human capital \(h_0 > 0\) is, the economy will increase its stock of knowledge in the future. The education level is so high that the human capital stock grows at any point in time. As a result of perpetual human capital accumulation, life expectancy converges towards its highest possible level \(2 + \widetilde{\pi}\), whatever the initial level of human capital is.

Finally, there exists also an intermediate case, Case 2, where the education level does not allow for human capital growth when the stock of human capital is low, but allows for human capital growth when the human capital stock is larger than the threshold \(h^{**}\). In that case, history matters: depending on whether the initial human capital stock \(h_0\) is lower or larger than the intermediate equilibrium \(h^{**}\), the economy is either trapped in poverty, and undergoes a convergence towards a zero level of human capital stock, or, alternatively, experiences perpetual growth of its human capital stock. Hence, in that intermediate case, there exists a threshold in human capital such that only economies with an initial level of human capital larger than that threshold will exhibit long-run economic growth, whereas the other economies will be trapped in poverty.\(^{15}\) That poverty trap has a demographic origin: when the initial human capital stock is low, agents have a very low life expectancy, so that the collectively chosen level of education is low, which implies, in the future, an even lower level of human capital, leading to even lower levels of life expectancy, and so forth. Those three cases are illustrated in Figure 2.\(^{16}\)

Hence, depending on the productivity parameter \(A\), on the parental taste for children’s education \(\gamma\), on the bounds of the survival probability \(\tilde{\pi}\) and \(\widetilde{\pi}\), and on the elasticity of human capital to \(\alpha\), and on the distribution of the bargaining power \(\varepsilon\), an economy may experience three distinct forms of long-run dynamics.

The influences of parental taste for education \(\gamma\) and of the elasticity \(\alpha\) are not surprising: these are major determinants of the education level, which directly influences the human capital accumulation process. More important is the role of the bargaining power \(\varepsilon\), whose influence depends on how large \(\gamma\) is. If \(\gamma > ((1 + \tilde{\pi})/\tilde{\pi})\), the higher the parent’s bargaining power \(\varepsilon\) is, the higher the likelihood of perpetual growth is. The reason is that, in that case, parents want a higher level of education for their children in comparison to what children themselves want. As a consequence, the higher the bargaining power of the parent, the stronger the human capital accumulation process. If,
on the contrary, $\gamma < ((1 + \bar{p})/\bar{p})$, the higher the child’s power $1 - \varepsilon$ is, the higher the likelihood of perpetual growth is. In that case, the child wants more education than what his parent wants for him. Hence, in that case, a rise in the child’s bargaining power would, by increasing education, favour human capital accumulation and growth.

Note that the limits of the survival function $\tilde{p}$ and $\bar{p}$ play a crucial role with respect to long-run dynamics. The higher $\bar{p}$ and $\tilde{p}$ are, the lower the likelihood of the existence of a poverty trap is (Cases 1 and 2). Finally, the higher the productivity parameter $A$ is, the higher the likelihood of perpetual growth is.

Proposition 3 shows that the Ben-Porath mechanism is at the very heart of the long-run dynamics of the economy. Indeed, the economy’s dynamics depend crucially on the shape of the survival function $p(\cdot)$, and, in particular, on its limit levels $\tilde{p}$ and $\bar{p}$, which strongly affect the education (collective) decision and the human capital accumulation process. The higher these limits are, the higher the probability of perpetual growth is. Moreover, whether the economy’s dynamics fall under one case or another depends on the distribution of the bargaining power $\varepsilon$, and more precisely on its interplay with other parameters of the model: $\gamma$, $\tilde{p}$ and $\bar{p}$. Hence the introduction of a collective education investment decision refines the form of the Ben-Porath effect in a dynamic setting, by modifying the link between lifetime horizon and education in the long-run.

4. Endogenous bargaining power

As shown in the previous section, the distribution of bargaining power within the family can have a substantial impact on human capital accumulation and growth. The reason has to do with the collective decision process concerning the child’s education. When the condition $1 > (\gamma \tilde{p}/(1 + \bar{p}))$ is satisfied, the child’s longer remaining lifetime makes him want an education investment that is larger than the one desired by his parent. In that case, children’s emancipation could have a positive impact on long-run economic growth. That result is in line with the recent literature emphasizing the role of emancipation as a factor of economic development (Rubalcava, Teruel, and Thomas 2009 and Doepke and Tertilt 2011), except that, in our model, we emphasize the role of children’s emancipation with respect to their parents, and not of women’s emancipation.

Note that the distribution of bargaining power within families is likely to vary over time. The goal of this section is precisely to examine the robustness of our analysis to...
the introduction of a _varying_ distribution of bargaining power. For simplicity, we modelize \( \varepsilon \) as a function of human capital

\[
\varepsilon_t = \varepsilon(h_t). \tag{12}
\]

That modeling is quite standard in the literature, which makes intrafamily bargaining power depend on the human capital level of individuals.\textsuperscript{17}

The precise form of the functional relationship linking bargaining power to human capital can hardly be known a priori. Two opposite effects are at work. On the one hand, a child born with a higher human capital is likely to be more emancipated, thanks to his larger knowledge (prior to education). This favours a declining parental bargaining power with the human capital of the child, i.e. \( \varepsilon'(h_t) < 0 \). On the other hand, the human capital \( h_t \) is also enjoyed by the parents, and results from their own education decision. Better educated parents can also use their knowledge to better influence their child: \( \varepsilon'(h_t) > 0 \).

Given that it is too early, at this stage, to know which effect dominates the other, we will, in the rest of this section, consider the two cases successively: first, the case in which \( \varepsilon'(h_t) < 0 \) (emancipation of the child thanks to a higher human capital at birth); second, the case in which \( \varepsilon'(h_t) > 0 \) (reinforcement of the parental authority through his own education).\textsuperscript{18}

4.1. **Child’s emancipation \( \varepsilon'(h_t) < 0 \)**

Let us first consider the case where human capital accumulation favours the child’s emancipation. In this case, when the human capital increases, the bargaining power of the parent decreases

\[
\frac{\partial \varepsilon_t}{\partial h_t} < 0.
\]

We will use, throughout this section, the following notations: \( \lim_{h_t \to 0} \varepsilon(h_t) = \bar{\varepsilon} \) and \( \lim_{h_t \to +\infty} \varepsilon(h_t) = \tilde{\varepsilon} \).

Once the distribution of bargaining power within the family is dependent on the level of human capital, the intertemporal human capital equation becomes

\[
h_{t+1} = A \left( \frac{\varepsilon(h_t) \gamma \pi(h_t) + (1 - \varepsilon(h_t))(1 + \pi(h_t))}{1 + \gamma \varepsilon(h_t) \pi(h_t) + (1 - \varepsilon(h_t))(1 + \pi(h_t))} \right) ^{1/\alpha} h_t = H(h_t). \tag{13}
\]

The issue of the existence of a steady-state equilibrium amounts to studying whether the transition function \( H(h_t) \) admits a fixed point. The following proposition summarizes our results.\textsuperscript{19}

**Proposition 4** _The long-run dynamics belongs to one of the following four cases:_

- **Case 1:**
  \[
  ((\bar{\varepsilon} \alpha \gamma \tilde{\pi} + \alpha(1 - \bar{\varepsilon})(1 + \tilde{\pi}))/1 + \bar{\varepsilon} \alpha \gamma \tilde{\pi} + \alpha(1 - \bar{\varepsilon})(1 + \tilde{\pi})) < (1/A)^{1/\alpha} \quad \text{and} \quad ((\bar{\varepsilon} \alpha \gamma \tilde{\pi} + \alpha(1 - \bar{\varepsilon})(1 + \tilde{\pi}))/1 + \bar{\varepsilon} \alpha \gamma \tilde{\pi} + \alpha(1 - \bar{\varepsilon})(1 + \tilde{\pi})) < (1/A)^{1/\alpha}. \]
  There exist either zero or an even number of positive stationary equilibria. There exist a poverty trap and no area of perpetual growth.
depends on the levels of parameters \(a\) equal to zero, or tends to infinity. Here again, the likelihood of the different cases expressed in terms of the levels of education when the human capital stock is either the economy will exhibit perpetual growth.

Capital such that, for any economy with a human capital stock higher than the threshold, the stock of human capital will converge, in the long-run, towards a constant level, which can be either in Cases 1 and 2, there exists no area of perpetual growth, and the stock of human education fall to even lower levels. On the other hand, the decline in parental bargaining power when human capital accumulates can, if sufficiently strong, prevent the economy from being trapped in poverty, by favouring an even higher education investment.

As in Proposition 3, the conditions characterizing the different cases are still expressed in terms of the levels of education when the human capital stock is either equal to zero, or tends to infinity. Here again, the likelihood of the different cases depends on the levels of parameters \(\{\alpha, A, \gamma\}\). But the major difference is that, given the postulated emancipation of children, the bargaining power of the parent is higher at lower levels of human capital, and is lower at higher levels of human capital, unlike in Proposition 3, where the bargaining power of the parent was the same whatever the level of \(h_t\) was. The variability of \(e\) can have ambiguous effects on the dynamics, depending on the structural parameters of the economy. On the one hand, a regression towards a low level of human capital can be reinforced by the associated rise in the parent’s bargaining power (in case of low \(\gamma\)), since that rise can make education fall to even lower levels. On the other hand, the decline in parental bargaining power when human capital accumulates can, if sufficiently strong, prevent the economy from being trapped in poverty, by favouring an even higher education investment.

Hence, endogenizing the distribution of bargaining power within the family has ambiguous effects on the long-run dynamics of the economy. Having stressed this, it remains true, as in the baseline model, that the long-run dynamics of the economy is strongly influenced by the lifetime horizons of parents and children, which both determine the education level. The Ben-Porath effect remains present in this extended

\[\text{Case 2:}\]
\[
((\bar{e}\alpha\gamma\bar{p} + \alpha(1 - \bar{e})(1 + \bar{p}))/ (1 + \bar{e}\alpha\gamma\bar{p} + \alpha(1 - \bar{e})(1 + \bar{p}))) > (1/A)^{1/\alpha}\]
\[
((\bar{e}\alpha\gamma\bar{p} + \alpha(1 - \bar{e})(1 + \bar{p}))/ (1 + \bar{e}\alpha\gamma\bar{p} + \alpha(1 - \bar{e})(1 + \bar{p}))) < (1/A)^{1/\alpha}.\]

There exist an odd number of positive stationary equilibria. There may exist no stable equilibrium and there is no area of perpetual growth.

\[\text{Case 3:}\]
\[
((\bar{e}\alpha\gamma\bar{p} + \alpha(1 - \bar{e})(1 + \bar{p}))/ (1 + \bar{e}\alpha\gamma\bar{p} + \alpha(1 - \bar{e})(1 + \bar{p}))) < (1/A)^{1/\alpha}\]
\[
((\bar{e}\alpha\gamma\bar{p} + \alpha(1 - \bar{e})(1 + \bar{p}))/ (1 + \bar{e}\alpha\gamma\bar{p} + \alpha(1 - \bar{e})(1 + \bar{p}))) > (1/A)^{1/\alpha}.\]

There exist an odd number of positive stationary equilibria. There exist a poverty trap and an area of perpetual growth.

\[\text{Case 4:}\]
\[
((\bar{e}\alpha\gamma\bar{p} + \alpha(1 - \bar{e})(1 + \bar{p}))/ (1 + \bar{e}\alpha\gamma\bar{p} + \alpha(1 - \bar{e})(1 + \bar{p}))) > (1/A)^{1/\alpha}\]
\[
((\bar{e}\alpha\gamma\bar{p} + \alpha(1 - \bar{e})(1 + \bar{p}))/ (1 + \bar{e}\alpha\gamma\bar{p} + \alpha(1 - \bar{e})(1 + \bar{p}))) < (1/A)^{1/\alpha}.\]

There exist either zero or an even number of positive stationary equilibria. There may exist no stable equilibrium and there exists an area of perpetual growth.

For every cases, if at some point \(h^{**}\) the transition function crosses the 45° line from above and \(|H'(h^{**})| < 1,\) then \(h^{**}\) is a locally stable equilibrium.

\[\text{Proof}\] See the appendix.

In comparison to the dynamics of the economy under a fixed distribution of bargaining power within the family (Proposition 3), the dynamics under a varying distribution of power admits more cases. The reason why the dynamics admits more cases has to do with the nonmonotonicity of the transition function \(H(h_t)\). Nonetheless, it is clear that, in Cases 1 and 2, there exists no area of perpetual growth, and the stock of human capital will converge, in the long-run, towards a constant level, which can be either zero or positive. On the contrary, in Cases 3 and 4, there exists a threshold in human capital such that, for any economy with a human capital stock higher than the threshold, the economy will exhibit perpetual growth.
increases, the power of the parent increases the bargaining power of parents within the family. In this case, when the human capital accumulation increases, the power of the parent increases

\[ \frac{\partial \varepsilon_t}{\partial h_t} > 0. \]

We use the same notations as above for the lower bound and the upper bound of parental power \( \varepsilon_t \): \( \lim_{h_t \to 0} \varepsilon_t(h_t) = \tilde{\varepsilon} \) and \( \lim_{h_t \to +\infty} \varepsilon_t(h_t) = \bar{\varepsilon} \).

The human capital accumulation equation is now

\[ h_{t+1} = A \left( \frac{\varepsilon(h_t)\alpha\gamma\pi(h_t) + \alpha(1 - \varepsilon(h_t))(1 + \pi(h_t))}{1 + \varepsilon(h_t)\alpha\gamma\pi(h_t) + \alpha(1 - \varepsilon(h_t))(1 + \pi(h_t))} \right)^\alpha h_t = J(h_t). \]  \hspace{1cm} (14)

The issue of the existence of a stationary equilibrium amounts to studying whether the transition function \( J(h_t) \) admits a fixed point. Proposition 5 summarizes our results.

**Proposition 5** The long-run dynamics belongs to one of the following cases:

- **Case 1:**
  
  \( ((\hat{\varepsilon}\alpha\gamma\pi + \alpha(1 - \hat{\varepsilon})(1 + \hat{\pi}))/ (1 + \hat{\varepsilon}\alpha\gamma\pi + \alpha(1 - \hat{\varepsilon})(1 + \hat{\pi}))) < (1/A)^{1/\alpha} \) and \( ((\hat{\varepsilon}\alpha\gamma\pi + \alpha(1 - \hat{\varepsilon})(1 + \hat{\pi}))/ (1 + \hat{\varepsilon}\alpha\gamma\pi + \alpha(1 - \hat{\varepsilon})(1 + \hat{\pi}))) < (1/A)^{1/\alpha} \). There exist zero or an even number of positive stationary equilibria. There exist a poverty trap and no area of perpetual growth.

- **Case 2:**
  
  \( ((\hat{\varepsilon}\alpha\gamma\pi + \alpha(1 - \hat{\varepsilon})(1 + \hat{\pi}))/ (1 + \hat{\varepsilon}\alpha\gamma\pi + \alpha(1 - \hat{\varepsilon})(1 + \hat{\pi}))) > (1/A)^{1/\alpha} \) and \( ((\hat{\varepsilon}\alpha\gamma\pi + \alpha(1 - \hat{\varepsilon})(1 + \hat{\pi}))/ (1 + \hat{\varepsilon}\alpha\gamma\pi + \alpha(1 - \hat{\varepsilon})(1 + \hat{\pi}))) < (1/A)^{1/\alpha} \). There exist an odd number of positive stationary equilibria. There may exist no stable equilibrium and there is no area of perpetual growth.

- **Case 3:**
  
  \( ((\hat{\varepsilon}\alpha\gamma\pi + \alpha(1 - \hat{\varepsilon})(1 + \hat{\pi}))/ (1 + \hat{\varepsilon}\alpha\gamma\pi + \alpha(1 - \hat{\varepsilon})(1 + \hat{\pi}))) < (1/A)^{1/\alpha} \) and \( ((\hat{\varepsilon}\alpha\gamma\pi + \alpha(1 - \hat{\varepsilon})(1 + \hat{\pi}))/ (1 + \hat{\varepsilon}\alpha\gamma\pi + \alpha(1 - \hat{\varepsilon})(1 + \hat{\pi}))) > (1/A)^{1/\alpha} \). There exist an odd number of positive stationary equilibria. There exist a poverty trap and an area of perpetual growth.

- **Case 4:**
  
  \( ((\hat{\varepsilon}\alpha\gamma\pi + \alpha(1 - \hat{\varepsilon})(1 + \hat{\pi}))/ (1 + \hat{\varepsilon}\alpha\gamma\pi + \alpha(1 - \hat{\varepsilon})(1 + \hat{\pi}))) > (1/A)^{1/\alpha} \) and \( ((\hat{\varepsilon}\alpha\gamma\pi + \alpha(1 - \hat{\varepsilon})(1 + \hat{\pi}))/ (1 + \hat{\varepsilon}\alpha\gamma\pi + \alpha(1 - \hat{\varepsilon})(1 + \hat{\pi}))) > (1/A)^{1/\alpha} \). There exist either zero or an even number of positive stationary equilibria. There may exist no stable equilibrium and there exists an area of perpetual growth.
For every cases, if at some point \( h^{**} \) the transition function crosses the 45° line from above and \( |J(h^{**})| < 1 \), then \( h^{**} \) is a locally stable equilibrium.

Proof See the appendix.

The four cases presented in Proposition 5 are quite close to the ones presented under children’s emancipation (Proposition 4). An important difference concerns how bargaining power and life expectancy are related. Under child’s emancipation, the highest parental bargaining power prevails when human capital is at its lowest level, and when life expectancy is also at its lowest level (i.e. \( 2 + \tilde{\pi} \)). Inversely, the lowest parental bargaining power prevails when life expectancy takes its highest level (i.e. \( 2 + \bar{\pi} \)). On the contrary, under parental authority reinforced, the parental bargaining power is positively correlated with life expectancy. It follows from that difference that the conditions defining the distinct cases are here not the same as in the child’s emancipation case.

That difference may matter a lot when considering the capacity of the economy to overcome a poverty trap. In the present case, even if low human capital also implies a low life expectancy, at least the parent’s bargaining power is also low, which can favour, provided the child wants more education than the parent (i.e. under a low \( \gamma \)), a sufficiently high education level, and, hence, the take-off of the economy. Such a take-off would have been less feasible under the child’s emancipation, since in that case parents have their largest bargaining power at low levels of human capital. Note, however, that the possibility to have perpetual growth may be here more limited than under child’s emancipation case, since here high levels of human capital, by reinforcing parental authority, will restraint education investment in comparison to the child’s emancipation case.

Hence the endogenization of the distribution of bargaining power within the family has quite ambiguous effects on long-run economic dynamics. The two alternative assumptions – child’s emancipation and parental authority reinforced – have distinct implications, which can locally encourage or discourage human capital accumulation and growth. Hence one could hardly overemphasize the impact of the distribution of bargaining power for long-run economic dynamics. The next section proposes to explore, by means of an empirical application, whether the introduction of family bargaining helps to fit the data, and evaluates the plausibility of the hypotheses of emancipation of the child and of reinforcement of parental authority.

5. Empirical illustration

In this section, we propose to use data on education and life expectancy in 16 OECD countries, for cohorts born between 1940 and 1980, in order to investigate whether the introduction of family bargaining on education can allow us to better fit the data on education patterns, in comparison to standard models where either the parent or the child chooses the child’s education.20

For that purpose, we will proceed in three stages. We will first try to replicate, from observed life expectancy patterns, the theoretical level of education when either the parent or the child decides alone on the child’s education, in line with existing models of education choice. Those two cases coincide with the model developed above, where either \( \varepsilon_r = 1 \) or \( \varepsilon_r = 0 \). Then, in a second stage, we will contrast those replicated education levels with the ones obtained under a collective choice model (\( 0 \leq \varepsilon_r \leq 1 \),
while allowing for varying family bargaining power on education. Our results show that the introduction of bargaining on education improves significantly the ability of our model to replicate the observed education patterns. Finally, in a third stage, we will compare the evolution of intrafamily bargaining power used in our simulations with data from the World Values Survey and also with legal data on the majority age.

5.1. Data and calibration

Our model is calibrated as follows. We assume that the implicit childhood period lasts 10 years, and that each subsequent period lasts 25 years. That modelling amounts to assume that the first period of the model starts when the agent has age 10, and that individuals have their children at the age of 25 years.

Education: Education in our model \((e_t)\) is the fraction of life spent at school during the first period, between the age of 10 and the age of 35. To compute it, we use the mean years of education per cohorts from Cohen and Leker (2013), as well as the age of enrollment to primary school in each of the 16 countries, to compute the mean time spent at school above the age of 10, for cohorts born between 1940 and 1980, and divide it by 25.

Survival probability: The survival probability taken into account in the education decision in our model \((\pi_t)\) is the expected probability to live the third period, computed at the beginning of the first period. It therefore refers to the expected probability to survive from the age of 60 to the age of 85, computed at the age of 10 (as there is an implicit period of childhood). We take those probabilities from the period life tables of the Human Mortality Database. For instance, for the cohort born in 1950, the survival probability considered is the expected probability to survive from 60 to 85 in 1960.

Calibration of parameters: When considering education decisions, a crucial parameter is the parameter \(\alpha\), which is the elasticity of future human capital to education. Throughout the rest of this section, we assume that \(\alpha = 0.10\), in line with the calibrations in Zhang, Zhang, and Lee (2001).

The parameter \(\gamma\), which captures the parental taste for children’s education, is a preference parameter, and, as such, is hardly directly observable. However, given that the purpose of this section is to compare models of education choices where either the parent and/or the child make the education decision, it makes sense to take, for the parameter \(\gamma\), the value that best fits the observed education pattern when the parent is the one who decides on education, that is, when \(e_t = 1\). Table 1 shows the selected value of \(\gamma\) for each country under study.

Regarding the calibration of the parent’s bargaining power \(e_t\), we will, for the sake of our comparisons of models of education decisions, consider successively three distinct calibrations: (1) \(e_t = 1\) (model where parents decide alone on education); (2) \(e_t = 0\) (model where children decide alone on education) and (3) \(e_t\), varying between 0 and 1 (model where both parents and children decide on education). The next section compares the observed education pattern with education patterns simulated under those three distinct calibrations of the distribution of intrafamily bargaining power.

5.2. Comparison of the models

Let us now turn to the comparison of the three education decision models. Note that, whatever the education decision is made by the parent and/or by the child, the lifecycle
model developed above suggests that the education investment is unambiguously increasing with the human life horizon. Hence, in order to compare the three education decision models, it makes sense to compare the actual patterns of education with the ones replicated from the models, under the observed evolution of survival conditions.

For that purpose, Figures 3 and 4 show, for the 16 OECD countries under study, the fraction of time spent at school above the age of 10 (denoted by $e$) and the survival probability from 60 to 85 at the age of 10 (denoted by $p$), for cohorts born between 1940 and 1980.

The actual education pattern, which appears as the thick curve in Figures 3 and 4, invites three comments. First, the overall pattern of the relation between education above age 10 and survival probabilities seems to be increasing for most countries, in line with the Ben-Porath hypothesis. The observed education investment is, in general, increasing and concave with the life expectancy level. Second, we observe that patterns are different from one country to another. In the USA, the relationship is flat, while it is much more increasing for countries such as France or Finland. Some countries exhibit strong discontinuities, such as Sweden and Ireland. Third, in many countries, education increases even during a period when expected survival probabilities do not evolve (frequently between cohorts born in 1950 and 1960).

In addition to the actual education patterns, Figures 3 and 4 show also, in dotted lines, the hypothetical education patterns replicated from two distinct models of education choice: on the one hand, the education pattern when the parent alone decides on education (i.e. $e_t = 1$); on the other hand, the education pattern when the child alone decides on education (i.e. $e_t = 0$). Those two hypothetical education patterns are, over the period considered, increasing, thanks to the rise in life expectancy. Nonetheless, although education patterns obtained from models with a single decision-maker are increasing over time, these are quite different from the observed education pattern.

Actually, the replicated education pattern under $e_t = 0$ is below the observed education path, and also flatters than the actual education path. That result is due to the relatively low level of the elasticity $\alpha$ (i.e. 0.1), which makes the child’s desired education level slowly increasing with life expectancy. On the contrary, the replicated education pattern under $e_t = 1$ is, in general, higher than the actual education pattern, to an extent varying across countries and periods. Hence, models of education choice with a single decision-maker can hardly replicate the observed rise in education, whatever the country is.

Having stressed this, let us now consider the third type of education models: the model of intrafamily bargaining on education. As we will now explain that alternative model of education choice leads, despite its simplicity, to a better replication of the observed education pattern, in comparison to models with a single decision-maker. More precisely, the

<table>
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model of intrafamily bargaining can, under an adequate calibration of the bargaining power $e_t$, replicate the observed education pattern for all countries and all periods.

The mere introduction of a collective decision with a varying bargaining power between the parent and the child enables us to reproduce the observed education
evolution. The intuition behind that result goes as follows. As shown in Figures 3 and 4, the actual education pattern lies between the hypothetical education levels obtained under the models with a single decision-maker. In other words, the actual education...
lies between, on the one hand, the optimal education from the perspective of the parent, and, on the other hand, the optimal education from the child’s perspective. The fact that the actual education lies between the optimal education from the perspectives of parents and children implies that it can be rationalized as a compromise between those different decision-makers. In other words, one can rationalize the observed education levels as the outcomes of an intrafamily bargaining process with some particular distribution of bargaining power in the family. Put it differently, since the actual education lies between the optimal education for the child and for the parent, there must necessarily exist a level of $\hat{\epsilon}_t$ for each period such that the hypothetical education chosen under the model of family bargaining coincides with the actual education level. Those levels of $\hat{\epsilon}_t$ are shown in Table 2.

Table 2 can be read as follows. In the case of Australia, for instance, the model of intrafamily bargaining can replicate the observed education level for the cohort born in 1940 when $\hat{\epsilon}$ equals 0.937. On the contrary, the replication for the cohort born in 1980 requires a lower $\hat{\epsilon}$, equal to 0.533.

In the light of Table 2, it appears that, even though there is no clear pattern for the evolution of $\hat{\epsilon}_t$ over time, there is, after the cohort born in 1960, an unambiguous decline of $\hat{\epsilon}_t$. That decline consists of a decline in the bargaining power of parents, and, as a consequence, a rise in the bargaining power of children. Hence, Table 2 is compatible with some form of emancipation of children. Is that dynamics of intrafamily bargaining power plausible? The rest of this section discusses that issue in the light of different sources of data.

5.3. Evidence on family bargaining power

As this is well known in the literature, bargaining power is something difficult to measure. In the development literature, it is often approximated by the level of education of individuals. But given that our theoretical framework uses a collective model of choice to determine the education of the child, that kind of proxy cannot be used here. In this section, we will use two distinct kinds of sources: on the one
hand, data from the World Values Survey; on the other hand, legal data on the majority age in the countries under study.

Within the World Values Survey, there exists no direct question concerning the distribution of bargaining power within the family. However, a particular question requires respondents to pick up some qualities of children that they consider to be important. That question is formulated as follows: ‘Here is a list of qualities that children can be encouraged to learn at home. Which, if any, do you consider to be especially important? Please choose up to five.’ Ten children qualities are proposed to them. Two of them are particularly relevant for our purpose: children’s obedience and children’s independence. Actually, if one postulates that more authoritarian parents tend to value children’s obedience to a larger extent but children’s independence to a lower extent, one can use the proportions of respondents choosing the quality ‘obedience’ and the proportions of respondents not choosing the quality ‘independence’ as two distinct indirect indicators of $\varepsilon_t$.

Tables 3 and 4 show, respectively, the proportions of respondents who pick up the quality ‘obedience’ and the proportions for respondents who do not pick up the quality ‘independence’. Those proportions are shown for five waves of the World Values Survey, for some of the countries in our sample.

Table 3. Proportion of respondents picking up the quality ‘obedience’.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.41</td>
<td>–</td>
<td>0.29</td>
<td>–</td>
<td>0.37</td>
</tr>
<tr>
<td>Canada</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.31</td>
<td>0.31</td>
</tr>
<tr>
<td>Finland</td>
<td>–</td>
<td>–</td>
<td>0.28</td>
<td>–</td>
<td>0.33</td>
</tr>
<tr>
<td>France</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.41</td>
</tr>
<tr>
<td>Italy</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.26</td>
</tr>
<tr>
<td>Japan</td>
<td>0.06</td>
<td>0.10</td>
<td>0.06</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>Netherlands</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.41</td>
</tr>
<tr>
<td>New Zealand</td>
<td>0.22</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.24</td>
</tr>
<tr>
<td>Norway</td>
<td>–</td>
<td>–</td>
<td>0.26</td>
<td>–</td>
<td>0.29</td>
</tr>
<tr>
<td>Sweden</td>
<td>–</td>
<td>–</td>
<td>0.16</td>
<td>0.12</td>
<td>0.16</td>
</tr>
<tr>
<td>UK</td>
<td>–</td>
<td>–</td>
<td>0.51</td>
<td>–</td>
<td>0.46</td>
</tr>
<tr>
<td>USA</td>
<td>–</td>
<td>–</td>
<td>0.37</td>
<td>0.32</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Table 4. Proportion of respondents not picking up the quality ‘independence’.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.75</td>
<td>–</td>
<td>0.47</td>
<td>–</td>
<td>0.37</td>
</tr>
<tr>
<td>Canada</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.38</td>
<td>0.42</td>
</tr>
<tr>
<td>Finland</td>
<td>–</td>
<td>–</td>
<td>0.44</td>
<td>–</td>
<td>0.31</td>
</tr>
<tr>
<td>France</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.62</td>
</tr>
<tr>
<td>Italy</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.42</td>
</tr>
<tr>
<td>Japan</td>
<td>0.53</td>
<td>0.36</td>
<td>0.40</td>
<td>0.18</td>
<td>0.20</td>
</tr>
<tr>
<td>Netherlands</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.36</td>
</tr>
<tr>
<td>New Zealand</td>
<td>–</td>
<td>–</td>
<td>0.46</td>
<td>–</td>
<td>0.47</td>
</tr>
<tr>
<td>Norway</td>
<td>–</td>
<td>–</td>
<td>0.11</td>
<td>–</td>
<td>0.10</td>
</tr>
<tr>
<td>Sweden</td>
<td>–</td>
<td>–</td>
<td>0.39</td>
<td>0.31</td>
<td>0.22</td>
</tr>
<tr>
<td>UK</td>
<td>–</td>
<td>–</td>
<td>0.51</td>
<td>–</td>
<td>0.42</td>
</tr>
<tr>
<td>USA</td>
<td>–</td>
<td>–</td>
<td>0.56</td>
<td>0.38</td>
<td>0.47</td>
</tr>
</tbody>
</table>
Table 3 shows mitigated trends. The proportion of respondents choosing the children’s quality ‘obedience’ decreases in four countries (Australia, Japan, the UK and the USA). That decrease is compatible with our calibrations: those four countries exhibit, in Table 2, a strong decline in $\varepsilon_t$ over time. However, the proportions of respondents choosing ‘obedience’ is constant in Canada and Sweden, and has increased — but to a small extent — in Finland, Norway and New Zealand. On the contrary, Table 4 provides a stronger support to our calibrations. Actually, Table 4 shows that the proportions of respondents not choosing ‘independence’ as an important quality of children has decreased over time in seven out of nine countries (Australia, Finland, Japan, Norway, Sweden, the UK and the USA), and has only slightly increased in Canada and New Zealand. Therefore, even though the two questions of the World Values Survey only provide indirect indicators of the evolution of intrafamily bargaining power over the last decades, the observed trends seem to be, at least qualitatively, roughly compatible with our calibrations, that is, with the emancipation of children.

Besides the World Values Survey, another source of information on the possible evolution of the distribution of intrafamily bargaining power over time consists of legal data on the majority age. Indeed, it is not unreasonable to assume that, when the majority age is lower, children have a larger bargaining power when discussing their educational perspectives with their parents. Hence, a decline in the majority age could be regarded as an indirect clue of a decrease in the parental bargaining power $\varepsilon_t$. Table 5, which shows the evolution of the majority age in the countries under study, highlights that the majority age has been reduced during the second part of the twentieth century.24

The decline in the majority age occurred in all countries, but the timing of the decline varies across countries. In general, the fall in the majority age occurred during the 1970s, and, hence, concerns cohorts born in the 1950s. If one assumes that a lower majority age implies a lower parental bargaining power, then the observed decline in the majority age must have affected the distribution of bargaining power in the family for cohorts born after 1950. That conjecture is not incompatible with our calibrations for the dynamics of parental bargaining power $\varepsilon_t$. Indeed, Table 2 shows a

Table 5. Majority age in the countries under study.

<table>
<thead>
<tr>
<th>Countries</th>
<th>Evolution of the majority age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>From 21 to 18 (1973)</td>
</tr>
<tr>
<td>Austria</td>
<td>From 21 to 20 (1949), from 20 to 19 (1970), from 19 to 18 (1992), from 18 to 17 (2007)</td>
</tr>
<tr>
<td>Belgium</td>
<td>From 21 to 18 (1981)</td>
</tr>
<tr>
<td>Canada</td>
<td>From 21 to 18 (1970)</td>
</tr>
<tr>
<td>Denmark</td>
<td>From 25 to 23 (1953), from 23 to 21 (1961), from 21 to 20 (1971), from 20 to 18 (1978)</td>
</tr>
<tr>
<td>Finland</td>
<td>From 24 to 21 (1944), from 21 to 20 (1968), from 20 to 18 (1972)</td>
</tr>
<tr>
<td>France</td>
<td>From 21 to 18 (1974)</td>
</tr>
<tr>
<td>Ireland</td>
<td>From 21 to 18 (1973)</td>
</tr>
<tr>
<td>Italy</td>
<td>From 21 to 18 (1975)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>From 23 to 21 (1965), from 21 to 18 (1971)</td>
</tr>
<tr>
<td>New Zealand</td>
<td>From 21 to 20 (1969), from 20 to 18 (1974)</td>
</tr>
<tr>
<td>Sweden</td>
<td>From 20 to 18 (1972)</td>
</tr>
<tr>
<td>UK</td>
<td>From 21 to 18 (1970)</td>
</tr>
<tr>
<td>USA</td>
<td>From 21 to 18 (1971)</td>
</tr>
</tbody>
</table>
decline in parental bargaining power in most countries for cohorts born after 1960. The start of the postulated decline in \( \varepsilon_t \) occurs slightly after the time of the reduction of the majority age. Hence the legal evidence shown in Table 5 is not incompatible with our calibrations.

In sum, the empirical material covered in this section suggests that the hypothesis of a decline in the parental bargaining power during the second part of the twentieth century is plausible. True, we do not have direct estimates of the extent of parental bargaining power, but only indirect indicators. However, those indirect indicators tend to provide some support to the hypothesis of a decline in the parental bargaining power. As such, these provide also some qualitative support for the calibration of \( \varepsilon_t \) that allow our model of intrafamily bargaining to replicate the observed dynamics of education.

Of course, we are aware that our theoretical model is a reduced form model with few variables and few parameters. We are also aware that our calibrated parameters \( \varepsilon_t \) may capture many other features than the distribution of bargaining power within the family (i.e. variables that are omitted in the theoretical model). However, our reduced form model is, despite its simple form, compatible with the data. In particular, our family bargaining model of education choices can fit the observed education patterns in a much better way than standard education models with a single decision-maker (either the child or the parent). The reason why it provides better results goes as follows. Once family bargaining on education is introduced, heterogeneity in terms of age comes into the education decision, and that particular ingredient – heterogeneity – provides the additional degree of freedom allowing us to better fit the observed education patterns, in comparison with models with a single decision-maker.

6. Conclusions

Demography and human capital accumulation are key factors for the understanding of long-run economic dynamics. According to Ben-Porath (1967), the longer the expected lifetime is, the higher the returns on education investment are. The demand for education is, ceteris paribus, increasing in life expectancy, which is thus likely to foster human capital accumulation and growth.

In this paper, we proposed to enhance the Ben-Porath model by developing a three-period OLG model with endogenous mortality, where the education investment results from bargaining within the family. We first introduced the bargaining power as an exogenous parameter, and, then, proposed an extension where the bargaining power depends on human capital. The intuition behind our model is the following. Education is a major decision in life, and that decision is made in a particular environment: the family. Hence, it makes sense to model education not as the outcome of a simple decision process, but, rather, as the outcome of a family bargaining process, where both the parent and the child have a word to say.

At the microeconomic level, the introduction of intrafamily bargaining on education refines the time-horizon effect pointed out by Ben-Porath: both the life expectancy of parents and the life expectancy of children determine here the education level resulting from bargaining. On the contrary, in the standard Ben-Porath, the life expectancy of children is the only demographic determinant of their demand for education. Hence, the intrafamily distribution of bargaining power, by affecting the chosen amount of education, matters also for human capital accumulation.
It follows from this that, at the macro-level, the distribution of bargaining power matters for long-run dynamics: depending on how the bargaining power is distributed in the family, education may be more or less high, leading to a more or less strong human capital accumulation. Hence the intrafamily distribution of bargaining power affects, through education, the likelihood for an economy to experience perpetual growth.

After having studied the dynamics of the economy under exogenous and endogenous bargaining power, we used data on education and life expectancy in 16 OECD countries for cohorts born between 1940 and 1980, in order to compare the capacity of different models of education choice (either with a single decision-maker or with intrafamily bargaining) to fit the observed education patterns. We showed that the model of intrafamily bargaining can better replicate the observed education patterns, provided we assume that the parental bargaining power has declined during the second part of the twentieth century. We also used data from the World Values Survey and from legal background to provide support for that hypothesis.

Thus, although the present model constitutes a simple extension of existing frameworks – introducing family bargaining on education decision – it allows us, despite its simplicity, to fit the observed education patterns in a much better way than models based on either the child or the parent’s education decision. That refinement of existing models takes a full advantage of the natural source of heterogeneity in OLG models: the age of agents coexisting at a given period. Using that source of heterogeneity in a collective decision model is fruitful when trying to replicate the dynamics of education as a function of life expectancy. The introduction of family bargaining on education allows us also to qualify the standard Ben-Porath effect. In our model, it remains true that a rise in life expectancy raises education thanks to a horizon effect, but that the impact of that effect depends on how bargaining power is distributed within the family.

Finally, it should be stressed that our simple model captures only some mechanisms explaining education patterns. Other mechanisms, such as access to credit, supply of education, endogenous fertility (i.e. ‘quantity’ versus ‘quality’ of children concerns) are not discussed here. We also ignored intracohort heterogeneity, for instance regarding lifestyles. Those extensions of our framework would further enrich our study of the determinants of education demand. Note, however, that the heterogeneity in life horizons between parents and children would still be at work in those contexts. Hence our requalification of the Ben-Porath effect would remain relevant in those extended models.

Acknowledgements
The authors are grateful to Daniel Cohen, Marion Davin, Colin Green, Volker Meier and to two anonymous referees for their comments on this paper.

Notes
1. The intuition for using period life expectancy for cohorts born in 1940–1980 goes as follows. Agents, when choosing their education ex ante, have access to the currently available demographic statistics, which are period lifetables. Cohort lifetables are only known ex post, that is, once the entire cohort is dead. Hence cohort lifetables are not really relevant for the Ben-Porath effect, which is about individuals making schooling decisions ex ante.
2. This is taken into account in models with endogenous mortality, such as Cervellati and Sunde (2005) and de la Croix and Licandro (2013).


4. In our model, education is time consuming. The education investment takes the form of a fraction of time that the parent and the child must spend together. This specification is general: one can think of a father watching his child, but also of a professor teaching a student, or of a senior worker helping a junior worker.

5. For simplicity, we assume here a fixed fertility, equal to the replacement level. On the determinants of education under endogenous fertility, see Barro and Becker (1989), Ehrlich and Lui (1991), Soares (2005) and de la Croix and Licandro (2013).

6. For simplicity, we assume that there is no savings decision. Although widespread in the literature (Blackburn and Cipriani 2002), that assumption is strong and requires some justification. In our context, the justification is merely that our model focuses on a part of the lifecycle where agents are either studying or working, without considering the retirement age. The absence of an inactivity period in our economy makes the savings issue less important.

7. Note that, for the sake of simplicity, we restrict ourselves here to a model with only two possible lengths of life. See Ponthiere (2011) for a study of asymptotic age structures in an OLG model with endogenous fertility.

8. Many studies show that the level of human capital has an important impact on longevity, through knowledge on prevention and treatments of diseases. See Easterlin (1999).

9. The parent’s motive to educate his child is here purely egoistic. This explains why the utility derived from the child’s education is conditional on survival of the parent, unlike in altruistic models, where this would be unconditional on the parent’s survival.

10. That objective function already includes the time constraints for the different periods. In period 1, the parent’s own education reduces his consumption, while in period 2 the parent’s consumption is reduced by the education he provides to his child, et+1.

11. The concavity of the time-horizon effect is standard in the literature. See for instance the increasing and concave effect of life expectancy on the propensity to save in Chakraborty (2004), which also relies on log linear utility.

12. The myopic anticipation assumption Et+1(πt+3) = πt+2 amounts to assume that human capital enters here the survival probability as an externality. Indeed, we have πt+3 = π(ht+2) = π(h(ht+1, et+1)), so that the child affects, by his education choice, his future survival chances. Assuming Et+1(πt+3) = πt+2 amounts to assume that children do not internalize the impact of their education choice on their life expectancy.

13. Given that the parent and his child bargain and discuss about the future, it makes sense to assume that they share the same beliefs on longevity prospects. In particular, it is plausible to suppose that the parent, thanks to his older age, is better informed about the survival process (explaining that his foresight is perfect), and shares that information with his child.

14. Note that, if the child faced, because of some cohort-specific circumstances (e.g. wars), a shorter expected life horizon than his parent, then the differential in remaining lifespans might go in the other direction. Throughout this paper, we rule out those cases, and focus on the general situations where children have a higher remaining life expectancy than parents.

15. Those dynamics is actually quite close to Azariadis and Drazen’s (1990) threshold effects in human capital accumulation.

16. Figure 2 shows the transition function G(ht) and the 45\(^{\circ}\) line.

17. See, for instance Lam and Schoeni (1993), who show that women with higher education and income have more bargaining power in their household, which implies higher schooling for their children.

18. The distribution of bargaining power may also vary in a nonmonotonic way. Yet, for simplicity, we will only consider a monotonic relation between the distribution of the bargaining power and human capital.
19. In that proposition, the reference to a ‘poverty trap’ means that there exists a threshold in human capital such that if the initial condition is below this threshold, then the human capital stock shrinks over time. On the contrary, an ‘area of perpetual growth’ refers to the existence of a threshold in human capital such that, if the initial condition is above that threshold, then the human capital stock will grow indefinitely in the future.

20. The considered countries are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Sweden, the UK and the USA.

21. We use period life expectancy, since cohort life expectancy requires the death of all cohort members before being known. That assumption is too strong in our context, since education decisions take place quite early during the lifecycle. We thus consider that period life expectancy statistics are more adequate than cohort life expectancy statistics.

22. The reason why we proceed in that way goes as follows. We would like to investigate here whether the model where the parent decides alone can better replicate the data than other models, including the one where only the child decides on his education. Given that the $\gamma$ does not affect education when the child decides alone, but that $\gamma$ strongly affects the chosen education when the parent decides alone, it makes sense to select the level of $\gamma$ that best fits the education data when only the parent decides.


24. Sources: for European countries, we use data from the study *Voter Turnout in Western Europe*, published by the International Institute for Democracy and Electoral Assistance, 2004. For Australia, we use data from the Australian Electoral Commission website ([http://www.aec.gov.au/About_AEC/25/theme1-voting-history.htm](http://www.aec.gov.au/About_AEC/25/theme1-voting-history.htm)). For Canada, we use the Amendment to Canada Election Act (1970). For New Zealand, we use the website [New Zealand history online](http://www.nzhistory.net.nz/politics/milestones). For the USA, we use the Amendment XXVI to the US Constitution and Voting Rights Act Amendments (1971).

25. On this, see Ponthiere (2010).

References


Appendix

A.1. Proof of Proposition 3

The transition function is

\[ h_{t+1} = A \left( \frac{e \alpha \gamma \pi(h_t) + \alpha(1 - \varepsilon)(1 + \pi(h_t))}{1 + e \alpha \gamma \pi(h_t) + \alpha(1 - \varepsilon)(1 + \pi(h_t))} \right)^\alpha h_t \equiv G(h_t). \]

Note first that, given \( A > 0 \), we have \( G(h_t) > 0 \). We can also see that \( G(0) = 0 \).

Moreover, we have, under \( \lim_{h_t \to +\infty} \pi(h_t) = \overline{\pi} < 1 \)

\[ \lim_{h_t \to \infty} \frac{G(h_t)}{h_t} = A \left( \frac{e \alpha \gamma \overline{\pi} + \alpha(1 - \varepsilon)(1 + \overline{\pi})}{1 + e \alpha \gamma \overline{\pi} + \alpha(1 - \varepsilon)(1 + \overline{\pi})} \right) \alpha \equiv 1. \]
The transition function is
\[ G'(h_t) = A \left( \frac{\varepsilon \alpha \gamma \pi(h_t) + \alpha(1 - \varepsilon)(1 + \pi(h_t))}{1 + \varepsilon \alpha \gamma \pi(h_t) + \alpha(1 - \varepsilon)(1 + \pi(h_t))} \right)^\alpha + \alpha Ah_t \left[ \frac{\varepsilon \alpha \gamma \pi(h_t) + \alpha(1 - \varepsilon)\pi(h_t)}{1 + \varepsilon \alpha \gamma \pi(h_t) + \alpha(1 - \varepsilon)(1 + \pi(h_t))} \right] \times \left( \frac{\varepsilon \alpha \gamma \pi(h_t) + \alpha(1 - \varepsilon)(1 + \pi(h_t))}{1 + \varepsilon \alpha \gamma \pi(h_t) + \alpha(1 - \varepsilon)(1 + \pi(h_t))} \right)^{\alpha-1}. \]

so that \( G'(h_t) > 0 \).

Note that, under \( \lim_{h_t \to 0} \pi(h_t) = \bar{\pi} > 0 \), \( 0 < \lim_{h_t \to 0} \pi'(h_t) < \infty \), we have
\[ \lim_{h_t \to 0} \frac{G(h_t)}{h_t} = A \left( \frac{\varepsilon \alpha \gamma \bar{\pi} + \alpha(1 - \varepsilon)(1 + \bar{\pi})}{1 + \varepsilon \alpha \gamma \bar{\pi} + \alpha(1 - \varepsilon)(1 + \bar{\pi})} \right)^\alpha \leq 1. \]

Finally, note that \((G(h_t)/h_t)\) is increasing in \( h_t \), as it is increasing in \( \pi(t) \) and \( \pi'(h_t) > 0 \). Hence \( \lim_{h_t \to 0}(G(h_t)/h_t) < \lim_{h_t \to \infty}(G(h_t)/h_t) \) and therefore there are three cases.

In case 1, \( G(h_t) \) is below the 45° line in the neighbourhood of 0 (as \( G(0) = 0 \) and \( \lim_{h_t \to 0}(G(h_t)/h_t) < 1 \), and remains below the 45° line when \( h_t \) tends to infinity (as \( \lim_{h_t \to \infty}(G(h_t)/h_t) < 1 \)). Thus, given that \( G \) is continuous and \( G'(h_t) > 0 \), \( G(h_t) \) always remain below the 45° line, so that no positive steady-state exists. \( h^* = 0 \) is the unique stationary equilibrium: any economy with \( h_0 > 0 \) will converge towards \( h^* = 0 \). That equilibrium is stable.

In case 2, \( G(h_t) \) is also below the 45° line in the neighbourhood of 0 (as \( G(0) = 0 \) and \( \lim_{h_t \to 0}(G(h_t)/h_t) < 1 \)), but lies above the 45° line when \( h_t \) tends to infinity (as \( \lim_{h_t \to \infty}(G(h_t)/h_t) > 1 \)). As a consequence, given the continuity of \( G(h_t) \), it must be the case that \( G(h_t) \) crosses the 45° line at least once at a positive \( h = h^{**} \). As \( G \) is increasing in \( h \), \( h^{**} \) is the unique positive steady-state equilibrium. It is unstable, so that if \( h_0 < h^{**} \), the economy tends to \( h^* = 0 \), while if \( h_0 > h^{**} \), the economy exhibits perpetual growth.

In case 3, \( G(h_t) \) is above the 45° line in the neighbourhood of 0 (as \( G(0) = 0 \) and \( \lim_{h_t \to 0}(G(h_t)/h_t) > 1 \)), and remains above the 45° line when \( h_t \) tends to infinity (as \( \lim_{h_t \to \infty}(G(h_t)/h_t) > 1 \)). Thus, given that \( G \) is continuous and \( G'(h_t) > 0 \), \( G(h_t) \) always remain above the 45° line so that no positive steady-state exists: the economy exhibits eternal growth.

A.2. Proof of Proposition 4

The transition function is
\[ h_{t+1} = A \left( \frac{\varepsilon(h_t)\alpha \gamma \pi(h_t) + \alpha(1 - \varepsilon(h_t))(1 + \pi(h_t))}{1 + \varepsilon(h_t)\alpha \gamma \pi(h_t) + \alpha(1 - \varepsilon(h_t))(1 + \pi(h_t))} \right)^\alpha h_t = H(h_t). \]

Note first that, given \( A > 0 \), we have \( H(h_t) > 0 \). We can also see that \( H(0) = 0 \).
Moreover, we have, under \( \lim_{h_t \to +\infty} \varepsilon(h_t) = \bar{\varepsilon} \) and \( \lim_{h_t \to +\infty} \pi(h_t) = \bar{\pi} < 1 \)
\[ \lim_{h_t \to \infty} \frac{H(h_t)}{h_t} = A \left( \frac{\bar{\varepsilon} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{\varepsilon})(1 + \bar{\pi})}{1 + \bar{\varepsilon} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{\varepsilon})(1 + \bar{\pi})} \right)^\alpha \leq 1. \]
The derivative $H'(h_i)$ is

$$H'(h_i) = A \left( \frac{e(h_i)\alpha\gamma\pi(h_i) + \alpha(1 - e(h_i))(1 + \pi(h_i))}{1 + e(h_i)\alpha\gamma\pi(h_i) + \alpha(1 - e(h_i))(1 + \pi(h_i))} \right)^\alpha$$

$$+ \alpha A h_i \frac{\varepsilon(h_i)[\alpha\gamma\pi(h_i) - \alpha - \alpha\pi(h_i)] + \pi'(h_i)[\varepsilon(h_i)\alpha\gamma + \alpha - \alpha\varepsilon(h_i)]}{[1 + \alpha\gamma\varepsilon(h_i)\pi(h_i) + \alpha(1 - e(h_i))(1 + \pi(h_i))]^2}$$

$$\times \left( \frac{\varepsilon(h_i)\alpha\gamma\pi(h_i) + \alpha(1 - e(h_i))(1 + \pi(h_i))}{1 + e(h_i)\alpha\gamma\pi(h_i) + \alpha(1 - e(h_i))(1 + \pi(h_i))} \right)^{\alpha - 1}.$$ 

Hence $H'(h_i) \leq 0$, so that the transition function can cross the $45^\circ$ line more than once. Note that, under $\lim_{h_i \to 0} H(h_i) = \pi > 0$, $0 < \lim_{h_i \to 0} \pi(h_i) < \infty$ and $\varepsilon(0) = 0$, we have

$$\lim_{h_i \to 0} \frac{H(h_i)}{h_i} = A \left( \frac{\varepsilon(h_i)\alpha\gamma\pi(h_i) + \alpha(1 - e)(1 + \pi)}{1 + \varepsilon(h_i)\alpha\gamma\pi(h_i) + \alpha(1 - e)(1 + \pi)} \right)^\alpha \leq 1.$$

Finally, the derivative of $(H(h_i))/h_i$ is

$$\frac{\varepsilon(h_i)[\alpha\gamma\pi(h_i) - \alpha - \alpha\pi(h_i)] + \pi'(h_i)[\varepsilon(h_i)\alpha\gamma + \alpha - \alpha\varepsilon(h_i)]}{[1 + \alpha\gamma\varepsilon(h_i)\pi(h_i) + \alpha(1 - e(h_i))(1 + \pi(h_i))]^2} \leq 0,$$

since $\varepsilon(h_i) < 0$ but $\gamma \leq (1 + \pi(h_i))/\pi(h_i))$.

Hence $\lim_{h_i \to 0} (H(h_i)/h_i) \geq \lim_{h_i \to 0} (H(h_i)/h_i)$, so that we have the following four cases.

In case 1, $H(h_i)$ is below the $45^\circ$ line in the neighbourhood of 0 (as $H(0) = 0$ and $\lim_{h_i \to 0} H(h_i)/h_i = A(\varepsilon(\alpha\gamma\pi + \alpha(1 - e)(1 + \pi)))/(1 + \varepsilon(\alpha\gamma + \alpha(1 - e)(1 + \pi))) < 1$), and still lies below the $45^\circ$ line when $h_i$ tends to infinity (as $\lim_{h_i \to \infty} (H(h_i)/h_i) = A(\varepsilon(\alpha\gamma\pi + \alpha(1 - e)(1 + \pi)))/(1 + \varepsilon(\alpha\gamma + \alpha(1 - e)(1 + \pi))) < 1$). Thus the transition function may never cross the $45^\circ$ line, but can cross it an even number of times since $H$ is nonmonotonic. If $H(h_i)$ crosses the $45^\circ$ line from above at a certain point $h^*$, if $|H'(h_i)| < 1$, then $h^*$ is a stable equilibrium. Moreover, 0 is a stable equilibrium and there exists therefore a poverty trap. Since $\lim_{h_i \to \infty} (H(h_i)/h_i) = A(\varepsilon(\alpha\gamma\pi + \alpha(1 - e)(1 + \pi)))/(1 + \varepsilon(\alpha\gamma + \alpha(1 - e)(1 + \pi))) < 1$, there exists no zone of perpetual growth.

In case 2, $H(h_i)$ is above the $45^\circ$ line in the neighbourhood of 0 (as $H(0) = 0$ and $\lim_{h_i \to 0} (H(h_i)/h_i) = A(\varepsilon(\alpha\gamma\pi + \alpha(1 - e)(1 + \pi)))/(1 + \varepsilon(\alpha\gamma + \alpha(1 - e)(1 + \pi))) > 1$), but lies below the $45^\circ$ line when $h_i$ tends to infinity (as $\lim_{h_i \to \infty} (H(h_i)/h_i) = A(\varepsilon(\alpha\gamma\pi + \alpha(1 - e)(1 + \pi)))/(1 + \varepsilon(\alpha\gamma + \alpha(1 - e)(1 + \pi))) > 1$). Thus the transition function crosses the $45^\circ$ line at least once, but can cross it more than once since $H$ is nonmonotonic. Since it begins from above the $45^\circ$ line and ends below, it crosses the $45^\circ$ line an odd number of times. If $H(h_i)$ crosses the $45^\circ$ line from above at a certain point $h^*$, if $|H'(h_i)| < 1$, then $h^*$ is a stable equilibrium. Zero is an unstable equilibrium, and there may exist no stable equilibrium. Since $\lim_{h_i \to \infty} (H(h_i)/h_i) = A(\varepsilon(\alpha\gamma\pi + \alpha(1 - e)(1 + \pi)))/(1 + \varepsilon(\alpha\gamma + \alpha(1 - e)(1 + \pi))) > 1$, there exists no zone of perpetual growth.

In case 3, $H(h_i)$ is below the $45^\circ$ line in the neighbourhood of 0 (as $H(0) = 0$ and $\lim_{h_i \to 0} (H(h_i)/h_i) = A(\varepsilon(\alpha\gamma\pi + \alpha(1 - e)(1 + \pi)))/(1 + \varepsilon(\alpha\gamma + \alpha(1 - e)(1 + \pi))) < 1$), but lies above the $45^\circ$ line when $h_i$ tends to infinity (as $\lim_{h_i \to \infty} (H(h_i)/h_i) = A(\varepsilon(\alpha\gamma\pi + \alpha(1 - e)(1 + \pi)))/(1 + \varepsilon(\alpha\gamma + \alpha(1 - e)(1 + \pi))) > 1$). Thus the transition function crosses the $45^\circ$ line at least once, but can cross it more than once since $H$ is nonmonotonic. Since it begins from below the $45^\circ$ line and ends above, it crosses the $45^\circ$ line an odd number of times. If $H(h_i)$ crosses the $45^\circ$ line from above at a certain point $h^*$, if $|H'(h_i)| < 1$, then $h^*$ is a stable equilibrium. Moreover, $H(h_i)$ crosses the $45^\circ$ line from below at least once, so that the first positive equilibrium is unstable. Hence, zero is a stable equilibrium and there exists therefore a poverty trap. Since $\lim_{h_i \to \infty} (H(h_i)/h_i) = A(\varepsilon(\alpha\gamma\pi + \alpha(1 - e)(1 + \pi)))/(1 + \varepsilon(\alpha\gamma + \alpha(1 - e)(1 + \pi))) > 1$, there exists a zone of perpetual growth.
A.3. Proof of Proposition 5

The transition function is

$$h_{t+1} = A \left( \frac{e(h_t)\alpha\gamma\pi(h_t) + \alpha(1 - e(h_t))(1 + \pi(h_t))}{1 + e(h_t)\alpha\gamma\pi(h_t) + \alpha(1 - e(h_t))(1 + \pi(h_t))} \right)^a h_t = J(h_t).$$

Note first that, given $A > 0$, we have $J(h_t) > 0$. We can also see that $J(0) = 0$.

Moreover, we have, under $l_{h_t^*, \infty} \pi(h_t) = \tilde{\pi} < 1$ and $l_{h_t^*, \infty} e(h_t) = e$

$$\lim_{h_t \to \infty} \frac{J(h_t)}{h_t} = A \left( \frac{e \alpha \gamma \tilde{\pi} + \alpha(1 - e)(1 + \tilde{\pi})}{1 + e \alpha \gamma \tilde{\pi} + \alpha(1 - e)(1 + \tilde{\pi})} \right)^a \leq 1.$$

The derivative $J'(h_t)$ is

$$J'(h_t) = A \left( \frac{e(h_t)\alpha\gamma\pi(h_t) + \alpha(1 - e(h_t))(1 + \pi(h_t))}{1 + e(h_t)\alpha\gamma\pi(h_t) + \alpha(1 - e(h_t))(1 + \pi(h_t))} \right)^a \alpha$$

$$+ \alpha Ah_t \left( \frac{e(h_t)\alpha\pi(h_t) - e - \alpha \pi(h_t)}{1 + e(h_t)\alpha\pi(h_t) + \alpha(1 - e(h_t))(1 + \pi(h_t))} \right) \left( \frac{e(h_t)\alpha\gamma\pi(h_t) + \alpha(1 - e(h_t))(1 + \pi(h_t))}{1 + e(h_t)\alpha\gamma\pi(h_t) + \alpha(1 - e(h_t))(1 + \pi(h_t))} \right)^{a-1}\right),$$

so that $J'(h_t) \leq 0$. Hence the transition function can cross the $45^\circ$ line more than once.

Note that, under $l_{h_t^*, \to} \pi(h_t) = \tilde{\pi} > 0$, $0 < l_{h_t^*, \to} \pi(h_t) < \infty$ and $e(0) = e$, we have

$$\lim_{h_t \to 0} \frac{J(h_t)}{h_t} = A \left( \frac{\tilde{\epsilon} \alpha \gamma \tilde{\pi} + \alpha(1 - \tilde{e})(1 + \tilde{\pi})}{1 + \tilde{\epsilon} \alpha \gamma \tilde{\pi} + \alpha(1 - \tilde{e})(1 + \tilde{\pi})} \right)^a \leq 1.$$

Finally, the derivative of $J(h_t)/(h_t)$ is

$$e'(h_t)\alpha\gamma\pi(h_t) - e - \alpha \pi(h_t) + \pi(h_t)e(h_t)\alpha\gamma + \alpha - \alpha e(h_t)$$

$$\left( 1 + e(h_t)\alpha\gamma\pi(h_t) + \alpha(1 - e(h_t))(1 + \pi(h_t)) \right)^2 \leq 0.$$

Hence $\lim_{h_t \to 0}(J(h_t)/(h_t)) \leq \lim_{h_t \to \infty}(J(h_t)/(h_t))$, so that we have the following four cases, similar to Proposition 4.

In case 4, $J(h_t)$ is below the $45^\circ$ line in the neighbourhood of $0$ (as $J(0) = 0$ and $A((\tilde{\epsilon} \alpha \gamma \tilde{\pi} + \alpha(1 - \tilde{e})(1 + \tilde{\pi}))(1 + \tilde{\epsilon} \alpha \gamma \tilde{\pi} + \alpha(1 - \tilde{e})(1 + \tilde{\pi}))^a < 1$), and still lies below the $45^\circ$ line when $h_t$ tends to infinity (as $\lim_{h_t \to \infty}(J(h_t)/(h_t)) = A((\tilde{\epsilon} \alpha \gamma \tilde{\pi} + \alpha(1 - \tilde{e})(1 + \tilde{\pi}))(1 + \tilde{\epsilon} \alpha \gamma \tilde{\pi} + \alpha(1 - \tilde{e})(1 + \tilde{\pi}))^a < 1$). Thus the transition function may never cross the $45^\circ$ line, but can cross it an even number of times since $J$ is nonmonotonic. If $J(h_t)$ crosses the $45^\circ$ line from above at a certain point $h^*$, if $|J'(h_t)| < 1$, then $h^*$ is a stable
equilibrium. Moreover, zero is a stable equilibrium and there exists therefore a poverty trap. Since 
\( \lim_{h_i \to \infty} (J(h_i)/h_i) = A((\bar{e} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{e})(1 + \bar{\pi}))/((1 + \bar{e} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{e})(1 + \bar{\pi})))^a < 1 \), there exists no zone of perpetual growth.

In case 2, \( J(h_i) \) is above the 45° line in the neighbourhood of 0 (as \( J(0) = 0 \) and 
\( \lim_{h_i \to 0} (J(h_i)/h_i) = A((\bar{e} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{e})(1 + \bar{\pi}))/((1 + \bar{e} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{e})(1 + \bar{\pi})))^a > 1 \), but lies below the 45° line when \( h_i \) tends to infinity (as 
\( A((\bar{e} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{e})(1 + \bar{\pi}))/((1 + \bar{e} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{e})(1 + \bar{\pi})))^a < 1 \)). Thus the transition function crosses the 45° line at least once, but can cross it more than once since \( J \) is nonmonotonic. Since it begins from above the 45° line and ends below, it crosses the 45° line an odd number of times. If \( J(h_i) \) crosses the 45° line from above at a certain point \( h^* \), if \( |J'(h_i)| < 1 \), then \( h^* \) is a stable equilibrium. Zero is an unstable equilibrium, and there may exist no stable equilibrium.

Since \( A((\bar{e} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{e})(1 + \bar{\pi}))/((1 + \bar{e} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{e})(1 + \bar{\pi})))^a < 1 \), there exists no zone of perpetual growth.

In case 3, \( J(h_i) \) is below the 45° line in the neighbourhood of 0 (as \( J(0) = 0 \) and 
\( \lim_{h_i \to 0} (J(h_i)/h_i) = A((\bar{e} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{e})(1 + \bar{\pi}))/((1 + \bar{e} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{e})(1 + \bar{\pi})))^a > 1 \), but lies above the 45° line when \( h_i \) tends to infinity (as 
\( \lim_{h_i \to \infty} (J(h_i)/h_i) = A((\bar{e} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{e})(1 + \bar{\pi}))/((1 + \bar{e} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{e})(1 + \bar{\pi})))^a > 1 \)). Thus the transition function crosses the 45° line at least once, but can cross it more than once since \( J \) is nonmonotonic. Since it begins from below the 45° line and ends above, it crosses the 45° line an odd number of times. If \( J(h_i) \) crosses the 45° line from above at a certain point \( h^* \), if \( |J'(h_i)| < 1 \), then \( h^* \) is a stable equilibrium. Moreover, \( J(h_i) \) crosses the 45° line from below at least once, so that the first positive equilibrium is unstable. Hence zero is a stable equilibrium and there exists therefore a poverty trap. Since \( \lim_{h_i \to \infty} (J(h_i)/h_i) = A((\bar{e} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{e})(1 + \bar{\pi}))/((1 + \bar{e} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{e})(1 + \bar{\pi})))^a > 1 \), there exists a zone of perpetual growth.

In case 4, \( J(h_i) \) is above the 45° line in the neighbourhood of 0 (as \( J(0) = 0 \) and 
\( \lim_{h_i \to 0} (J(h_i)/h_i) = A((\bar{e} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{e})(1 + \bar{\pi}))/((1 + \bar{e} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{e})(1 + \bar{\pi})))^a > 1 \), and still lies above the 45° line when \( h_i \) tends to infinity (as 
\( \lim_{h_i \to \infty} (J(h_i)/h_i) = A((\bar{e} \alpha \gamma \bar{\pi} + (1 - \bar{e})(1 + \bar{\pi}))/((1 + \bar{e} \alpha \gamma \bar{\pi} + (1 - \bar{e})(1 + \bar{\pi})))^a > 1 \)). Thus the transition function may never cross the 45° line, but can cross it an even number of times since \( J \) is nonmonotonic. If \( J(h_i) \) crosses the 45° line from above at a certain point \( h^* \), if \( |J'(h_i)| < 1 \), then \( h^* \) is a stable equilibrium. Zero is an unstable equilibrium and there may exist no stable equilibrium. Since \( \lim_{h_i \to \infty} (J(h_i)/h_i) = A((\bar{e} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{e})(1 + \bar{\pi}))/((1 + \bar{e} \alpha \gamma \bar{\pi} + \alpha(1 - \bar{e})(1 + \bar{\pi})))^a > 1 \), there exists a zone of perpetual growth.